MECCANICA delle STRUTTURE AA. 2017-18

Corso P. FUSCHI - prova scritta del 11.04.2018

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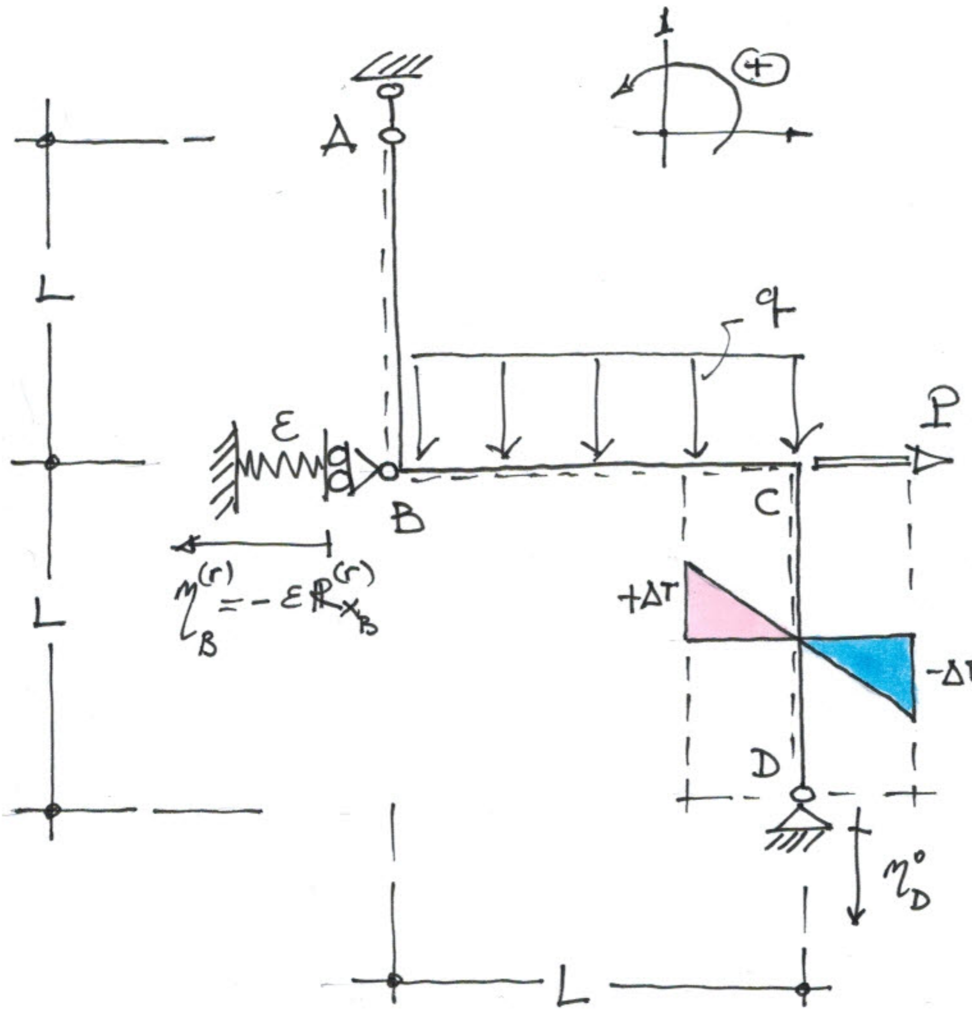
Quesito n. 1 (2CFU)

RISOLVERE LA STRUTTURA IPERSTATICA SEGUENTE TRACCIANDO IL DIAGRAMMA DEI MOMENTI:



Posizioni:

$|P| = qL$

$|\varepsilon| = \dfrac{L^3}{3EI}$

$|\eta_D^{o}| = \dfrac{qL^4}{24EI}$

$\left|\dfrac{\alpha \Delta T}{h}\right| = \dfrac{qL^2}{EI}$

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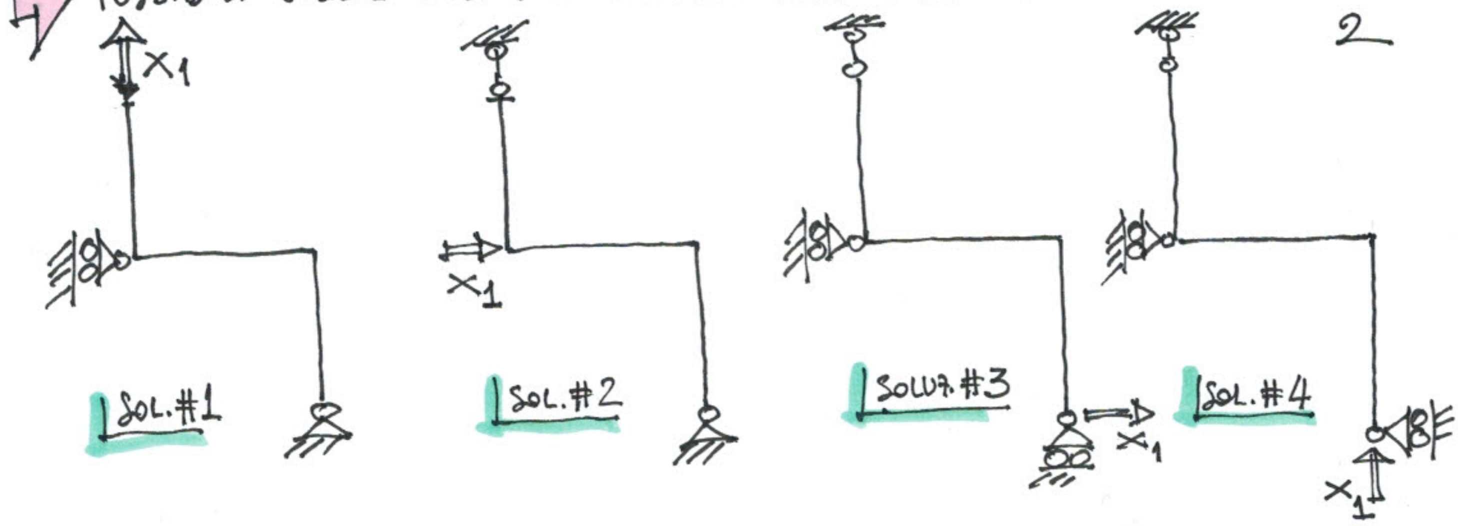
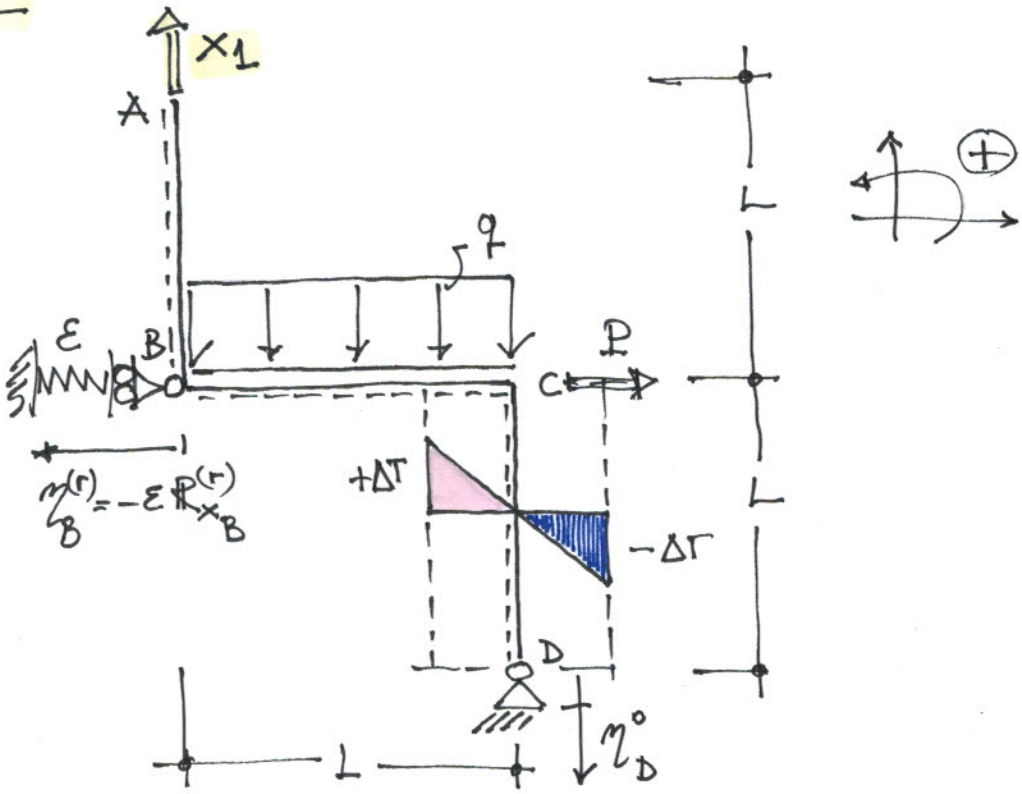
POSSIBILI SCELTE DEL SISTEMA PRINCIPALE ISOSTATICO



