MECCANICA delle STRUTTURE - A.A. 2017-18 — P. FUSCHI, A. PISANO — 1

CORSO P. FUSCHI - prova scritta del 24.01.2018

SOLUZIONE

Quesito n. 1 (2 CFU)

RISOLVERE LA STRUTTURA IPERSTATICA SEGUENTE TRACCIANDO IL DIAGRAMMA DEI MOMENTI:

Posizioni:

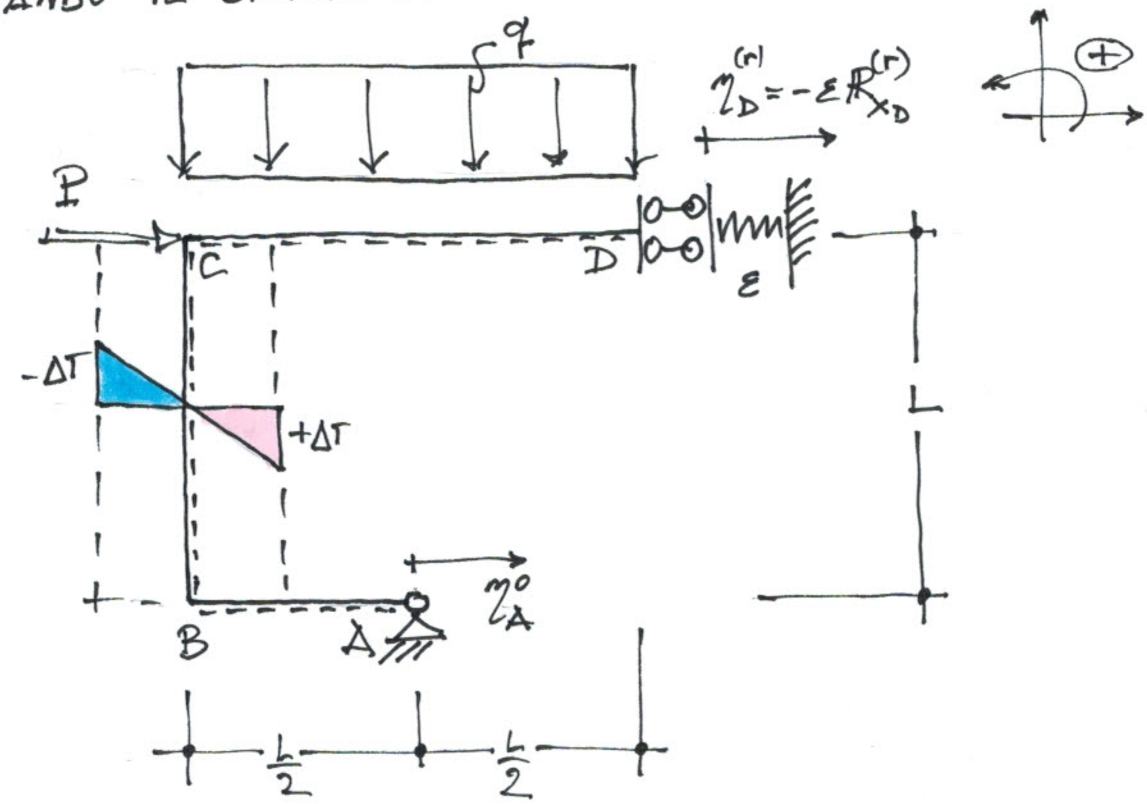
$|P| = qL$

$|\mathcal{E}| = \frac{2}{3}\frac{L^3}{EI}$

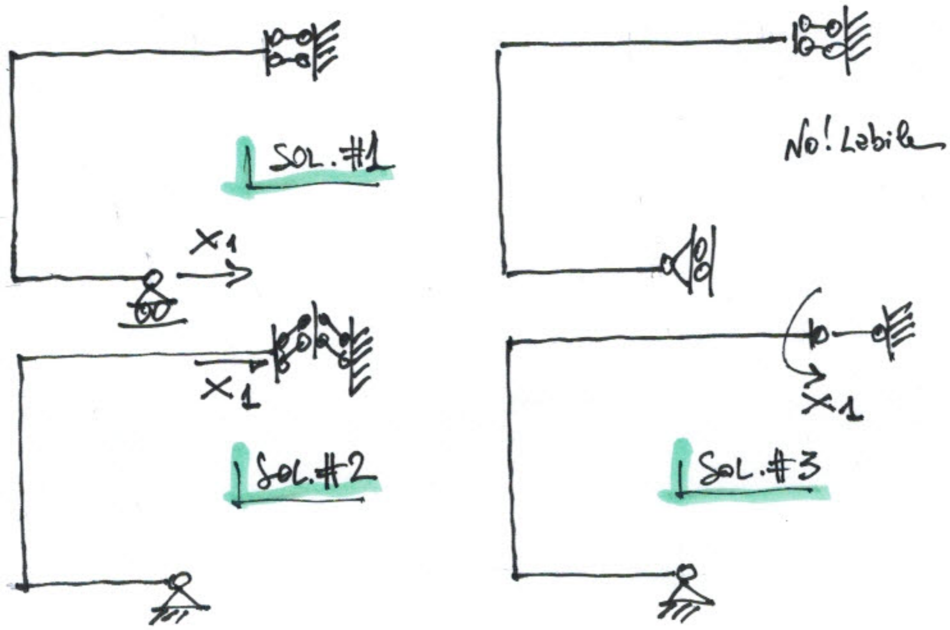
$|\eta_A^0| = \frac{5}{12}\frac{qL^4}{EI}$

