6 – Structural element behaviour

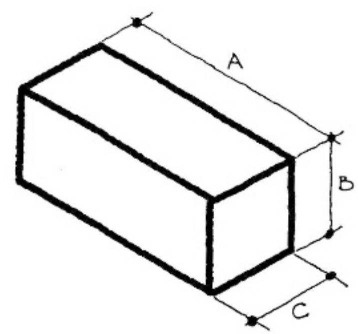
(Rev 02/2020)

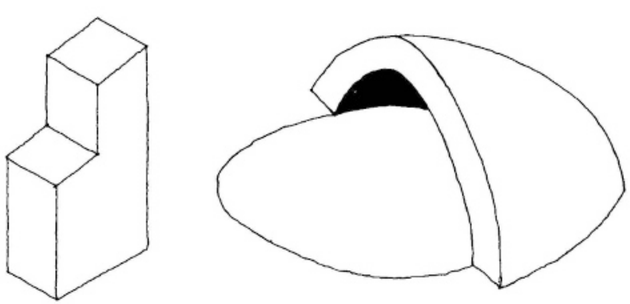
The following slides are only a basis for the development of the lessons and, therefore, do not replace the recommended texts

The choice of overall structural form for any structure is the most important task of the structural designer. However, before the behaviour of whole structural forms can be understood, the behaviour of very simple structures must be clear. To do this it is helpful to think of structures being assemblies of elements.

In this lesson the behaviour of structural elements which are part of a load path is examined in detail. The understanding that is obtained from this examination makes it clear how parts of structures resist the internal forces. It also gives guidance on the best shape for any particular part of the load path. Structural elements are considered to be one-dimensional, two-dimensional or three-dimensional.

The basic element can be thought of as a rectangular block, with sides of dimensions A, B and C.



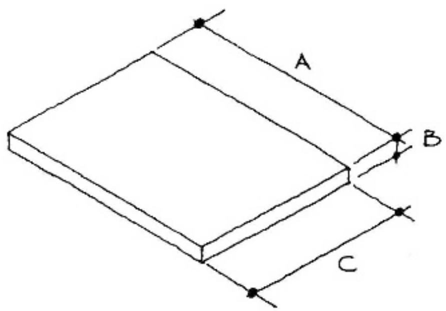


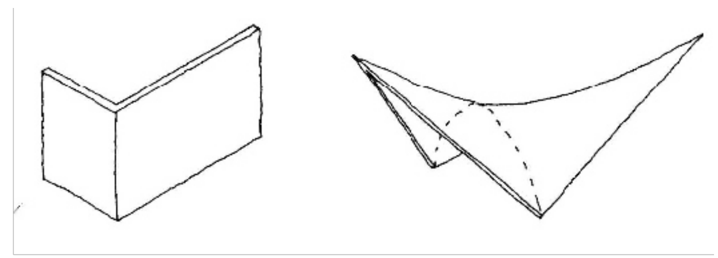
If the three dimensions are approximately equal, then such an element is a **three-dimensional element**.

Examples of three-dimensional elements are rare in modern building structures but often occur in older buildings, such as wall buttresses or thick stone domes.

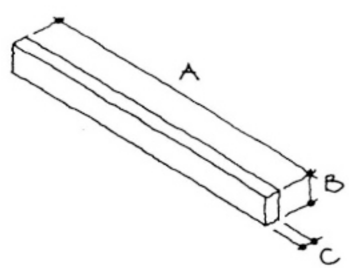
Structural elements 2/3

If one of the dimensions, say dimension B, is small compared with dimensions A and C, then the element is a **two-dimensional element**. Many parts of modern building structures are two-dimensional elements such as floor slabs, walls or shell roofs.



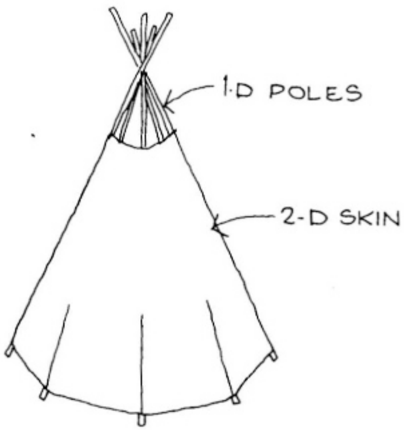


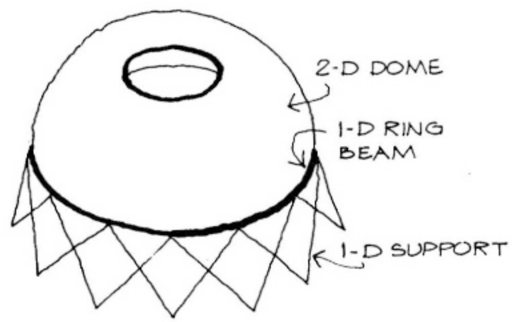
If two of the dimensions of the basic element, say B and C, are small compared with dimension A, then the element is a **one-dimensional element**. One-dimensional elements are used abundantly in nearly all buildings; examples are beams, bars, cables and columns.

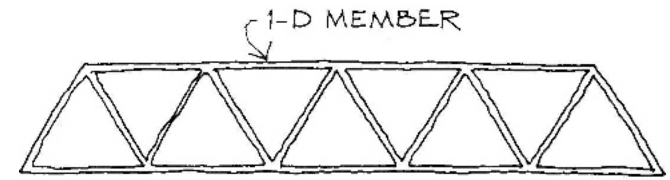


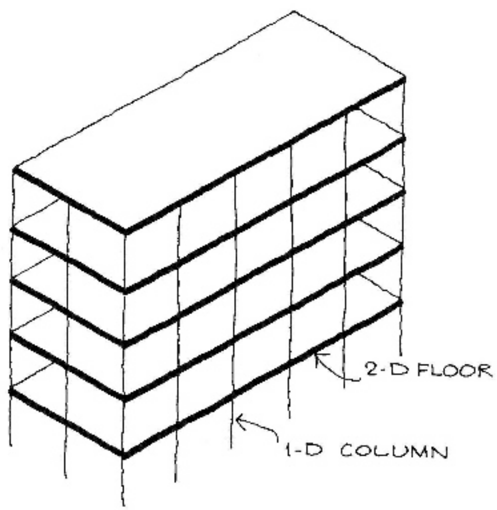
Structural elements

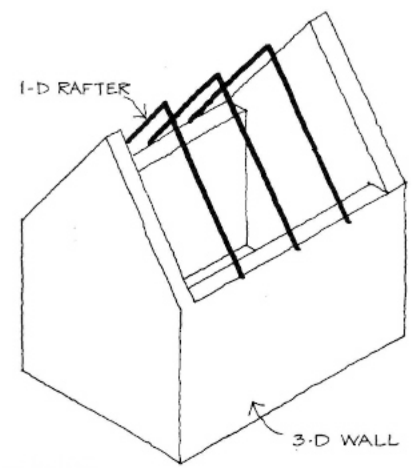
Using the concept of elements, structures can be conceived as assemblies of elements. Examples can be found both in traditional and modern structures.