SOLUZIONE #1

SISTEMA PRINCIPALE ISOSTATICO



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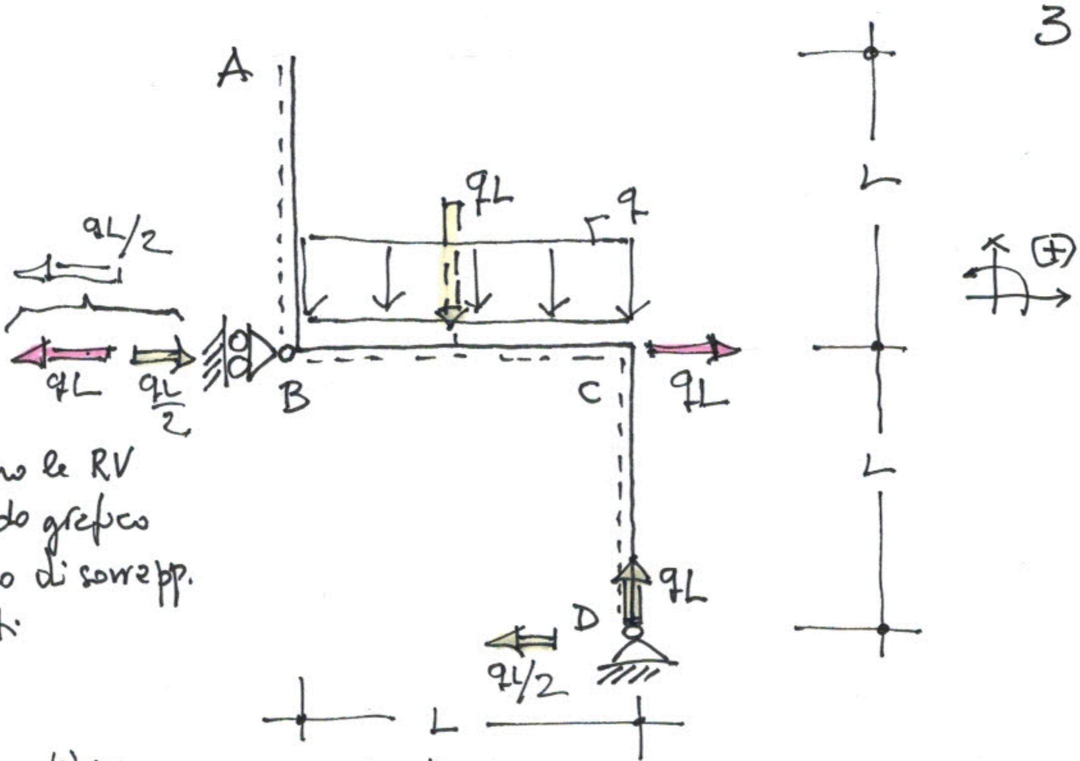
SCHEMA [0]

SOLO CARICHI ESTERNI

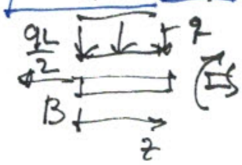


I. Si calcolano le RV con metodo grafico e principio di sovrapp. degli effetti

II. Si calcola $M^{(0)}(z)$ sui singoli tratti. Si ha:

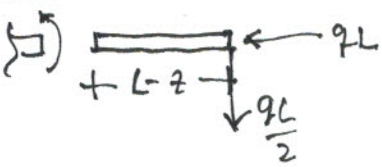
TRATTO AB $0 \le z \le L$ $\boxed{M^{(0)}(z) = \phi}$ scarico

TRATTO BC $0 \le z \le L$



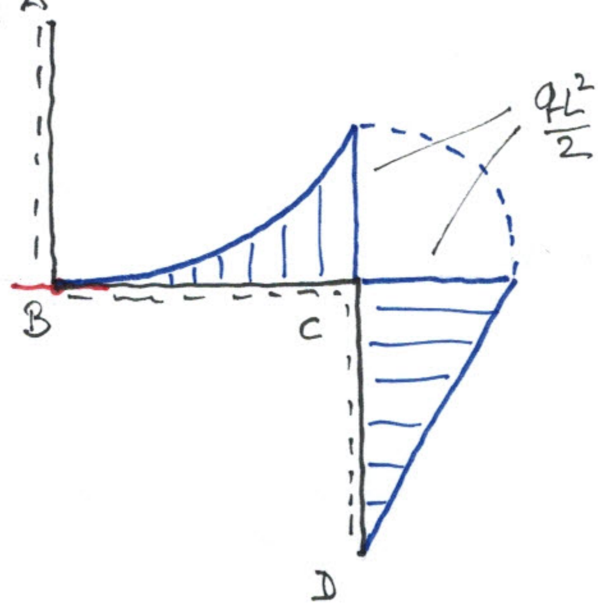
$$\boxed{M^{(0)}(z) = -\frac{qz^2}{2}} \quad \begin{cases} M_B = \phi \\ M_C = -\frac{qL^2}{2} \end{cases}$$

TRATTO CD $0 \le z \le L$



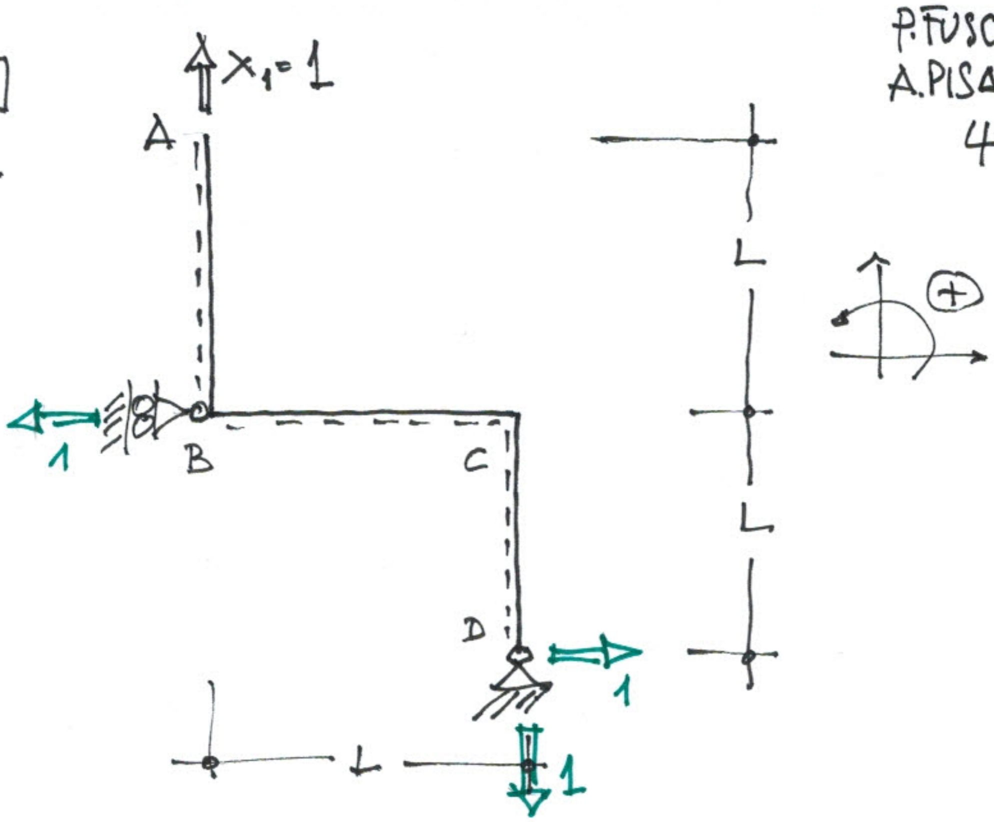
$$\boxed{M^{(0)}(z) = -\frac{qL}{2}(L-z)} \quad \begin{cases} M_C = -\frac{qL^2}{2} \\ M_D = \phi \end{cases}$$

diagramma $M^{(0)}(z)$:



SCHEMA [1]

Solo $X_1 = 1$

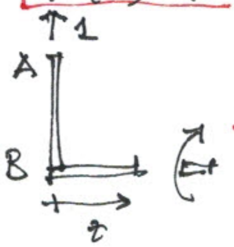


I. Si calcolano le RV con metodo grafico.

II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

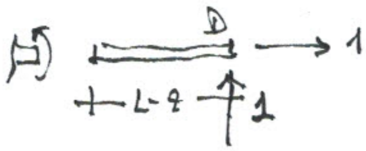
TRATTO AB $\quad 0 \le z \le L \quad \boxed{M^{(1)}(z) = \phi}$

TRATTO BC $\quad 0 \le z \le L$



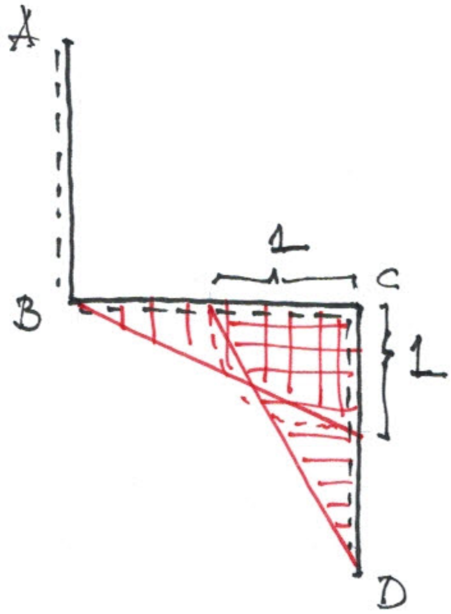
$$\boxed{M^{(1)}(z) = z} \quad \begin{cases} M_B = \phi \\ M_C = L \end{cases}$$