$\left|\frac{\alpha \Delta T}{h}\right| = \frac{1}{3}\frac{qL^2}{EI}$

$\eta_D^{(r)} = -\mathcal{E} R_{X_D}^{(r)}$

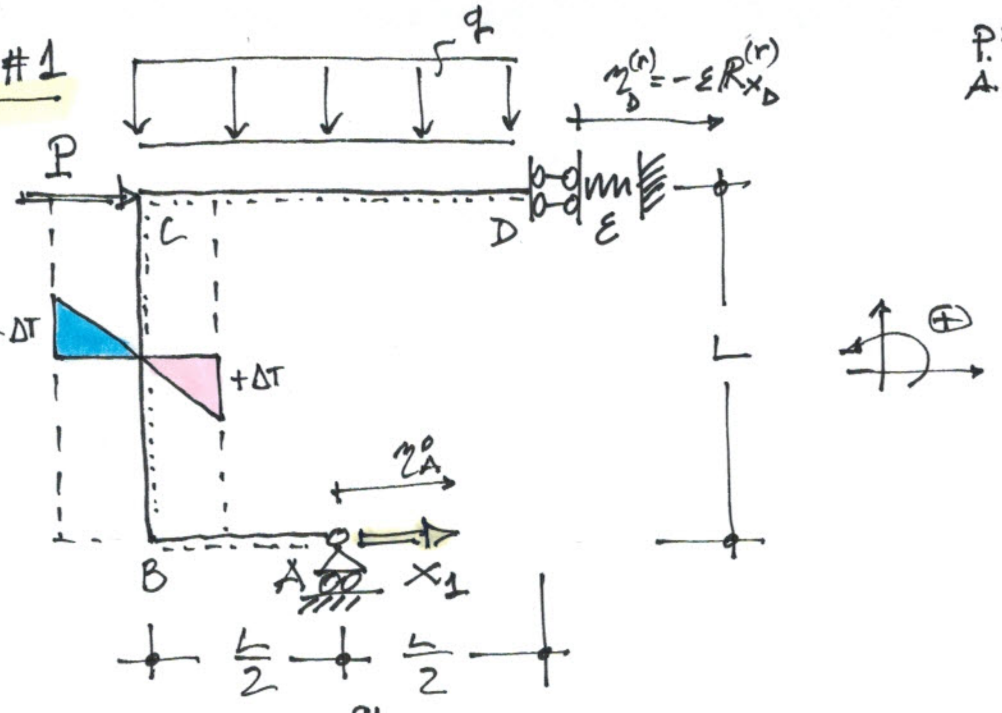


POSSIBILI SCELTE DEL SISTEMA PRINCIPALE ISOSTATICO:

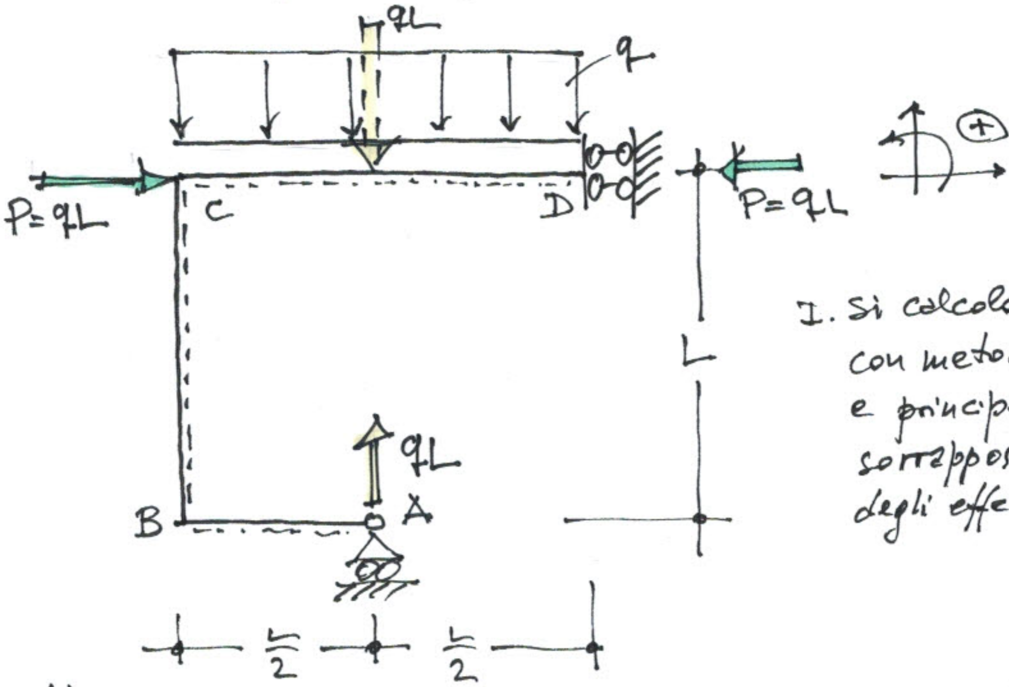


SOLUZIONE #1

SISTEMA PRINCIPALE ISOSTATICO



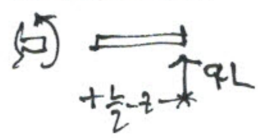
SCHEMA [0]
SOLO CARICHI ESTERNI



I. Si calcolano le RV con metodo grafico e principio di sovrapposizione degli effetti!

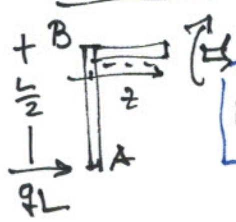
II. Si calcola $M^{(0)}(z)$ sui singoli tratti. Si ha:

TRATTO BA $0 \le z \le \frac{L}{2}$



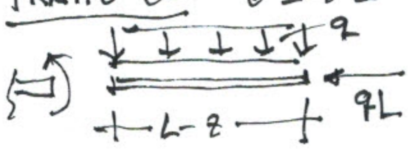
$$\boxed{M^{(0)}(z) = qL\left(\frac{L}{2} - z\right)} \quad \begin{cases} M_B = \frac{qL^2}{2} \\ M_A = \phi \end{cases}$$

TRATTO BC $0 \le z \le L$



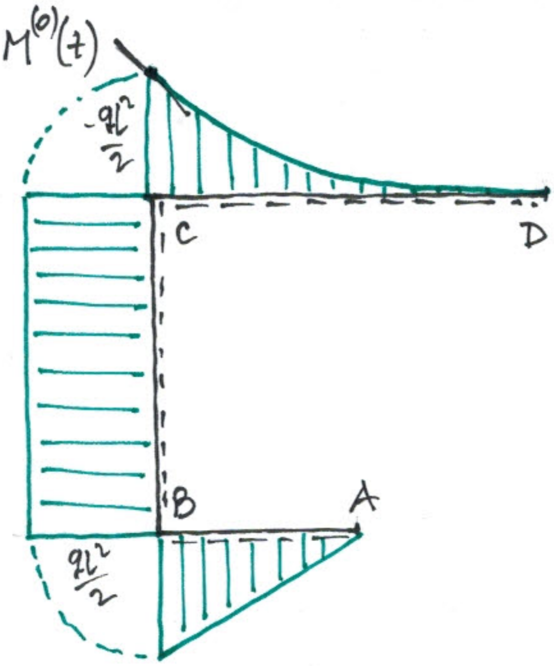
$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2}}$$ costante!

TRATTO CD $0 \le z \le L$

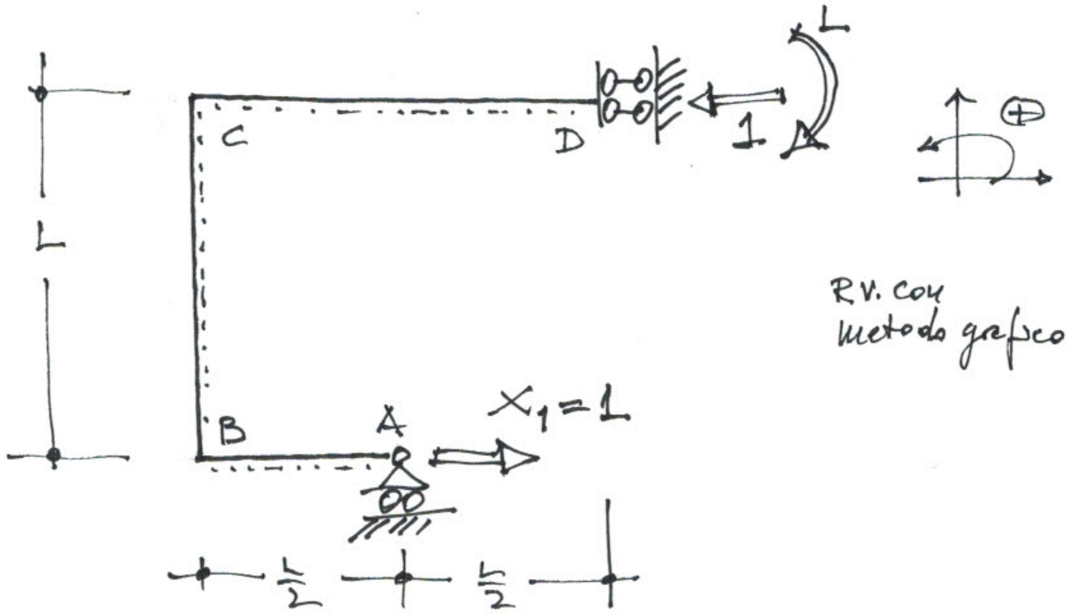


$$\boxed{M^{(0)}(z) = -\frac{q(L-z)^2}{2}} \quad \begin{cases} M_C = -\frac{qL^2}{2} \\ M_D = \phi \end{cases}$$

Diagramma $M^{(0)}(z)$



SCHEMA [1]
solo $X_1 = 1$

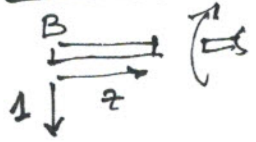


R.V. con metodo grafico!

II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

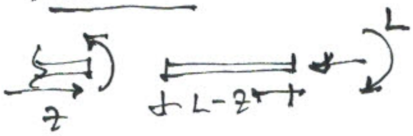
TRATTO BA $0 \le z \le \frac{L}{2}$ $\boxed{M^{(1)}(z) = \phi}$