Nowadays structures are usually conceived and designed as assemblies of structural elements. This means the structural behaviour can be identified by considering the behaviour of each **structural element** in each load path.

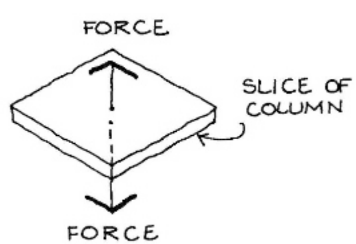
Concept of stress and stress distribution

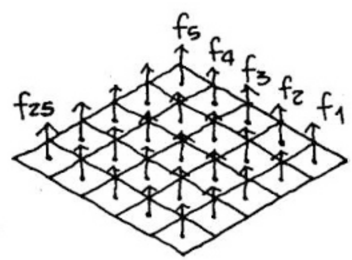
For any structure, all the elements that make up each load path must be strong enough to resist the internal structural actions caused by the loads. This means detailed information is required about the structural behaviour of structural elements.

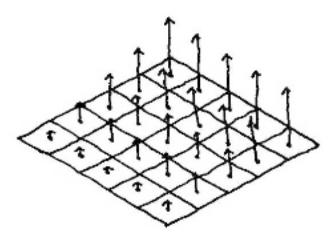
To obtain this knowledge a new concept has to be introduced, which is the concept of stress and the related idea of stress distribution. Stress is force per unit area. Stress distribution describes how the sizes of stresses vary from unit area to unit area.

To understand these concept, it is helpful to look at the slice of a column subjected to an axial force. Suppose the cross-section of the slice is gridded into squares of the same size (unit squares), then a small force can be attached to each square. If the slice is divided into 25 unit squares, so the axial force is divided into 25 forces per unit area, $\mathbf{f}_1$ to $\mathbf{f}_{25}$.

For equilibrium, the numerical sum of the sizes of the twenty-five forces per unit area must equal the total force on the cross-section. So far there is no requirement that any of the forces per unit area, $\mathbf{f}_1$ to $\mathbf{f}_{25}$, are numerically equal. The last figure a possible pattern of variation for $\mathbf{f}_1$ to $\mathbf{f}_{25}$. The length of each force arrow indicates the size of the force in each unit square and it can be seen that these forces (stresses) vary in a pattern.

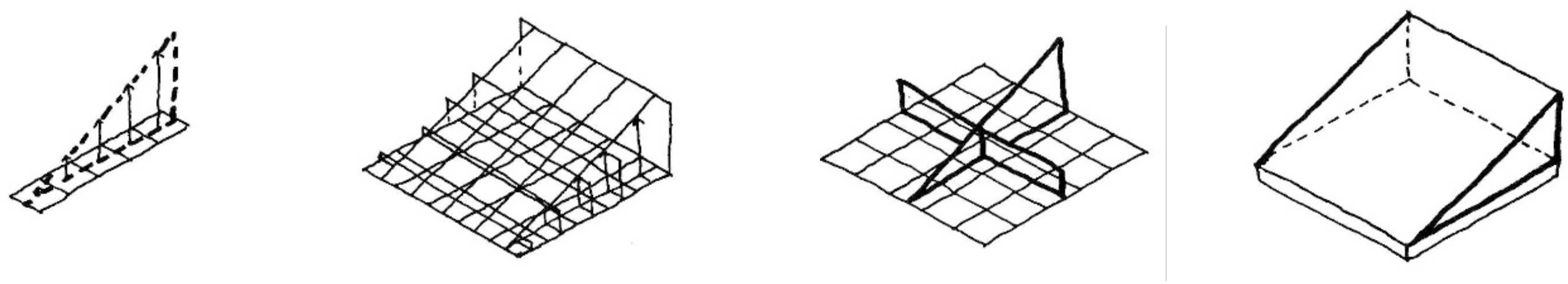






Concept of stress and stress distribution

Suppose, for clarity, just one strip of squares is drawn, and the tops of the arrows are joined with a line. As can be seen the resulting shape is a triangle, so along this strip there is a **triangular stress distribution**. The subsequent figure displays the stresses varying in both directions across the cross-section so the tops of all the arrows can be joined with lines as shown. These lines show triangular shapes in one direction and rectangular ones in the other direction. It usual to simplify these diagrams of stress distribution by just drawing the outline along the edges.

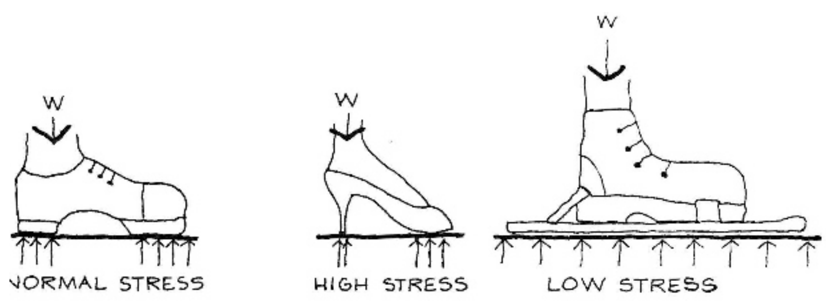


In general there is no restriction on how stresses vary across any cross-section of any structure, except that the sum of the stresses must be equal to the internal force acting at the section and that the internal force acts at the **center of gravity** of the stresses.

The concept of stress requires checks along each load path to ensure that structural elements are strong enough to resist the internal forces caused by the loads. This is checked by making sure that the stresses in the structural elements that are in the load path are less than the maximum allowable stress allowed for the structural material being used. The structure must not be over-stressed.