TRATTO CD $\quad 0 \le z \le L$



$$\boxed{M^{(1)}(z) = L - z} \quad \begin{cases} M_C = L \\ M_D = \phi \end{cases}$$

diagramma $M^{(1)}(z)$



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{re} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{re} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} = 1 \cdot \underset{\eta_A^{(r)}}{\phi} + \overbrace{R_{X_B}^{(1)}}^{-1} \cdot \overbrace{\eta_B^{(r)}}^{-\varepsilon R_B^{(r)}} + R_{y_D}^{(1)} \cdot \eta_D^{0} =$$

$$= +\varepsilon \Big[\underbrace{R_{X_B}^{(0)}}_{-\frac{qL}{2}} + X_1 \underbrace{R_{X_B}^{(1)}}_{-1} \Big] - 1 \left(-\eta_D^0 \right) = \varepsilon \left\{ -\frac{qL}{2} - X_1 \right\} + \eta_D^0$$

$$L_{vi} = \int_{Str} \underset{M^{(1)}}{M^{(f)}} \frac{\overset{M^{(0)}+X_1 M^{(1)}}{M^{(r)}}}{EI} \, dStr + \int_{Str} \overset{M^{(1)}}{M^{(f)}} \frac{\alpha \overline{\Delta T}}{h} \, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} \, dStr + \frac{X_1}{EI} \int_{Str} \left[M^{(1)} \right]^2 dStr + \frac{\alpha \overline{\Delta T}}{h} \int_{Str} M^{(1)} dStr =$$

$$= \frac{1}{EI} \left\{ \int_{\overline{BC}} M^{(1)} M^{(0)} dz + \int_{\overline{CD}} M^{(1)} M^{(0)} dz \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \int_{\overline{BC}} \left[M^{(1)} \right]^2 dz + \int_{\overline{CD}} \left[M^{(1)} \right]^2 dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_{\overline{CD}} M^{(1)} dz =$$

$$= \frac{1}{EI} \left\{ \int_0^L z \cdot \left[-\frac{q z^2}{2} \right] dz + \int_0^L (L-z) \cdot -\frac{qL}{2} (L-z) \, dz \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \int_0^L z^2 dz + \int_0^L (L-z)^2 dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_0^L (L-z) dz =$$

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$$= \frac{1}{EI}\left\{\int_0^L -\frac{q}{2}\, z^3 dz + \int_0^L -\frac{qL}{2}\left[L^2+z^2-2Lz\right]dz\right\} +$$

$$+ \frac{X_1}{EI}\left\{\int_0^L z^2 dz + \int_0^L \left[L^2+z^2-2Lz\right]dz\right\} + \frac{\alpha \Delta T}{h}\int_0^L (L-z)\,dz =$$

$$= \frac{1}{EI}\left\{-\frac{q}{2}\left[\frac{z^4}{4}\right]_0^L - \frac{qL^3}{2}\left[z\right]_0^L - \frac{qL}{2}\left[\frac{z^3}{3}\right]_0^L + qL^2\left[\frac{z^2}{2}\right]_0^L\right\} +$$

$$+ \frac{X_1}{EI}\left\{\left[\frac{z^3}{3}\right]_0^L + L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L\right\} + \frac{\alpha \Delta T}{h}\left\{L\left[z\right]_0^L - \left[\frac{z^2}{2}\right]_0^L\right\} =$$

$$= \frac{1}{EI}\left\{-\frac{qL^4}{8} - \cancel{\frac{qL^4}{2}} - \frac{qL^4}{6} + \cancel{\frac{qL^4}{2}}\right\} +$$

$$+ \frac{X_1}{EI}\left\{\frac{L^3}{3} + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right\} + \frac{\alpha \Delta T}{h}\left[L^2 - \frac{L^2}{2}\right] =$$

$$= -\frac{7}{24}\frac{qL^4}{EI} + \frac{2}{3}\frac{L^3}{EI}X_1 + \frac{\alpha \Delta T}{h}\frac{L^2}{2}$$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$\underbrace{\varepsilon}_{\frac{1}{3}\frac{L^3}{EI}}\left\{-\frac{qL}{2} - X_1\right\} + \underbrace{\eta_D^0}_{\frac{1}{24}\frac{qL^4}{EI}} = -\frac{7}{24}\frac{qL^4}{EI} + \frac{2}{3}\frac{L^3}{EI}X_1 + \underbrace{\frac{\alpha \Delta T}{h}}_{\frac{qL^2}{EI}}\cdot\frac{L^2}{2}$$

tenendo conto delle posizioni di pag. 1 si ottiene:

$$\boxed{X_1 = -\frac{qL}{3}}$$

NEGATIVA! ⇒ Verso opposto a quello ipotizzato!

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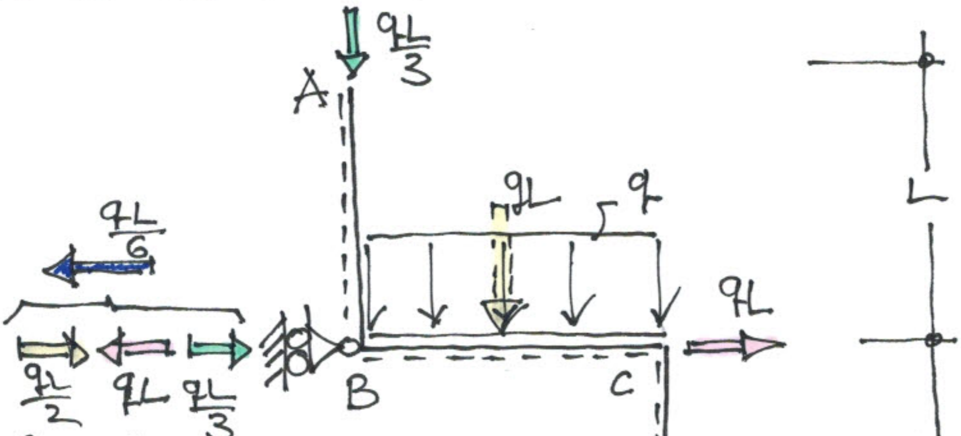
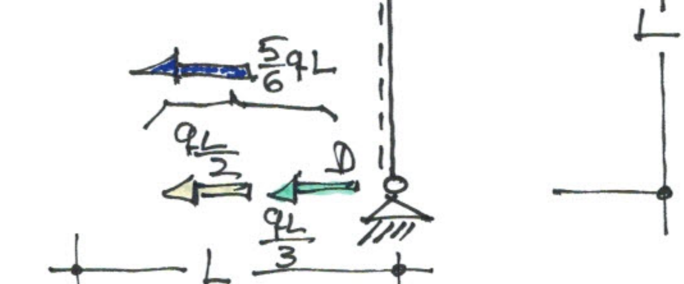
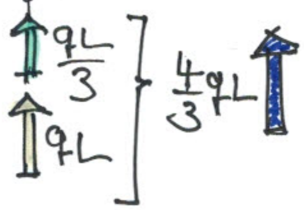
SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO & DIAGRAMMA DEI MOMENTI DELLA STRUTTURA IPERSTATICA ANALIZZATA



I. Si calcolano le RV con metodo grafico e principio di sovrapp. degli effetti.

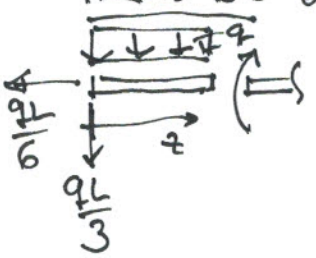




II. Si calcola $M(z)$ sui singoli tratti.

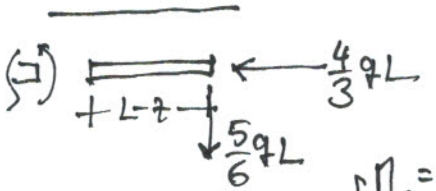
TRATTO AB $0 \le z \le L$ $M^{(r)}(z) = \emptyset$

TRATTO BC $0 \le z \le L$

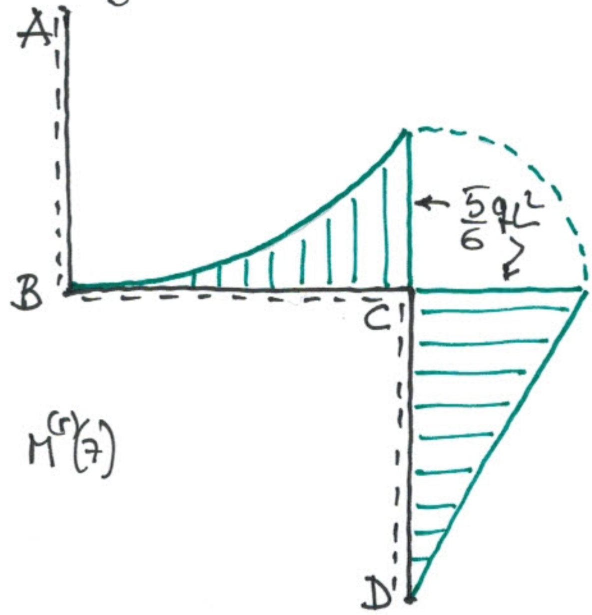


$$M^{(r)}(z) = -\frac{qL}{3}z - \frac{qz^2}{2} \quad \begin{cases} M_B = \emptyset \\ M_C = -\frac{5}{6}qL^2 \end{cases}$$