TRATTO BC $0 \le z \le L$



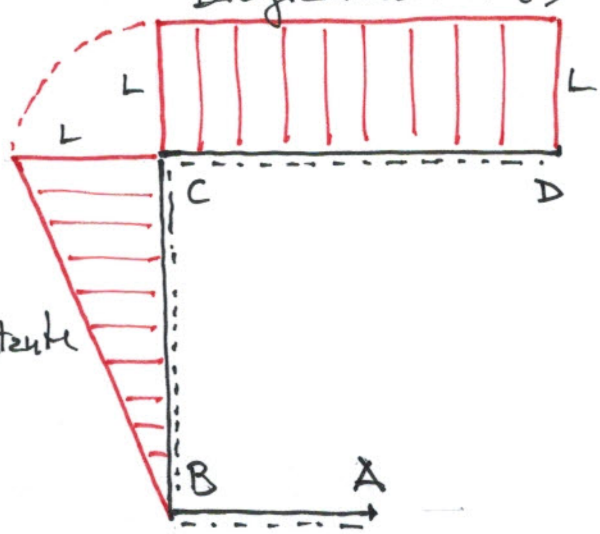
$\boxed{M^{(1)}(z) = -z}$ $\begin{cases} M_B = \phi \\ M_C = -L \end{cases}$

TRATTO CD $0 \le z \le L$



$\boxed{M^{(1)}(z) = -L}$ costante

Diagramma $M^{(1)}(z)$:



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L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data.

Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} = 1 \cdot \eta_A^{\circ} + \underbrace{R_{x_D}^{(1)}}_{=-1} \cdot \underbrace{\eta_D^{(r)}}_{-\varepsilon R_{x_D}^{(r)}} = \eta_A^{\circ} - \varepsilon\left[qL + X_1\right]$$

$$R_{x_D}^{(r)} = \underbrace{R_{x_D}^{(0)}}_{-qL} + X_1 \underbrace{R_{x_D}^{(1)}}_{=-1}$$

$$L_{vi} = \int_{Str} \underbrace{M^{(f)}}_{M^{(1)}} \frac{\overbrace{M^{(r)}}^{M^{(0)}+X_1 M^{(1)}}}{EI}\, ds_{tr} + \int_{Str} M^{(f)} \alpha \frac{\overline{\Delta T}}{h}\, ds_{tr} =$$

$$= \int_{Str} \frac{M^{(1)} M^{(0)}}{EI}\, ds_{tr} + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI}\, ds_{tr} + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h}\, ds_{tr} =$$

$$= \frac{1}{EI}\left\{ \int_{\overline{BC}} [-z]\left[-\frac{qL^2}{2}\right] dz + \int_{\overline{CD}} -L\left[-\frac{q}{2}(L-z)^2\right] \right\} +$$

$$+ \frac{X_1}{EI}\left\{ \int_{\overline{BC}} z^2 dz + \int_{\overline{CD}} L^2\, dz \right\} + \int_{\overline{BC}} [-z]\, \alpha \frac{\overline{\Delta T}}{h}\, dz =$$

$$= \frac{1}{EI}\left\{ \int_0^L \frac{qL^2}{2} z\, dz + \int_0^L \left[\frac{qL^3}{2} + \frac{qL}{2} z^2 - qL^2 z \right] dz \right\} +$$

$$\frac{X_1}{EI}\left\{ \int_0^L z^2 dz + \int_0^L L^2 dz \right\} - \alpha \frac{\overline{\Delta T}}{h} \int_0^L z\, dz =$$

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$$= \frac{1}{EI}\left\{ \frac{qL^2}{2}\left[\frac{z^2}{2}\right]_0^L + \frac{qL^3}{2}\left[z\right]_0^L + \frac{qL}{2}\left[\frac{z^3}{3}\right]_0^L - qL^2\left[\frac{z^2}{2}\right]_0^L \right\} +$$

$$+ \frac{X_1}{EI}\left\{\left[\frac{z^3}{3}\right]_0^L + L^2\left[z\right]_0^L\right\} - \alpha\frac{\Delta T}{h}\left[\frac{z^2}{2}\right]_0^L =$$

$$= \frac{1}{EI}\left\{ \frac{qL^4}{4} + \cancel{\frac{qL^4}{2}} + \frac{qL^4}{6} - \cancel{\frac{qL^4}{2}} \right\} + \frac{X_1}{EI}\left\{\frac{L^3}{3} + L^3\right\} - \alpha\frac{\Delta T}{h}\frac{L^2}{2} =$$

$$= \frac{5}{12}\frac{qL^4}{EI} + \frac{4}{3}X_1\frac{L^3}{EI} - \alpha\frac{\Delta T}{h}\frac{L^2}{2}$$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$\eta_A^0 - \varepsilon\left[qL + X_1\right] = \frac{5}{12}\frac{qL^4}{EI} + \frac{4}{3}X_1\frac{L^3}{EI} - \alpha\frac{\Delta T}{h}\frac{L^2}{2}$$

quest'ultima, tenendo conto delle posizioni a pag. 1

fornisce:

$$\boxed{X_1 = -\frac{qL}{4}}$$

NEGATIVA! ⇒ Verso opposto a quello ipotizzato!!