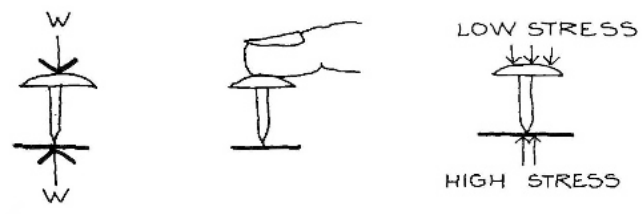
The main point about the size of stresses is that they can be varied without altering the force. If some part of a load path is over-stressed, it may be convenient to alter the structure locally, by altering the geometry, so that the stress is reduced below the maximum stress that is allowed.

This idea is used widely in everyday life: stresses are increased or reduced purposely. For example, the weight of a person may be constant, but the stress under the person's feet will vary with the area of the shoe in contact with the ground. This variation may have good or bad effects. Next figure shows three types of shoes, normal shoes, high heeled shoes and snowshoes.



Normal shoes cause normal stresses and can be used on surfaces that can resist these stresses. High heeled shoes, as they provide a much smaller area to carry the same weight, cause higher stresses under the shoe, particularly under the heel. Where stresses must be kept low, for walking on snow for instance, the area under the foot must be increased. This is why snowshoes prevent people from sinking into snow.

The idea of deliberately altering stress sizes by geometric methods is also widely used in many other objects used by humans. For example drawing pins are provided with large heads, to allow comfortable stresses on the thumb, and pointed shafts to cause high stresses under the point. The point stress is so high that the base surface fails and allows the drawing pin to be driven in.



The important idea is that for equilibrium, the force on the head must equal the force on the point, but the stresses vary. The stresses are varied by changing the geometry (of the drawing pin). The provision of handles, points, sharp edges and wide shoulder straps are all familiar devices for deliberately raising or lowering stresses.

The task of the structural designer task is to provide a structure that will carry the prescribed loads down the load path with **comfortable stresses everywhere**. Depending on the material used, the size of the comfortable stress will vary. For instance, as steel is stronger than timber, the allowable (comfortable) stress for steel is higher than for timber. So, in a general, timber structures will have larger structural elements than steel structures if they are to carry the same load.

Stress distribution in structural elements

Simplifying assumptions about the nature of material:

- ***The material is isotropic:*** the mechanical behaviour of the material is the same in all directions.
- ***The material is linear elastic:*** after deforming under load, the material returns to the same state when the load is removed.

Simplifying assumptions about the geometry of the structure:

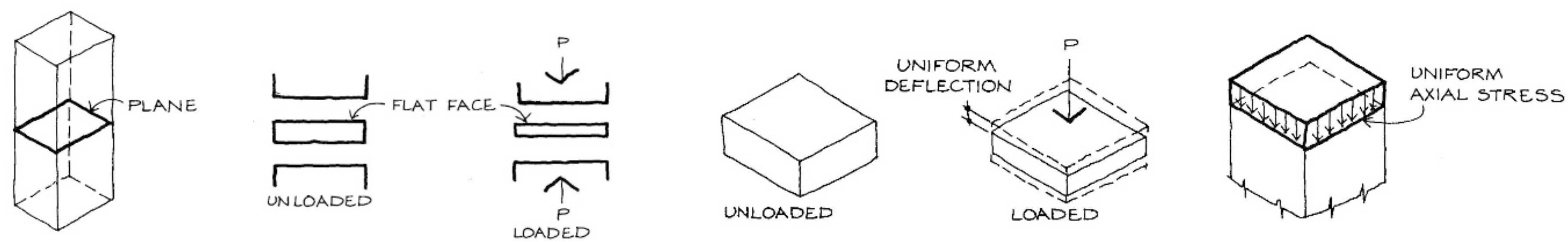
- ***The deflections of the loaded structure are small:*** using the shape of the unloaded structure for calculations to determine structural behaviour will not lead to any significant errors.
- ***Plane sections remain plane:*** certain parts of a structure that are flat before loading are still flat after loading.

Axial stresses

Axial stresses

An axially loaded structural element has axial internal forces and these cause axial stresses across the element. The assumption that plane cross-sections remain plane leads to a very simple stress distribution.

Because flat faces of the unloaded slice are flat after the slice is loaded, all parts of the column cross-section deflect by the same amount. Because the deflections are equal over the cross-section, the stress (load/unit area) is the same everywhere, in other words there is a uniform stress distribution.



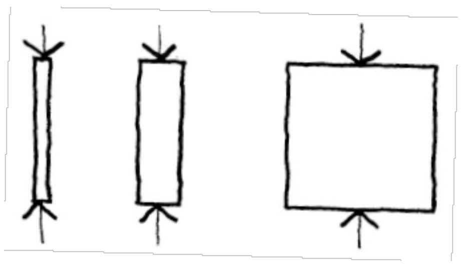
The uniform stress over the cross-section of an axially loaded column gives a very simple relationship between force and stress and this is:

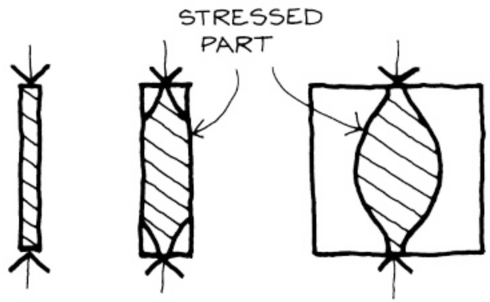
$$\sigma = \frac{P}{A}$$

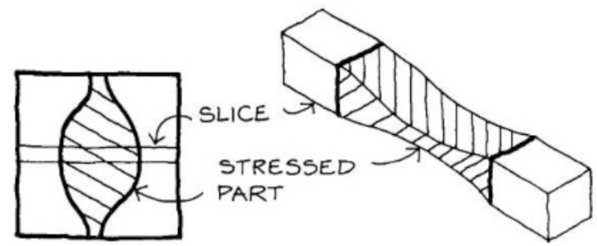
This means that for a given force, the size of the stress can be varied by increasing or decreasing the cross-sectional area of the column.

Axial stresses 2/2

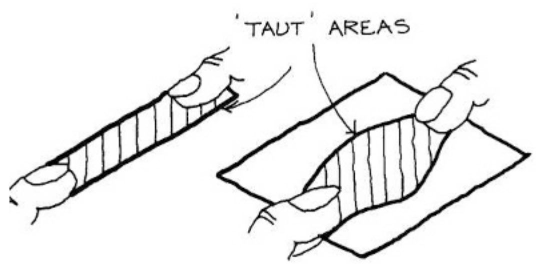
The plane cross-section assumption implies that the whole cross-section is equally stressed. However for wide columns it does not seem reasonable to assume that the whole cross-section is equally stressed or even that the whole cross-section is stressed. Very approximately the stress spreads out at about 60º. This means that for the widest column, plane sections **do not remain plane**.