TRATTO CD $0 \le z \le L$



$$M^{(r)}(z) = -\frac{5}{6}qL(L-z) \quad \begin{cases} M_C = -\frac{5}{6}qL^2 \\ M_D = \emptyset \end{cases}$$



$M^{(r)}(z)$

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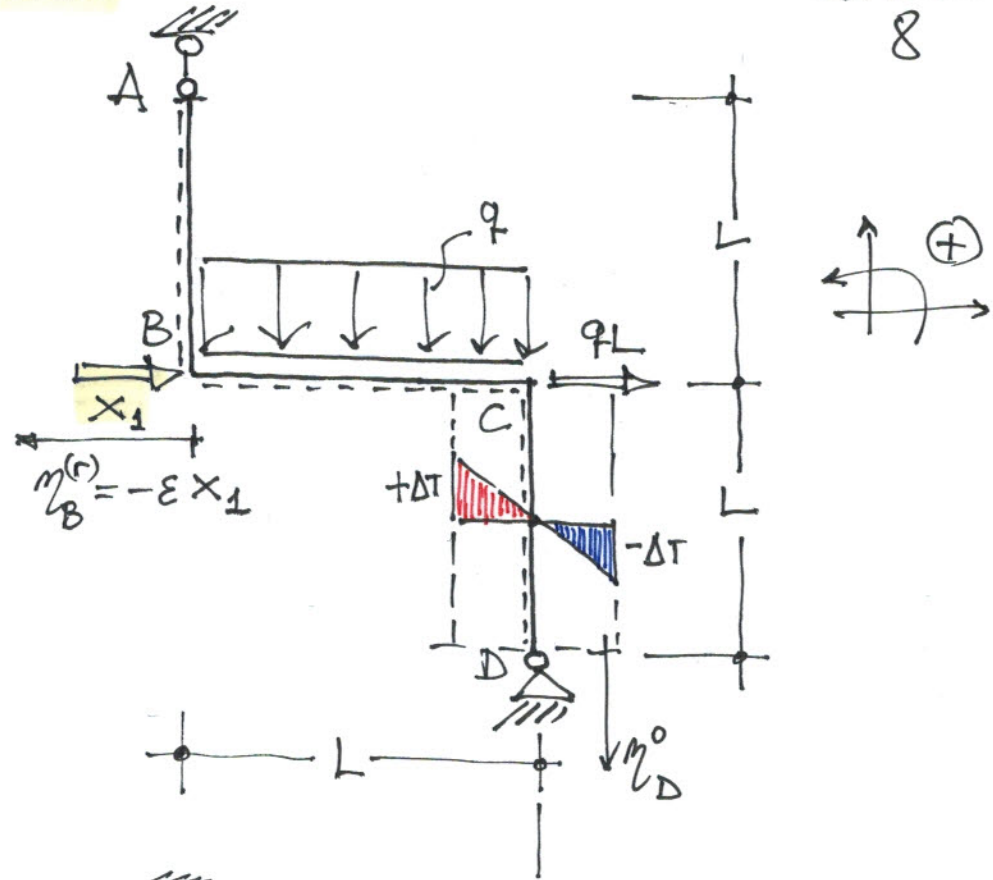
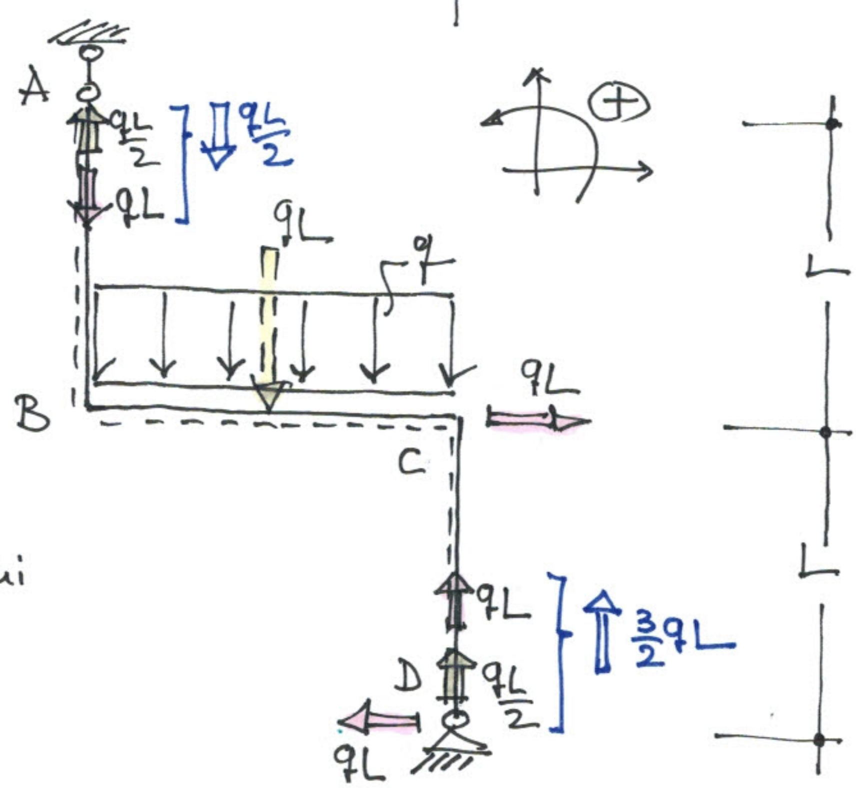
SOLUZIONE # 2

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]
SOLO CARICHI ESTERNI

I. Si calcolano le RV con metodo grafico e principio di sovrapp. degli effetti.



II. Si calcola $M^{(0)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $0 \le z \le L$

$$\boxed{M^{(0)}(z) = \phi}$$

TRATTO BC $0 \le z \le L$

$$\boxed{M^{(0)}(z) = -\frac{qL}{2}z - \frac{qz^2}{2}} \quad \begin{cases} M_B = \phi \\ M_C = -qL^2 \end{cases}$$

TRATTO CD $0 \le z \le L$

$$\boxed{M^{(0)}(z) = -qL(L-z)} \quad \begin{cases} M_C = -qL^2 \\ M_D = \phi \end{cases}$$

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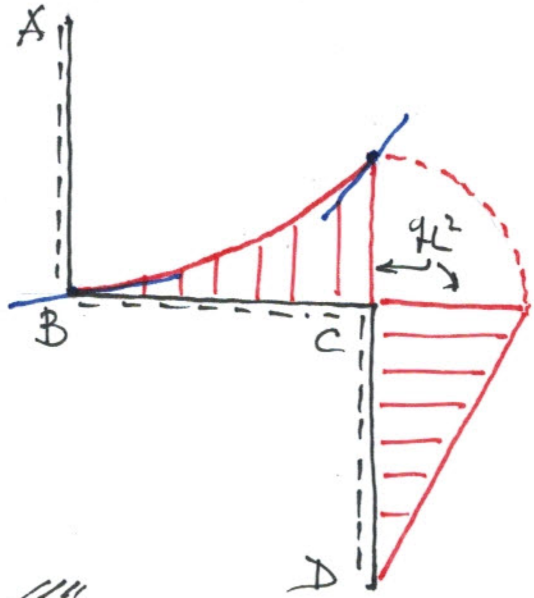
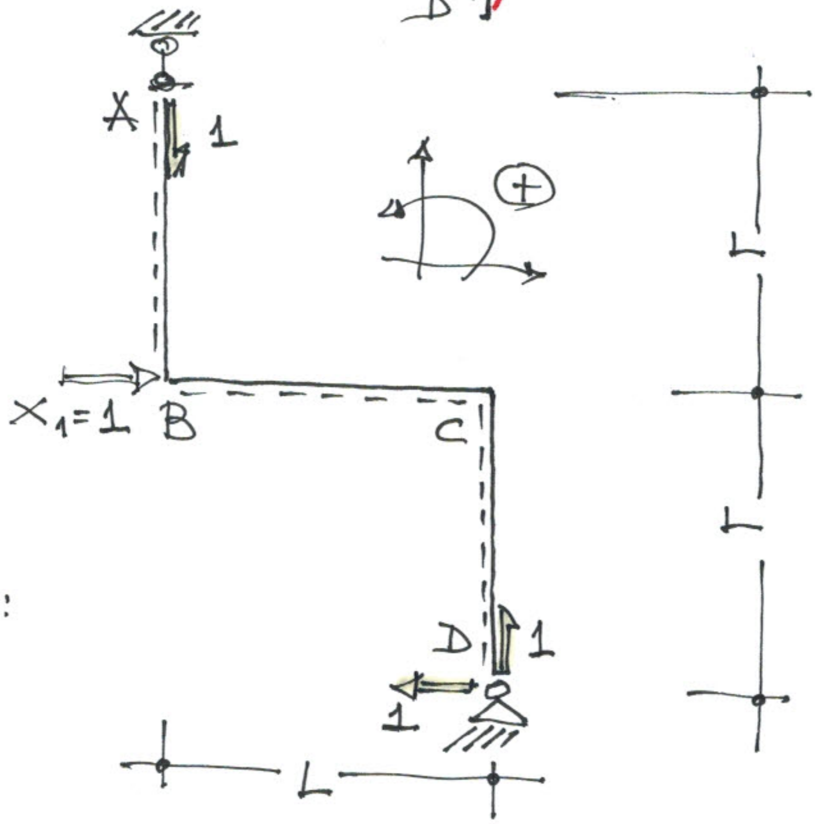
diagramma $M^{(0)}(z)$:



SCHEMA [1]

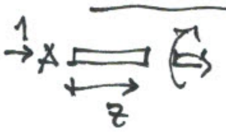
Solo $X_1 = 1$



I. Si calcolano le RV con metodo grafico.

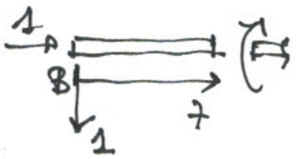
II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $0 \le z \le L$



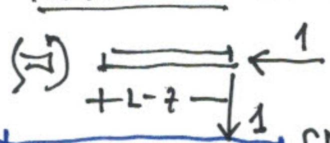
$\Rightarrow \boxed{M^{(1)}(z) = \phi}$

TRATTO BC $0 \le z \le L$



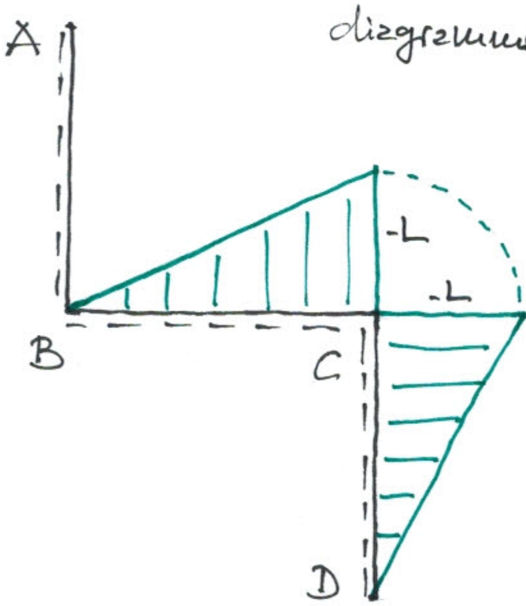
$\Rightarrow \boxed{M^{(1)}(z) = -z}$ $\begin{cases} M_B = \phi \\ M_C = -L \end{cases}$

TRATTO CD $0 \le z \le L$



$\boxed{M^{(1)}(z) = -(L-z)}$ $\begin{cases} M_C = -L \\ M_D = \phi \end{cases}$

diagramma $M^{(1)}(z)$



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve}=L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)}\eta_i^{(r)} + \sum_j R_j^{(f)}\eta_j^{(r)} =$$

$$= 1\cdot \underbrace{\eta_B^{(r)}}_{=-\varepsilon X_1} + R_{y_D}^{(1)}\cdot\left(-\eta_D^0\right) = -\varepsilon X_1 - \eta_D^0$$

$$L_{vi} = \int_{Str} \underbrace{M^{(f)}}_{M^{(1)}} \frac{\overbrace{M^{(r)}}^{M^{(0)}+X_1M^{(1)}}}{EI}\, dStr + \int_{Str} \overbrace{M^{(f)}}^{M^{(1)}} \frac{\alpha\Delta T}{h}\, dStr =$$

$$= \int_{Str} M^{(1)}\frac{M^{(0)}}{EI}\, dStr + X_1\int_{Str}\frac{\left[M^{(1)}\right]^2}{EI}\, dStr + \frac{\alpha\Delta T}{h}\int_{Str} M^{(1)}\, dStr =$$

$$= \frac{1}{EI}\int_{Str} M^{(1)}M^{(0)}\, dStr + \frac{X_1}{EI}\int_{Str}\left[M^{(1)}\right]^2 dStr + \frac{\alpha\Delta T}{h}\int_{Str} M^{(1)}\, dStr =$$

$$= \frac{1}{EI}\left[\int_{\overline{BC}}[-z]\left[-\frac{qL}{2}z - \frac{qz^2}{2}\right] + \int_{\overline{CD}} \overbrace{[z-L]\left[-qL(L-z)\right]}^{-qL^2z + qLz^2 + qL^3 - qL^2z}\right] +$$

$$+ \frac{X_1}{EI}\left[\int_{\overline{BC}} z^2 dz + \int_{\overline{CD}}(z-L)^2 dz\right] + \frac{\alpha\Delta T}{h}\int_{\overline{CD}}(z-L)\, dz =$$

$$= \frac{1}{EI}\left[\int_0^L\left(\frac{qL}{2}z^2 + \frac{qz^3}{2}\right)dz + \int_0^L\left(-2qL^2z + qLz^2 + qL^3\right)dz\right] +$$

$$+ \frac{X_1}{EI}\left[\int_0^L z^2 dz + \int_0^L\left(z^2 + L^2 - 2Lz\right)dz\right] + \frac{\alpha\Delta T}{h}\int_0^L (z-L)\, dz =$$

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$$= \frac{1}{EI}\left\{ \frac{qL}{2}\left[\frac{z^3}{3}\right]_0^L + \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L - 2qL^2\left[\frac{z^2}{2}\right]_0^L + qL\left[\frac{z^3}{3}\right]_0^L + qL^3\left[z\right]_0^L \right] +$$

$$+ \frac{X_1}{EI}\left\{ \left[\frac{z^3}{3}\right]_0^L + \left[\frac{z^3}{3}\right]_0^L + L^2\left[z\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L \right] + \frac{\alpha\overline{\Delta T}}{h}\left\{ \left[\frac{z^2}{2}\right]_0^L - L\left[z\right]_0^L \right] =$$

$$= \frac{1}{EI}\left\{ \frac{qL^4}{6} + \frac{qL^4}{8} - \cancel{qL^4} + \frac{qL^4}{3} + \cancel{qL^4} \right] + \frac{X_1}{EI}\left\{ \frac{L^3}{3} + \frac{L^3}{3} + \cancel{L^3} - \cancel{L^3} \right] + \frac{\alpha\overline{\Delta T}}{h}\left[\frac{L^2}{2} - L^2\right] =$$

$$= \frac{15}{24}\frac{qL^4}{EI} + \frac{2}{3}\frac{L^3}{EI}X_1 - \frac{L^2}{2}\frac{\alpha\overline{\Delta T}}{h}$$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\underset{\uparrow\ \frac{L^3}{3EI}}{\varepsilon} X_1 - \underset{\uparrow\ \frac{qL^4}{24EI}}{\eta^0_D} = \frac{15}{24}\frac{qL^4}{EI} + \frac{2}{3}\frac{L^3}{EI}X_1 - \frac{L^2}{2}\underset{\uparrow\ \frac{qL^2}{EI}}{\frac{\alpha\overline{\Delta T}}{h}}$$

POSIZIONI INIZIALI

$$X_1\underbrace{\left[\frac{2}{3}L^3 + \frac{L^3}{3}\right]}_{L^3} = \underbrace{-\frac{16}{24}qL^4 + \frac{qL^4}{2}}_{-\frac{1}{6}qL^4}$$

$$\boxed{X_1 = -\frac{1}{6}qL}$$

NEGATIVA! Verso opposto a quello ipotizzato!