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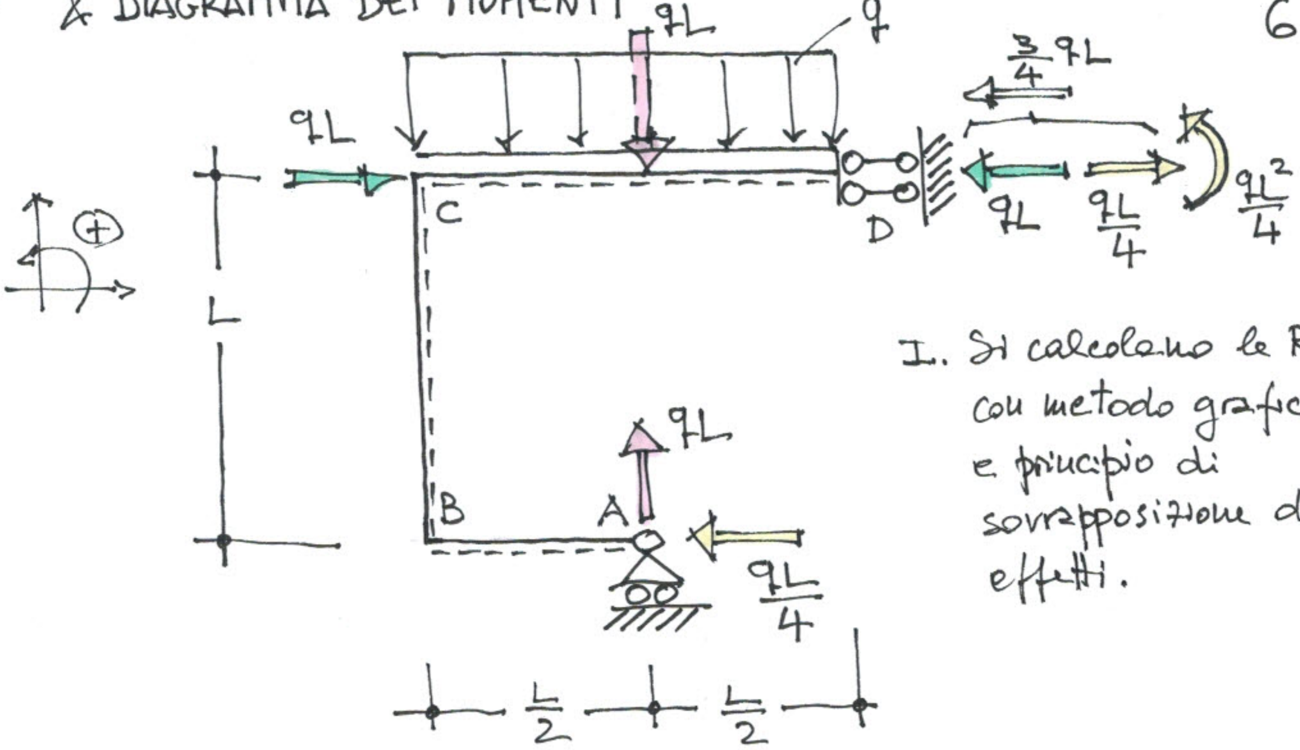
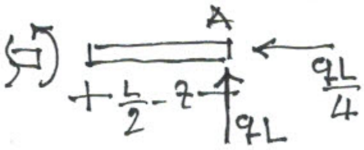
SISTEMA PRINCIPALE ISOSTATICO & DIAGRAMMA DEI MOMENTI



I. Si calcolano le RV con metodo grafico e principio di sovrapposizione degli effetti.

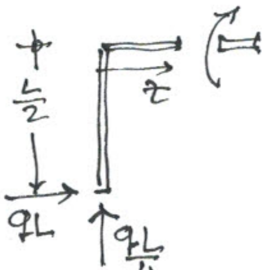
II. Si calcola $M(z)$ sui tratti, si ha:

TRATTO BA $0 \le z \le \frac{L}{2}$



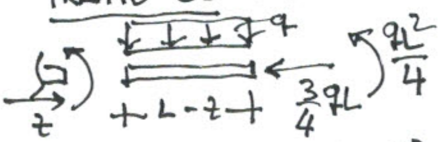
$$M(z) = qL(L-z) \begin{cases} M_B = qL^2 \\ M_A = \phi \end{cases}$$

TRATTO BC $0 \le z \le L$

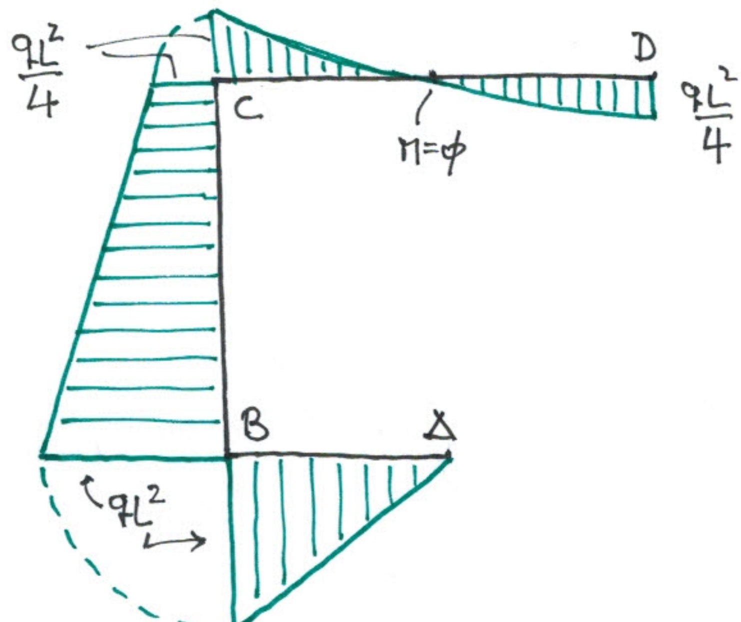


$$M(z) = \frac{qL}{4}\cdot z - \frac{qL^2}{2} \begin{cases} M_B = -\frac{qL^2}{2} \\ M_C = -\frac{1}{4}qL^2 \end{cases}$$

TRATTO CD $0 \le z \le L$



$$M(z) = \frac{qL^2}{4} - \frac{q(L-z)^2}{2} \begin{cases} M_C = -\frac{1}{4}qL^2 \\ M_D = \frac{qL^2}{4} \\ M(z)\big|_{z=\frac{L}{2}} = \phi \end{cases}$$