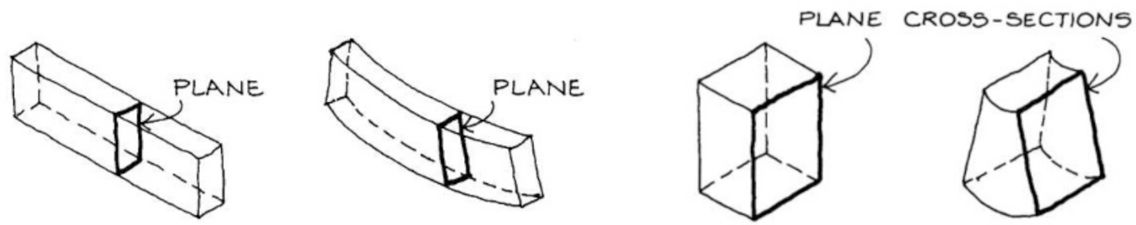


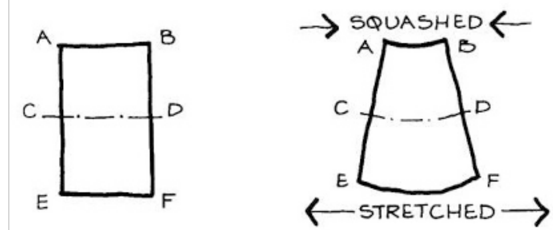


From a technical point of view, this gives guidance as to whether structural elements are one, two or three-dimensional. Where simple stress distributions are reasonable, then elements can be regarded as one-dimensional, but where the stress distributions are no longer simple, the elements are two or three-dimensional. The widest column has to be regarded as a two-dimensional element. This effect can be seen by pulling on progressively wider and wider sheets of paper. The stressed part of the paper will become taut; the unstressed areas will remain floppy.

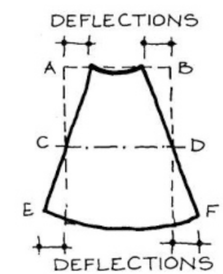


Where parts of the load path are beams and slabs, the elements will have internal bending forces (moments). The top and bottom surfaces of these elements become curved; however, plane cross-sections remain plane. Again looking at unloaded and loaded slices, the plane sections can be identified.

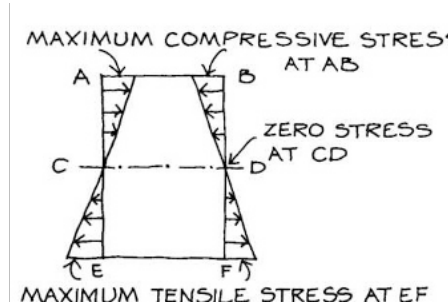




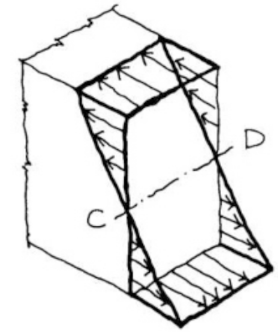
When the slice is bent by an internal bending moment, AB is squashed, EF is stretched, and CD remains the same length. Because the cross-sections remain plane, the amount each part of the slice is squashed or stretched varies directly with the distance it is from CD.



As the structural material is linear elastic, the force is directly proportional to the deflection, so the maximum compression is at AB and the amount of compression decreases constantly from AB to CD. Similarly the maximum tension is at EF and the tension decreases constantly from EF to CD. The maximum compression is at the top of the slice and the maximum tension is at the bottom of the slice and at CD, the change point, there is neither compression nor tension.

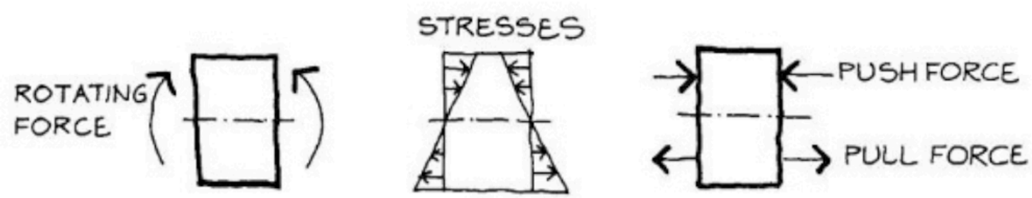


Using this information a stress distribution diagram can be drawn for the side view of the slice.



If it is also assumed that these stresses that are caused by an internal bending moment do not vary across the beam, a three-dimensional diagram of the stress distribution of the compressive and tensile stresses can be drawn.

This stress distribution, which is based on the assumptions of **linear elasticity** and **plane sections remaining plane**, is widely used in structural design. It can be viewed as being in two parts, a triangular distribution of compressive stress and a triangular distribution of tensile stress. The two parts of the stress distribution give a new concept which is the **moment as a pair of forces**. Now the bending moment acting on a slice of a beam can be thought of in three alternative ways: as a rotating force, as a double triangular distribution of compressive and tensile stresses, or as a pair of forces.

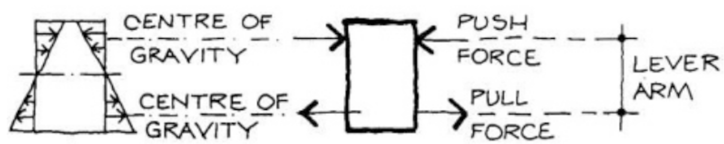


These three alternative views are logically connected by the various concepts that have been introduced:

The **first step** connects the idea of a bending moment in a beam with plane sections remaining plane and the sides of the slice of a beam rotating.

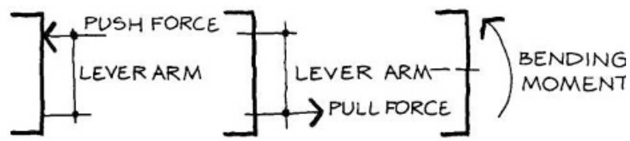
The **second step** connects the deflection of the slice caused by the rotation of the sides to ideas of linear elasticity and stress distribution.