ok! cfr. RV soluz. #1 a pag 7

SOLUZIONE # 3

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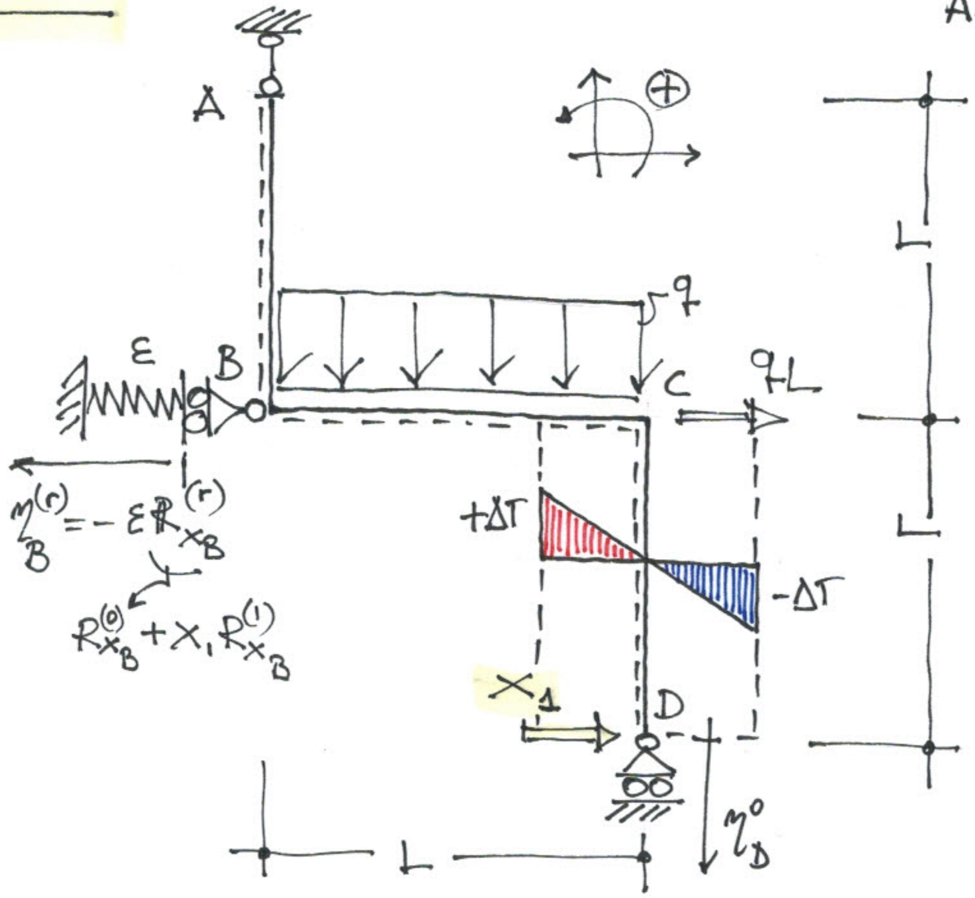
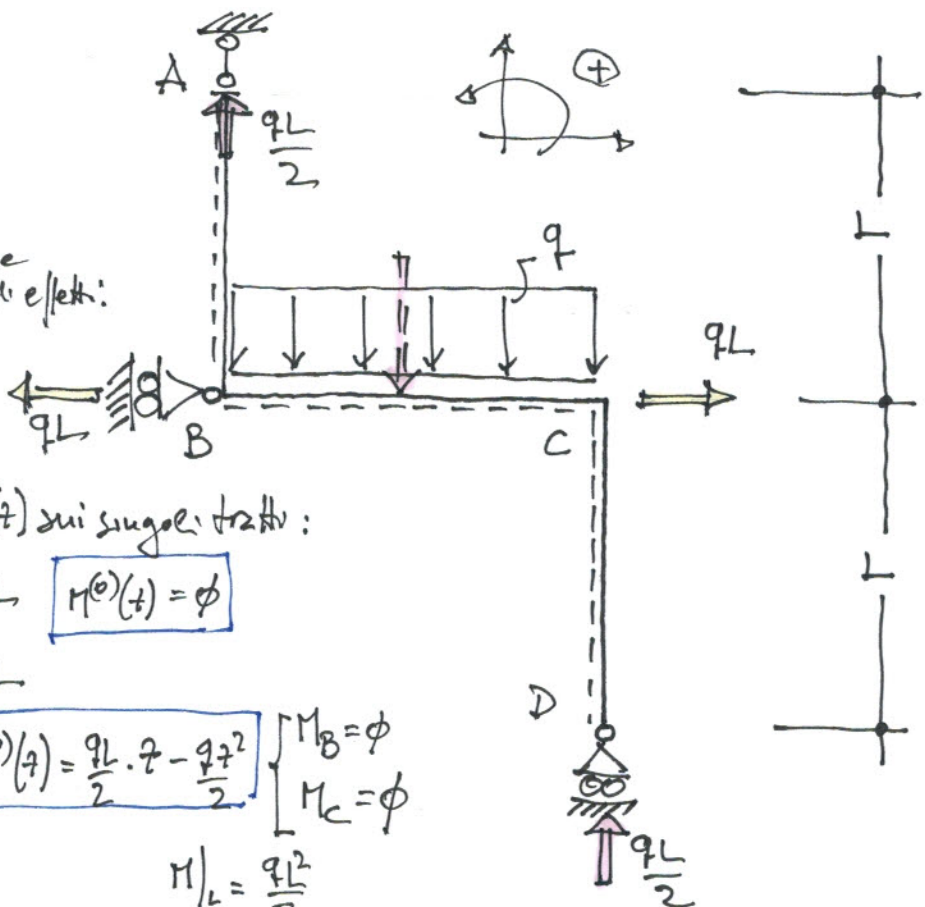
SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]

SOLO CARICHI ESTERNI

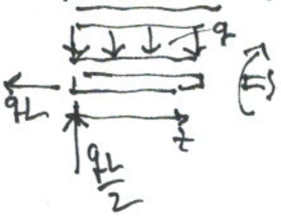
I. Si calcolano le RV con metodo grafico e princ. di sovrapp. degli effetti:



II. Si calcola $M^{(0)}(z)$ sui singoli tratti:

TRATTO AB $0 \le z \le L$ $\boxed{M^{(0)}(z) = \phi}$

TRATTO BC $0 \le z \le L$



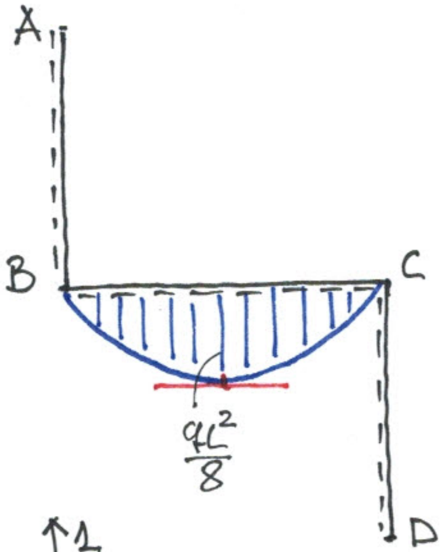
$$\boxed{M^{(0)}(z) = \frac{qL}{2} \cdot z - \frac{qz^2}{2}} \quad \begin{cases} M_B = \phi \\ M_C = \phi \end{cases}$$

$$M\Big|_{\frac{L}{2}} = \frac{qL^2}{8}$$

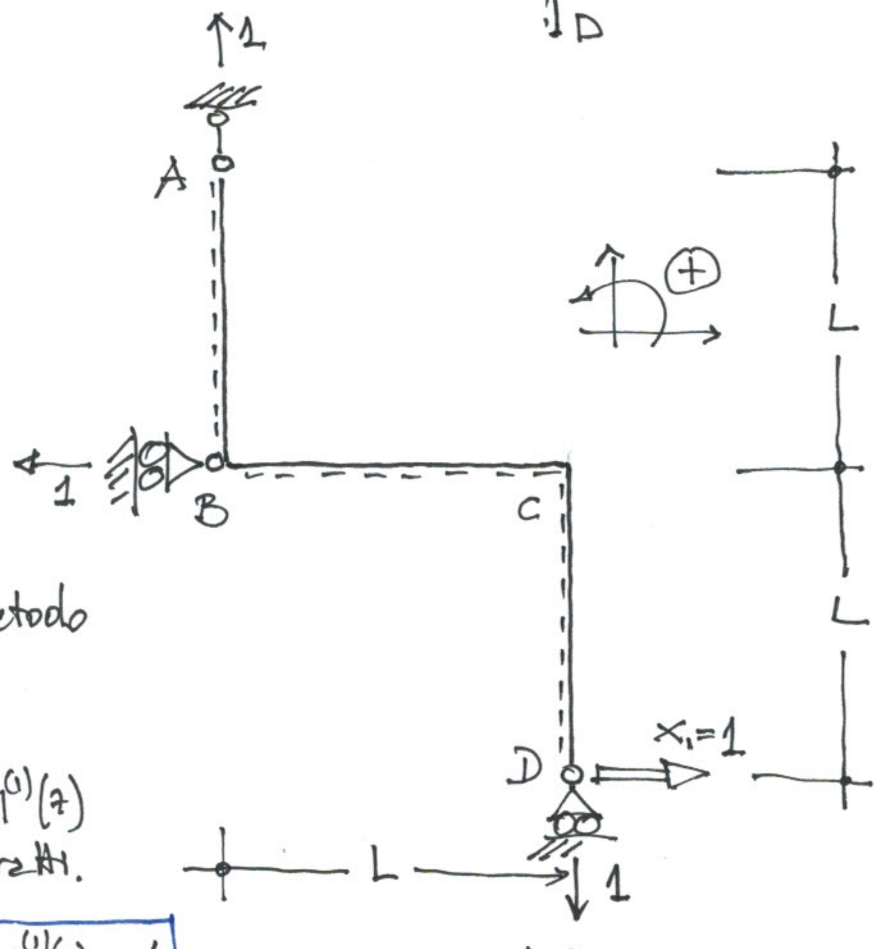
TRATTO CD $0 \le z \le L$

$\boxed{M^{(0)}(z) = \phi}$

diagramma $M^{(0)}(z)$



SCHEMA [1]
solo $X_1 = 1$



I. Si calcolano le RV con metodo grafico.

II. Si calcola $M^{(1)}(z)$ sui singoli tratti.

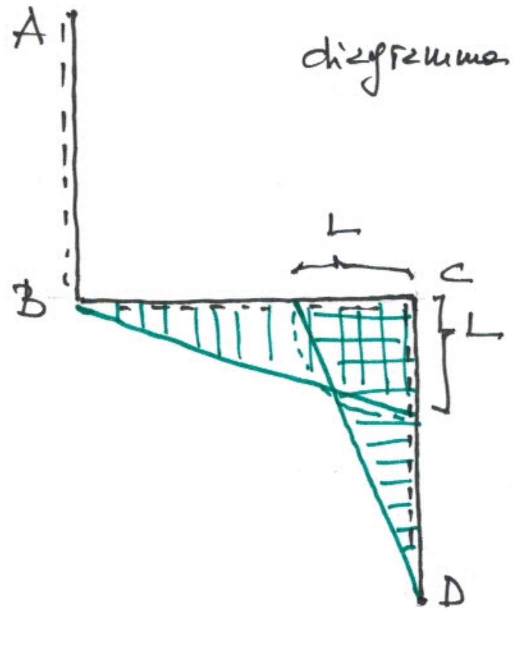
TRATTO AB $\boxed{M^{(1)}(z) = \phi}$

TRATTO BC $\Rightarrow \boxed{M^{(1)}(z) = z}$ $\begin{cases} M_B = 0 \\ M_C = L \end{cases}$