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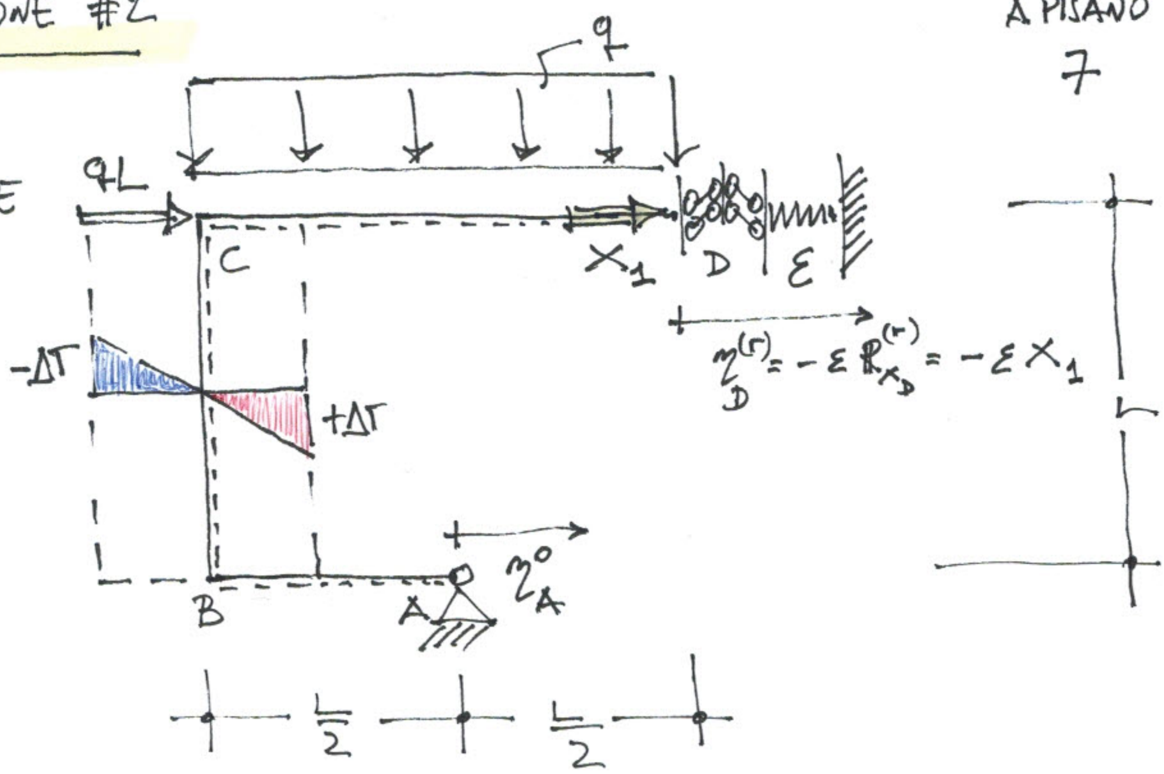
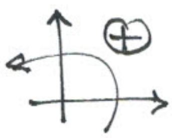
SOLUZIONE #2

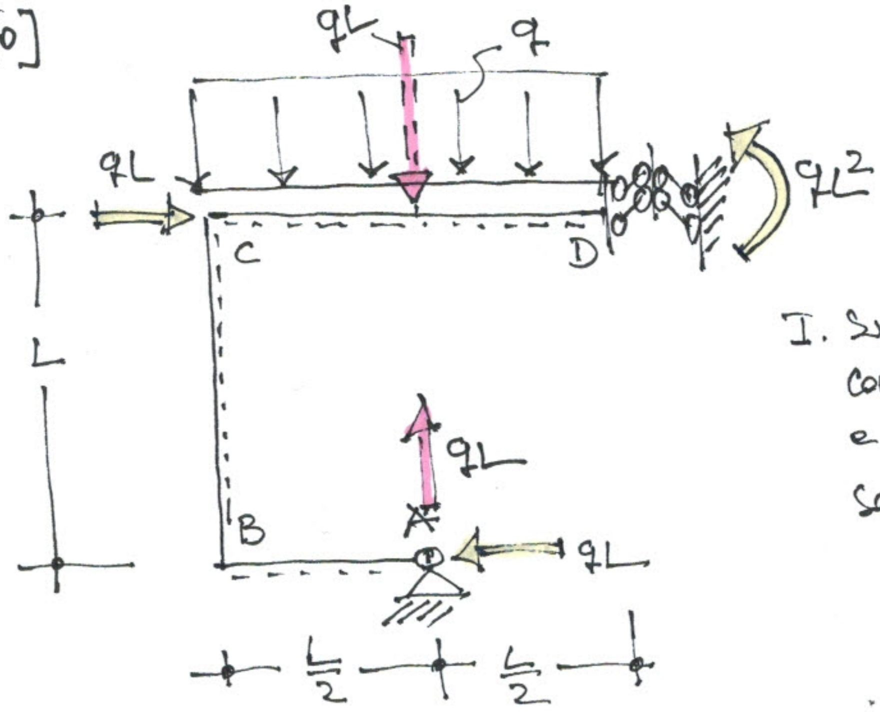
SISTEMA PRINCIPALE ISOSTATICO





SCHEMA [0]