The **third step** uses the idea that if a force causes a stress distribution, then where there is a stress distribution there must be a force. And this force must act at the center of gravity of the stress distribution.



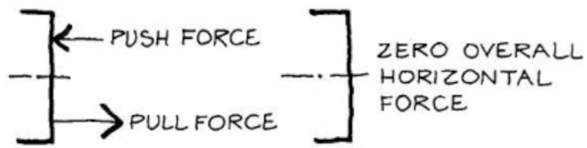
The distance between the push force, which is the effect of the compressive stresses, and the pull force, which is the effect of the tensile stresses, is called the **lever arm**.

Because any moment is a force times a distance, the push and the pull forces give back the bending moment.



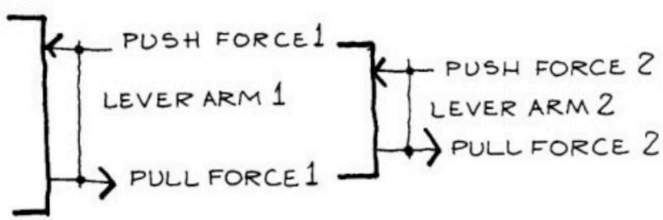
The push and pull forces and the lever arm show how by altering the local geometry of the beam, the size of the stresses can be altered for any bending moment. In fact, two statements can be made about the sizes of the forces from the requirements of equilibrium.

Firstly the forces on each face must be in horizontal equilibrium: the size of the push force must equal the size of the pull force.

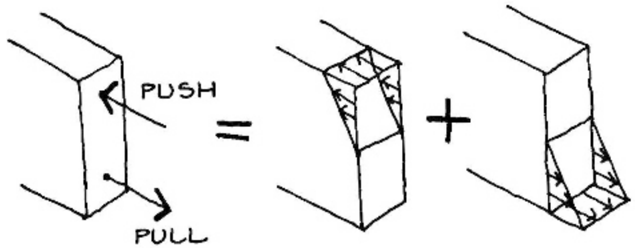


Secondly, from moment equilibrium, the size of the bending moment is equal the size of the push force times the lever arm, or the size of the pull force times the lever arm.

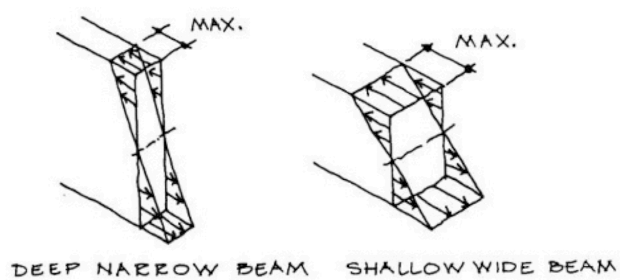
As a consequence, if the lever arm is made bigger, the push (or pull) force is smaller and vice versa.



The relationship between the size of stresses and forces is dependent, for any force, on the area and the shape of the distribution. All the compressive stresses (force per unit area) on the upper part of the beam must add up to the push force, and all the tensile stresses on the lower part of the beam must add up to the pull force.



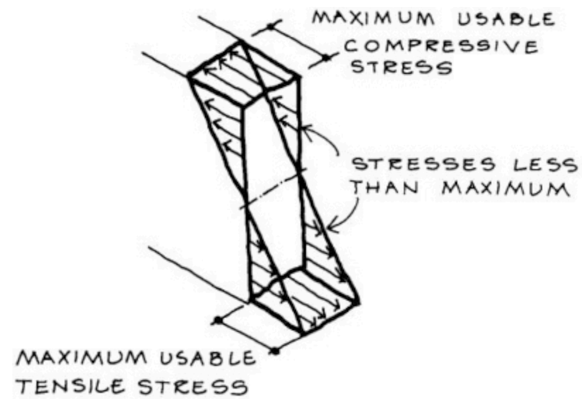
By varying the **depth** and therefore the lever arm, the size of the push and pull forces can be altered, which means the sizes of the stresses can be altered. This is only true if the width of the beam is not altered. The size of the stresses can also be altered by varying the width because this alters the area. Or the size of the stresses can be altered by varying both the depth and the width.



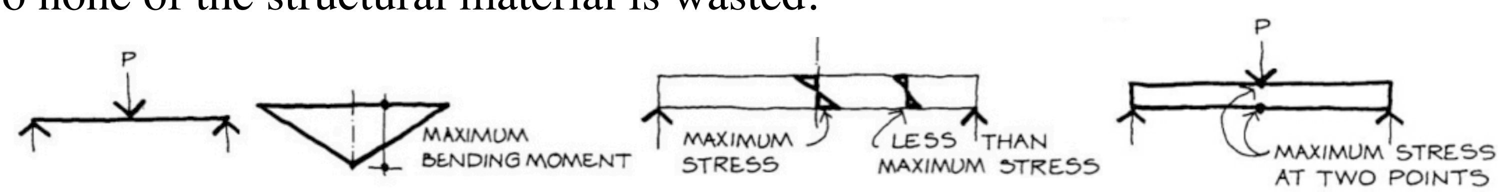
Bending stresses

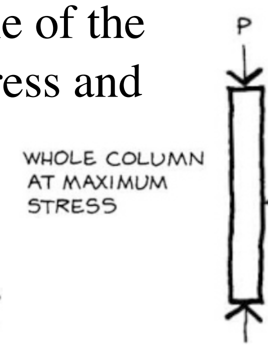
Beams bent by moments have varying stresses that are at a maximum at the top and bottom. As all structural materials have a maximum usable stress, rectangular solid beams are under-stressed except for the top and bottom faces.

It is one ambition of structural design to try and stress all parts of a structure to the maximum usable stress of the structural material being used. In this way no structural material is wasted. This is a sensible ambition provided it does not lead to geometrically complex structures that are expensive to build.



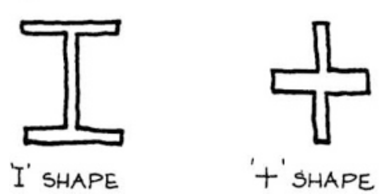
Not only can material be wasted within the depth of a beam, but it can also be wasted along its length. Suppose a beam of constant depth and rectangular cross-section is used to carry a load over a simple span. The size of the bending moment will vary along the length of the beam. For this simple structure, the maximum stress only occurs at one place where the bending moment is at its maximum. Almost the whole of the beam has bending stresses less than the maximum. This contrasts sharply with a column with end loads. Here the whole of the cross-section and the whole of the length of the column can be at maximum stress and so none of the structural material is wasted.