TRATTO CD $0 \le z \le L$

$\boxed{M^{(1)}(z) = L - z}$ $\begin{cases} M_C = L \\ M_D = \phi \end{cases}$

diagramma $M^{(1)}(z)$



P.FUSCHI
A.PISANO

L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \phi + \underbrace{R_{y_D}^{(1)} \cdot \eta_D^{o}}_{(-1)(-\eta_D^{o})} + \underbrace{R_{x_B}^{(1)}}_{-1} \underbrace{\eta_B^{(r)}}_{-\varepsilon\left(\underset{-qL}{R_B^{(0)}} + X_1 \underset{-1\cdot X_1}{R_B^{(1)}}\right)} = \eta_D^{o} - \varepsilon\left[qL + X_1\right]$$

$$L_{vi} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(f)} \alpha \frac{\overline{\Delta T}}{h} =$$

$$= \frac{1}{EI}\int_{Str} M^{(1)} M^{(0)} dStr + \frac{X_1}{EI}\int_{Str} \left[M^{(1)}\right]^2 dStr + \frac{\alpha \overline{\Delta T}}{h}\int_{Str} M^{(1)} dStr =$$

$$= \frac{1}{EI}\int_{\overline{BC}} z\left[\frac{qL}{2}z - \frac{qz^2}{2}\right] dz + \frac{X_1}{EI}\left\{\int_{\overline{BC}} z^2 dz + \int_{\overline{CD}} (L-z)^2 dz\right\} + \frac{\alpha \overline{\Delta T}}{h}\int_{\overline{CD}} (L-z) dz =$$

$$= \frac{1}{EI}\int_0^L \left[\frac{qL}{2}z^2 - \frac{qz^3}{2}\right] dz + \frac{X_1}{EI}\left\{\int_0^L z^2 dz + \int_0^L (L^2 + z^2 - 2Lz) dz\right\} +$$

$$+ \frac{\alpha \overline{\Delta T}}{h}\int_0^L (L-z) dz =$$

$$= \frac{1}{EI}\left\{\frac{qL}{2}\left[\frac{z^3}{3}\right]_0^L - \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L\right\} + \frac{X_1}{EI}\left\{\left[\frac{z^3}{3}\right]_0^L + L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L\right\} +$$

$$+ \frac{\alpha \overline{\Delta T}}{h}\left\{L\left[z\right]_0^L - \left[\frac{z^2}{2}\right]_0^L\right\} =$$

$$= \frac{1}{EI}\left\{\frac{qL^4}{6} - \frac{qL^4}{8}\right\} + \frac{X_1}{EI}\left\{\frac{L^3}{3} + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right\} + \frac{\alpha \overline{\Delta T}}{h}\left\{L^2 - \frac{L^2}{2}\right\} =$$

$$= \frac{qL^4}{24EI} + \frac{2}{3}X_1\frac{L^3}{EI} + \frac{\alpha \overline{\Delta T}}{h}\frac{L^2}{2}$$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$\underset{\substack{\uparrow \\ \frac{qL^4}{24EI}}}{\eta_D^0} - \underset{\substack{\nwarrow \\ \frac{L^3}{3EI}}}{\varepsilon}\left[qL + X_1\right] = \frac{qL^4}{24EI} + \frac{2}{3}\frac{L^3}{EI}X_1 + \alpha\frac{\Delta T}{\underset{\substack{\nearrow \\ \frac{qL^2}{EI}}}{h}}\frac{L^2}{2}$$

tenendo conto delle posizioni iniziali si ha:

$$\cancel{\frac{qL^4}{24EI}} - \frac{qL^4}{3EI} - \frac{L^3}{3EI}X_1 = \cancel{\frac{qL^4}{24EI}} + \frac{2L^3}{3EI}X_1 + \frac{qL^4}{2EI}$$

$$\cancel{\frac{L^3}{EI}}X_1 = \cancel{\frac{qL^4}{EI}}\left[-\frac{1}{3} - \frac{1}{2}\right]$$

$$\boxed{X_1 = -\frac{5}{6}qL}$$

NEGATIVA! verso opposto a quello ipotizzato!

OK! cfr. RV di pag. 7

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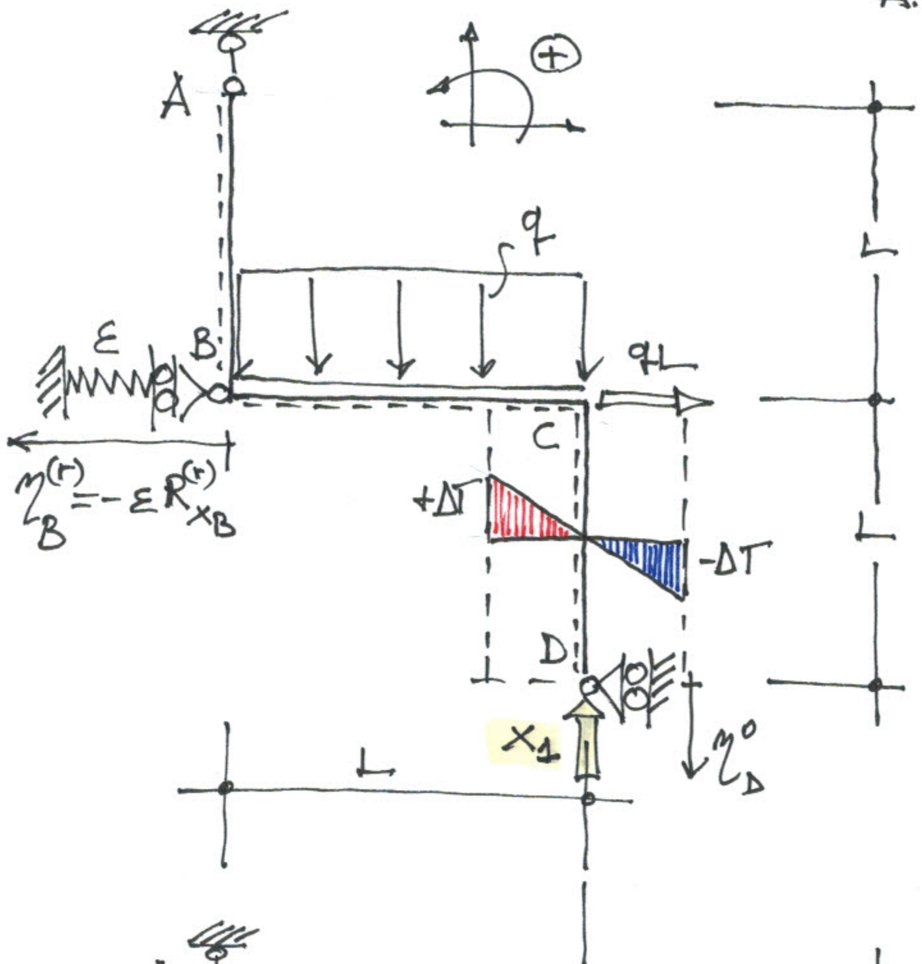
SOLUZIONE #4

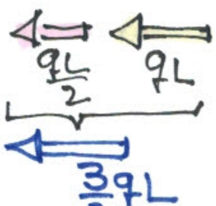
SISTEMA PRINCIPALE ISOSTATICO

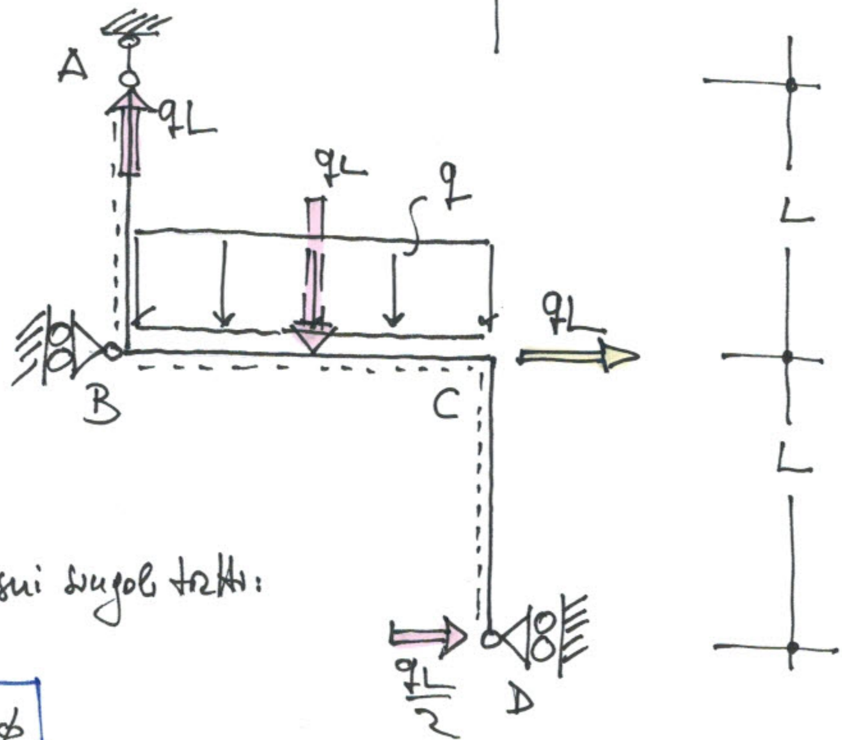


SCHEMA [0]