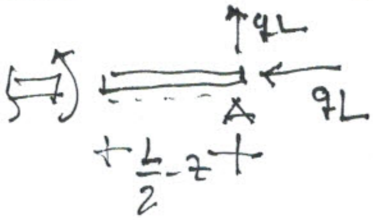
Solo carichi esterni



I. Si calcolano le RV con metodo grafico e principio di sovrapp. degli effetti.

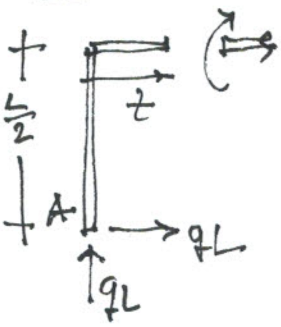
II. Si calcola $M^{(0)}(z)$ sui singoli tratti:

TRATTO BA $\quad 0 \le z \le \frac{L}{2}$



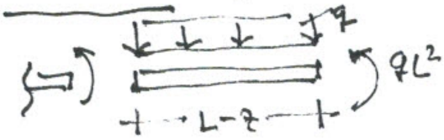
$$\boxed{M^{(0)}(z) = qL\left(\frac{L}{2} - z\right)} \qquad \begin{cases} M_B = \dfrac{qL^2}{2} \\ M_A = \emptyset \end{cases}$$

TRATTO BC $0 \le z \le L$



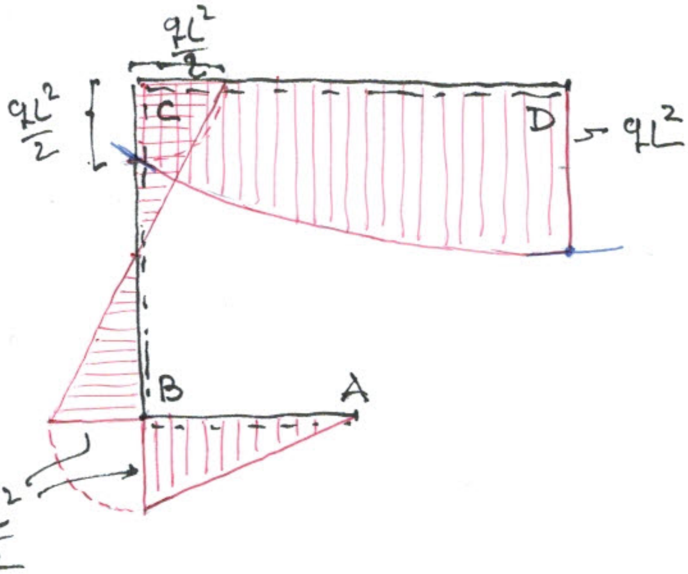
$$M^{(0)}(z) = \boxed{qL \cdot z - \frac{qL^2}{2}} \quad \begin{cases} M_B = -\frac{qL}{2} \\ M_C = \frac{qL^2}{2} \end{cases}$$

TRATTO CD $0 \le z \le L$



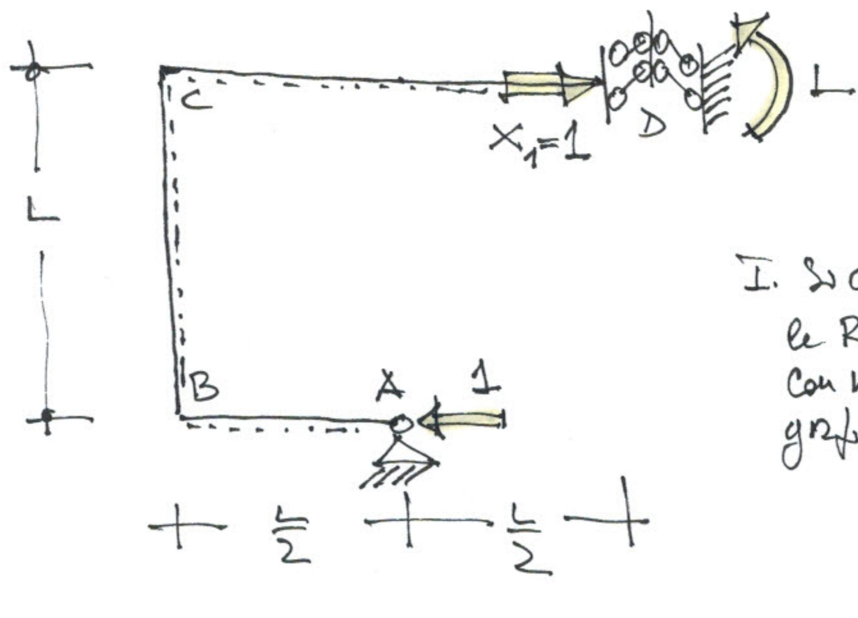
$$\boxed{M^{(0)}(z) = qL^2 - q\frac{(L-z)^2}{2}} \quad \begin{cases} M_C = \frac{qL^2}{2} \\ M_D = qL^2 \end{cases}$$

Diagramma $M^{(0)}(z)$



SCHEMA [1]

Solo $X_1 = 1$



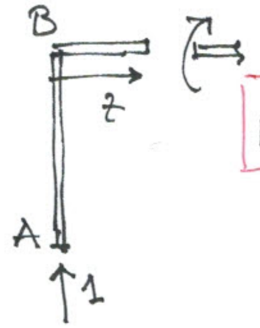
I. Si calcolano le RV con metodo grafico.