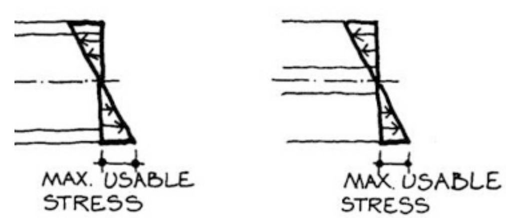




To try and make beams more stress effective, non-rectangular shapes have been developed. As the maximum stresses for bending are at the top and bottom of a beam, more efficient beam sections have more structural material here. These efficient sections are I, channel or box sections.

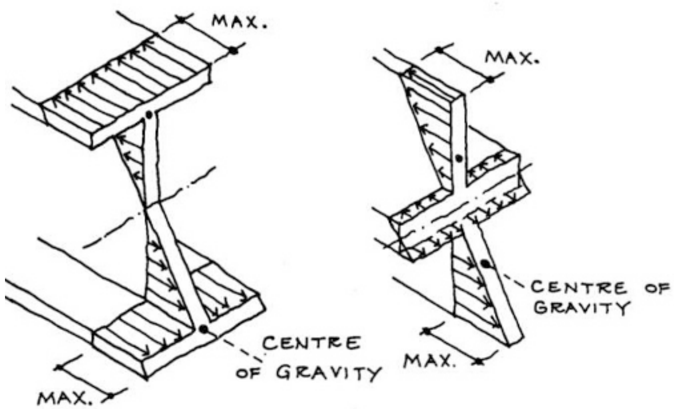
The exact details of these shapes depend on the structural material used, as the methods of construction are different. Furthermore where bending efficiency is not of paramount importance or for a variety of other reasons, such as cost and speed of construction, other shapes such as tubes, rods and angles may be used. To understand why the previous shapes are bending efficient, it is helpful to compare an I shaped section with a + shaped section. Both have the same depth and the same cross-sectional area. As plane sections are assumed to remain plane, and both sections are assumed to have the same maximum usable stress, the side view of the stress distribution is the same for both the sections.

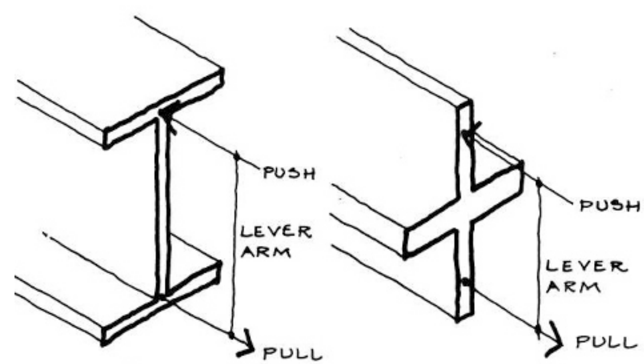




Bending stresses

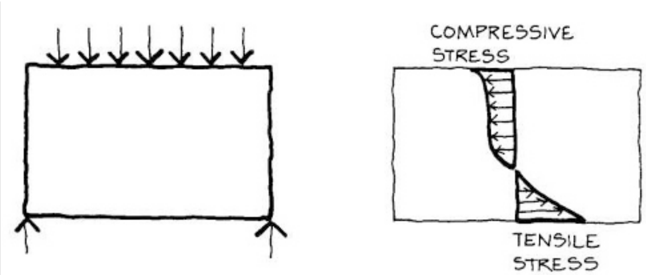
However, if the three-dimensional stress diagrams are drawn, dramatic differences appear. The I section has large areas of the cross-section with stresses near to the maximum, but the + section has large areas with stresses near to zero. This means that the push and pull forces are much bigger for the I section than for the + section. Also the positions of the centers of gravity of these stresses are different and this gives the I section a larger lever arm than the + section.



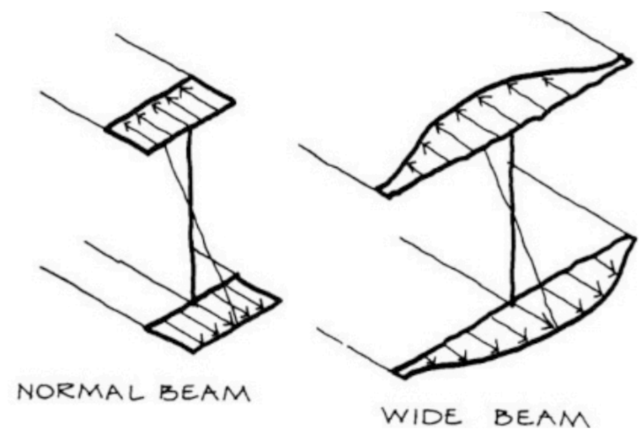


The maximum bending moment a beam can carry is given by the push (or the pull) force with the maximum usable stress, times the lever arm. Both the lever arm and the push force (or pull force) are greater for the I section than for the + section. Because of this, if beams have the same depth, the same cross-sectional area and the same maximum usable stress, then those with I sections will be able to resist larger bending moments than those with + sections. Although I beams, as they are called, can be made from timber or reinforced concrete they are readily made from steel. Due to the bending efficiency of I beams they are verywidely used in steel construction.

Bending stresses



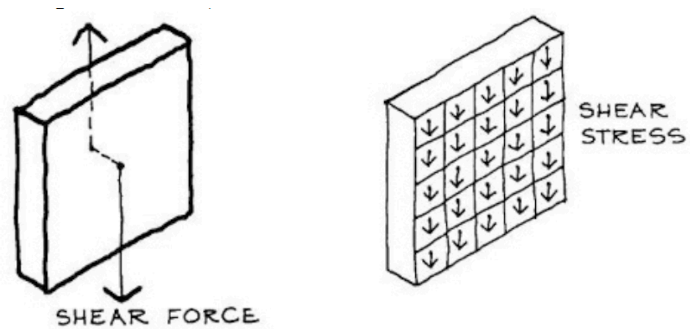
As with columns, the assumption that plane sections remain plane is not always valid. There are two situations where it may not apply. The first is when the span of the beam is not more than about five times the depth of the beam. As the plane sections are no longer plane, the bending stress distribution is not linear. These beams, called **deep beams**, cannot be regarded as one-dimensional elements but are two-dimensional elements.