SOLO CARICHI ESTERNI

I. Si calcolano le RV con metodo grafico!





II. Si calcola $M^{(0)}(z)$ sui singoli tratti:

TRATTO AB $0 \le z \le L$

$$\boxed{M^{(0)}(z) = \emptyset}$$

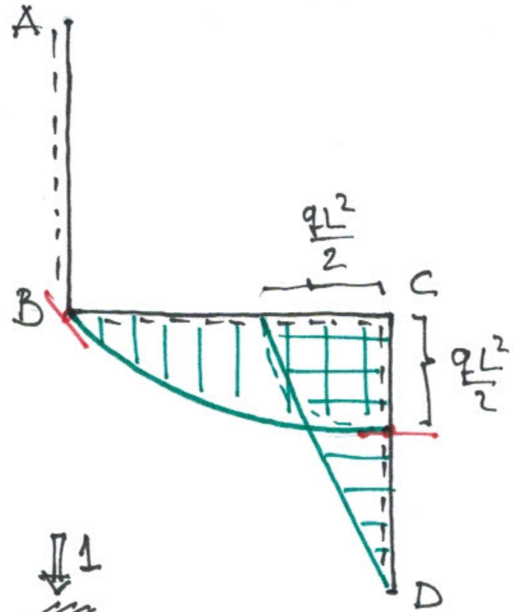
TRATTO BC $0 \le z \le L$

$$\boxed{M^{(0)}(z) = qL \cdot z - q\frac{z^2}{2}} \quad \begin{cases} M_B = \emptyset \\ M_C = \frac{qL^2}{2} \end{cases}$$

TRATTO CD $0 \le z \le L$

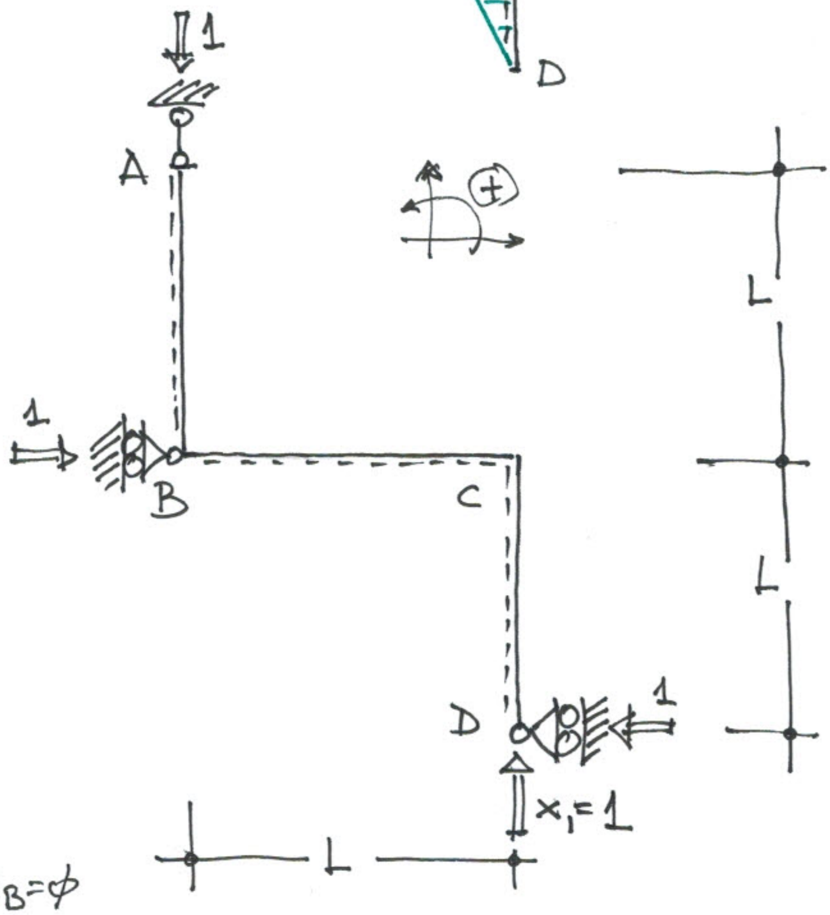
$$\boxed{M^{(0)}(z) = \frac{qL}{2}(L - z)} \quad \begin{cases} M_C = \frac{qL^2}{2} \\ M_D = \emptyset \end{cases}$$

diagramma $M^{(0)}(z)$



SCHEMA [1]

solo $X_1 = 1$



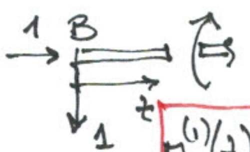
I. RV con metodo grafico.

II. Si calcola $M^{(1)}(z)$ sui singoli tratti.

TRATTO AB $0 \le z \le L$

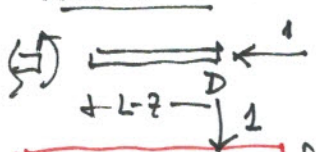
$$\boxed{M^{(1)}(z) = \phi}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -z} \quad \begin{cases} M_B = \phi \\ M_C = -L \end{cases}$$

TRATTO CD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -(L-z)} \quad \begin{cases} M_C = -L \\ M_D = \phi \end{cases}$$

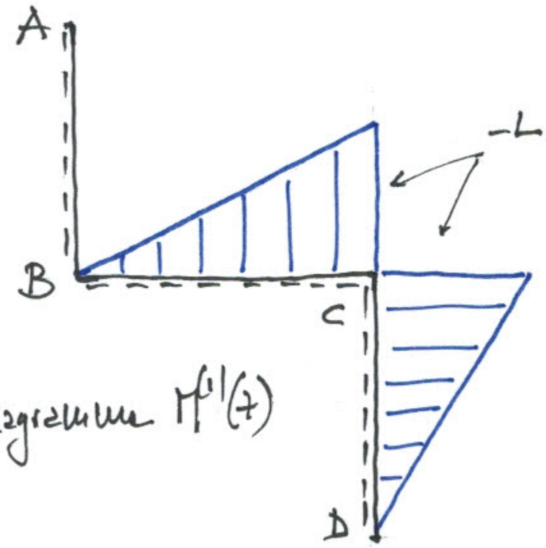


diagramma $M^{(1)}(z)$

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L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= \underbrace{1}_{} \cdot \left(-\eta_D^0\right) + \underbrace{R_{x_B}^{(1)}}_{1} \cdot \underbrace{\eta_B}_{-\varepsilon \underbrace{R_{x_B}^{(r)}}_{\underbrace{R_{x_B}^{(0)}}_{-\frac{3}{2}qL} + X_1 \underbrace{R_{x_B}^{(1)}}_{1}}} = -\eta_D^0 - \varepsilon\left[X_1 - \frac{3}{2}qL\right]$$

$$L_{vi} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \frac{1}{EI}\int_{Str} M^{(1)} M^{(0)} dStr + \frac{X_1}{EI}\int_{Str}\left[M^{(1)}\right]^2 dStr + \frac{\alpha \Delta T}{h}\int_{Str} M^{(1)} dStr =$$

$$= \frac{1}{EI}\left\{\int_{BC} -z\left[qLz - q\frac{z^2}{2}\right] + \int_{CD} (z-L)\left[\frac{qL}{2}(L-z)\right] dz\right\} +$$

$$+ \frac{X_1}{EI}\left\{\int_{BC} z^2 dz + \int_{CD} (z-L)^2 dz\right\} + \int_{CD} -(L-z)\frac{\alpha \Delta T}{h} dz =$$

$$= \frac{1}{EI}\left\{-qL\left[\frac{z^3}{3}\right]_0^L + \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L - \frac{qL}{2}\left[\frac{z^3}{3}\right]_0^L - \frac{qL^3}{2}\left[z\right]_0^L + qL^2\left[\frac{z^2}{2}\right]_0^L\right\} +$$

$$+ \frac{X_1}{EI}\left\{\left[\frac{z^3}{3}\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L + L^2\left[z\right]_0^L\right\} +$$

$$- \frac{\alpha \Delta T}{h}\left\{L\left[z\right]_0^L - \left[\frac{z^2}{2}\right]_0^L\right\} =$$

$$= \frac{1}{EI}\left\{-\frac{qL^4}{3}+\frac{qL^4}{8}-\frac{qL^4}{6}-\cancel{\frac{qL^4}{2}}+\cancel{\frac{qL^4}{2}}\right\}+$$

$$+\frac{X_1}{EI}\left[\frac{L^3}{3}+\frac{L^3}{3}-\cancel{L^3}+\cancel{L^3}\right]-\alpha\frac{\overline{\Delta T}}{h}\left[L^2-\frac{L^2}{2}\right]=$$

$$=-\frac{9}{24}\frac{qL^4}{EI}+\frac{2}{3}\frac{L^3}{EI}X_1-\frac{L^2}{2}\alpha\frac{\overline{\Delta T}}{h}$$

$\Rightarrow$ In definitiva $L_{ve}=L_{vi}$ fornisce:

$$-\eta_D^0-\varepsilon\left[X_1-\frac{3}{2}qL\right]=-\frac{9}{24}\frac{qL^4}{EI}+\frac{2}{3}\frac{L^3}{EI}X_1-\frac{L^2}{2}\alpha\frac{\overline{\Delta T}}{h}$$

quest'ultima, tenendo conto delle posizioni iniziali di pag. 1, fornisce:

$$X_1=\frac{4}{3}qL$$

POSITIVA $\Rightarrow$ Verso ipotizzato corretto!

OK! cfr. RV di pag. 7