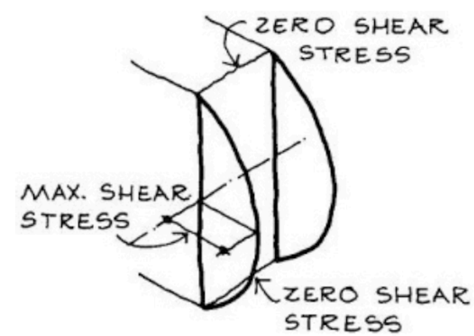
If a beam is not deep but is made from an I or similar section, again plane sections may not remain plane. If the widths of the top and bottom parts of the section are increased, eventually they will become **too wide** and not all the section will be stressed by the bending moment. For a normal I beam the bending stresses are assumed to be constant across the top and bottom parts, but for wide beams only part of the beam may be stressed, and the stress is not constant across the beam. The effect that causes this varying stress across wide beams is called **shear lag** and the part of the beam that is stressed is often called the **effective width**.

Shear stresses

Axial forces and bending moments cause axial stresses, shear forces cause shear stresses. Because shear stresses resist vertical loads, it is to be expected that shear stresses act vertically. On the face of a beam slice, unlike the column, shear stresses (force per unit area) act in line with the face of the slice.

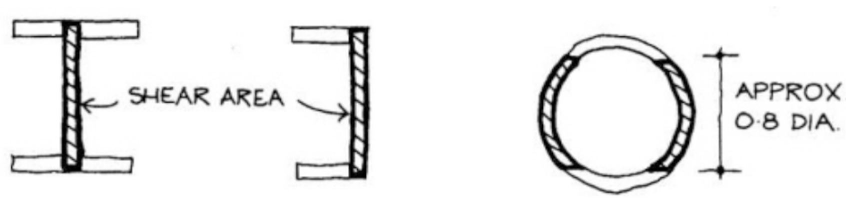


The distribution of shear stress cannot be deduced from the straightforward assumptions that were used for axial and bending stress. At the top and the bottom the shear stress must be zero, otherwise there would be vertical shear stresses on the surface of the beam, which is impossible. Mathematical analysis shows that for a rectangular beam the shear stress distribution has a curved shape, accurately described as parabolic distribution.



The maximum is at the middle of the beam, it is zero at the top and bottom and is constant across the width of the beam.

For typical beam cross-sections, it is usually assumed that the shear stress distribution is constant rather than curved and refers only to the vertical part of the section, A_v, called ***shear area***.



According to this assumption, the average shear stress can be evaluated by the equation

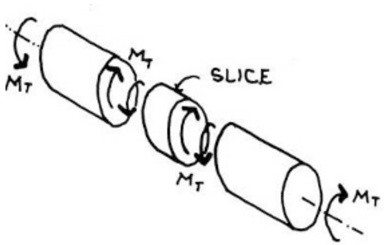
$$\tau = \frac{V}{A_v}$$

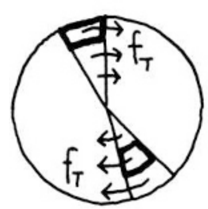
Torsional stresses

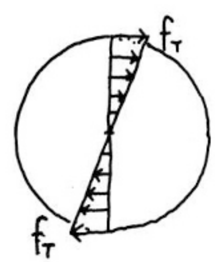
When a one-dimensional element is twisted by torsional moments the internal forces in the element cause torsional stresses. To see what is happening, cut a slice from a circular bar twisted by torsional moments.

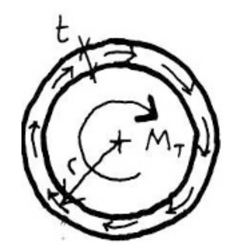
If the circular cross-section is divided into small areas by radial and circumferential lines, then each area has a force in the tangential direction to the circumference. At the center, these tangential forces are zero and it is assumed that they increase linearly towards the outside of the bar.

If a circular tube, with a wall thickness that is small compared with its diameter, is twisted by torsional moments, then it could be considered reasonable that the tangential stresses are constant across the wall of thickness t.



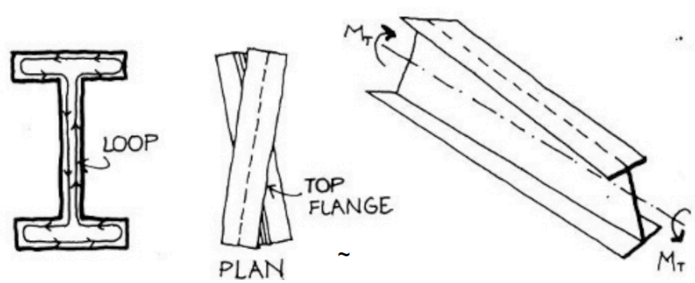






Torsional stresses

Where cross-sections of an element are made up of a number of rectangular elements which do not form any type of tube - I beams and channels for example - they are called ***open sections***. For these sections, the torsional stresses are loops round the whole section and the cross-section warps.

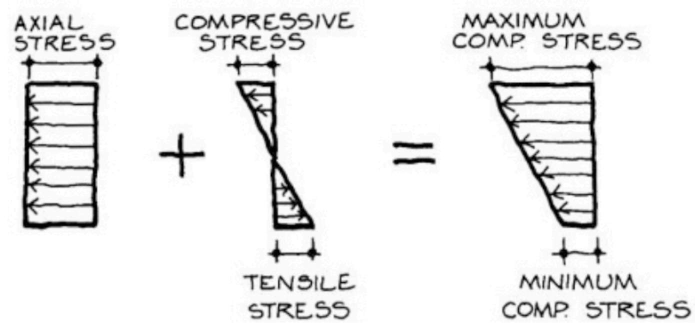


The torsional behaviour just described has two important features that are:

- the torsional stresses form ‘loops’ within the section
- in general a plane cross-section warps when the element is twisted

When a one-dimensional element is part of a load path, it will have internal forces and these may be axial forces, bending moments or shear forces. These internal forces can be thought of as distributions of axial stress, bending stress and shear stress. These stresses can be combined to give the total stress distribution.

This way of combining stresses is relatively straightforward as it just adds stresses that are in the same direction on the face of the slice. Both axial force and bending moment stresses act at right angles to the face of the beam, which is along the beam, so they are combined by adding the stress distributions together.



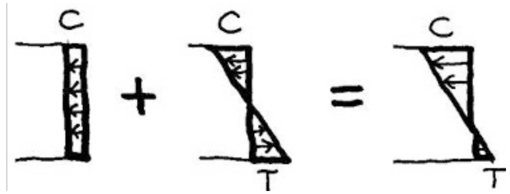
In this figure because the size of the axial compressive stress is bigger than the maximum tensile bending stress, the whole of the cross-section is in compression. The effect of combining the stresses gives a combined maximum stress and a combined minimum stress. The sizes of these stresses are:

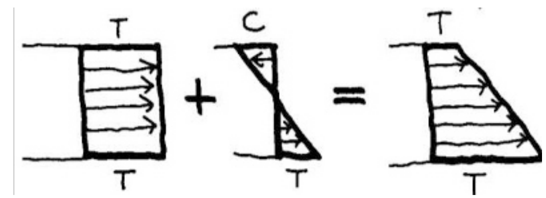
- Maximum stress = Axial compressive stress plus maximum compressive bending stress
- Minimum stress = Axial compressive stress minus maximum tensile bending stress

Because the shear stress is parallel to the face of the slice, it is not added to the axial and bending stresses, but is kept separate.

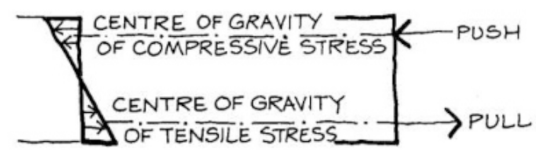
Combined stresses

Depending on the relative sizes of the axial and bending stresses and whether the axial stress is tensile or compressive, the combined stress distribution is all tensile, tensile and compressive, or all compressive.

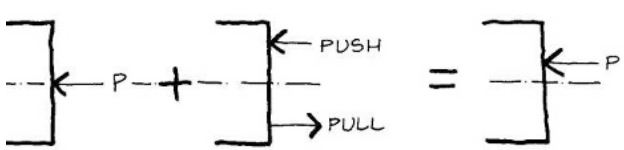


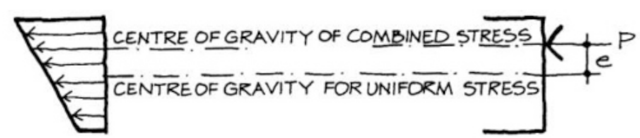


The axial stress distribution can be thought of as an axial force acting at the centre of gravity of the axial stress distribution and the bending stress distribution can be thought of as a pair of ***push-pull forces*** acting at the centres of gravity of the tensile and compressive parts of the bending stress distribution.



Because the push equals the pull, the combined force can only be an axial force. But this force must act at the centre of gravity of the combined stress distribution.



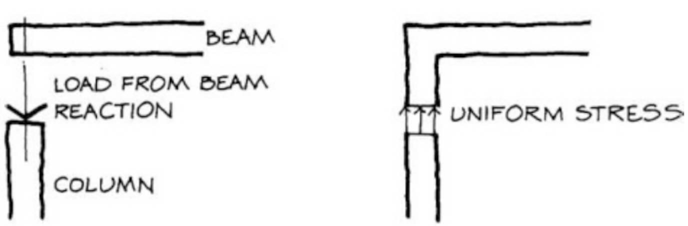


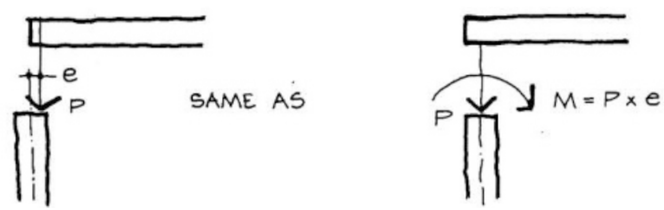
Combined stresses

The effect of the moment is to 'move' the axial force by a distance, e, from the center of gravity for uniform axial stress. This distance e is called **eccentricity**.

Before combining the forces there was an axial force P and a bending moment M. Now there is an axial force P that has 'moved' by a distance, the eccentricity e. What has happened to M, the bending moment? The bending moment still exists but now as P times e.

Suppose a beam is supported on a wall. Then, for the wall only to have uniform axial stress from the reaction of the beam, the beam must be supported exactly at the position of the centre of gravity for this uniform stress distribution. This is usually impossible in any real structure unless very precise precautions are taken. This means the reaction from the beam that the wall is supporting, will be applied to the wall at an eccentricity. So the wall is loaded by an axial load plus a bending moment.

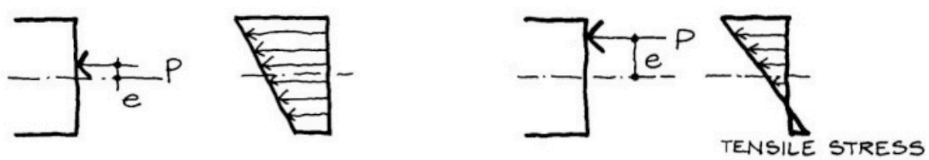




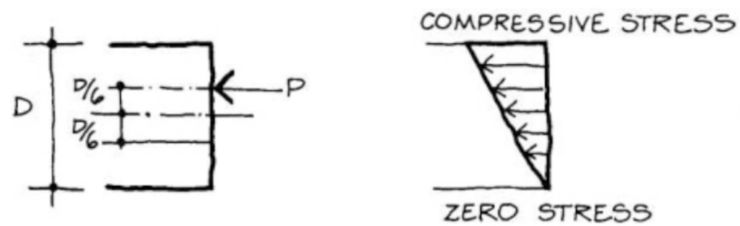
Combined stresses

What happens at the base of a garden wall or any other free-standing wall, when the wind blows? The axial force is caused by the weight of the wall itself and the bending moment is caused by the wind blowing horizontally on the wall.

Here the eccentricity could be of any size depending on the relative sizes of the axial force caused by the weight of the wall and the moment caused by the wind. The left diagram show a cross-section with only compression stresses, whilst the right diagram shows compressive and tensile stresses. This means that the eccentricity is greater in the right diagram.



For rectangular sections the eccentricity must be kept within the **middle third** of the cross-section if there is to be **no tensile stress**.



This has very important consequences for structures made from structural materials such as masonry or mass concrete that cannot carry significant tensile stresses. For structures made from these materials, axial forces must be kept within the central part of the cross-section or the structure will crack or collapse. This is why brick chimneys and walls sometimes blow over in high winds.