II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

TRATTO BA $0 \le z \le \frac{L}{2}$

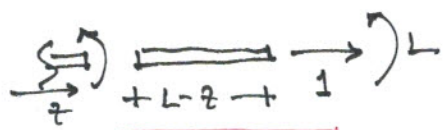
$$\boxed{M^{(1)}(z) = \phi}$$

TRATTO BC $0 \le z \le L$



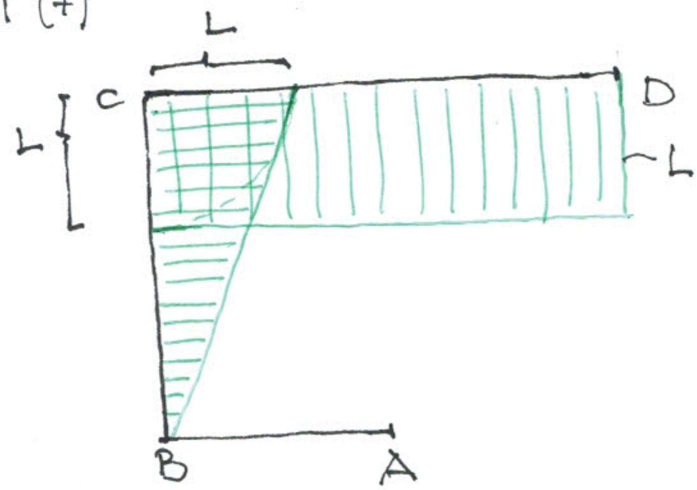
$$\boxed{M^{(1)}(z) = z} \quad \begin{cases} M_B = \phi \\ M_C = L \end{cases}$$

TRATTO CD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = L} = \text{costante}$$

Diagramma $M^{(1)}(z)$



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \cdot \eta_j^{(r)} = 1 \cdot (-\varepsilon X_1) + \underbrace{R_{X_A}}_{-1} \cdot \eta_A^0 = -\varepsilon X_1 - \eta_A^0$$

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$$L_{vi}=\int_{Str}\overset{M^{(1)}}{M^{(f)}}\frac{\overset{M^{(0)}+X_1M^{(1)}}{M^{(r)}}}{EI}\,dStr+\int_{Str}M^{(f)}\alpha\frac{\overline{\Delta T}}{h}\,dStr=$$

$$=\int_{Str}M^{(1)}\frac{M^{(0)}}{EI}\,dStr+X_1\int_{Str}\frac{\left[M^{(1)}\right]^2}{EI}\,dStr+\int_{Str}M^{(1)}\alpha\frac{\overline{\Delta T}}{h}\,dStr=$$

$$=\frac{1}{EI}\left[\int_{\overline{BC}}z\cdot\left[qLz-\frac{qL^2}{2}\right]dz+\int_{\overline{CD}}L\cdot\underbrace{\left[qL^2-\frac{q(L-z)^2}{2}\right]}_{\frac{qL^2}{2}-\frac{qz^2}{2}+qLz}dz\right]+$$

$$+\frac{X_1}{EI}\left[\int_{\overline{BC}}z^2\,dz+\int_{\overline{CD}}L^2\,dz\right]+\int_{\overline{BC}}z\cdot\frac{\alpha\overline{\Delta T}}{h}\,dz=$$

$$=\frac{1}{EI}\left[\int_0^L\left[qLz^2-\frac{qL^2}{2}z\right]dz+\int_0^L\left[\frac{qL^3}{2}-\frac{qL}{2}z^2+qL^2z\right]dz\right]+$$

$$+\frac{X_1}{EI}\left[\int_0^L z^2\,dz+\int_0^L L^2\,dz\right]+\alpha\frac{\overline{\Delta T}}{h}\int_0^L z\,dz=$$

$$=\frac{1}{EI}\left\{qL\left[\frac{z^3}{3}\right]_0^L-\frac{qL^2}{2}\left[\frac{z^2}{2}\right]_0^L+\frac{qL^3}{2}\left[z\right]_0^L-\frac{qL}{2}\left[\frac{z^3}{3}\right]_0^L+qL^2\left[\frac{z^2}{2}\right]_0^L\right\}+$$

$$+\frac{X_1}{EI}\left\{\left[\frac{z^3}{3}\right]_0^L+L^2\left[z\right]_0^L\right\}+\alpha\frac{\overline{\Delta T}}{h}\left[\frac{z^2}{2}\right]_0^L=$$

$$=\frac{1}{EI}\left[\frac{qL^4}{3}-\frac{qL^4}{4}+\frac{qL^4}{2}-\frac{qL^4}{6}+\frac{qL^4}{2}\right]+\frac{X_1}{EI}\left[\frac{L^3}{3}+L^3\right]+\alpha\frac{\overline{\Delta T}}{h}\cdot\frac{L^2}{2}=$$

$$=\frac{11}{12}\frac{qL^4}{EI}+\frac{4}{3}\frac{L^3}{EI}X_1+\alpha\frac{\overline{\Delta T}}{h}\frac{L^2}{2}$$

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In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\varepsilon X_1 - \eta^0_A = \frac{11}{12}\frac{qL^4}{EI} + \frac{4}{3}\frac{L^3}{EI}X_1 + \alpha\frac{\Delta T}{h}\frac{L^2}{2}$$

tenendo conto delle posizioni iniziali si ha:

$$-\frac{2}{3}\frac{\cancel{L^3}}{\cancel{EI}}X_1 - \frac{5}{12}\frac{q\cancel{L^4}}{\cancel{EI}} = \frac{11}{12}\frac{q\cancel{L^4}}{\cancel{EI}} + \frac{4}{3}\frac{\cancel{L^3}}{\cancel{EI}}X_1 + \frac{q\cancel{L^2}}{3\cancel{EI}}\frac{\cancel{L^2}}{2}$$

$$-\frac{5}{12}qL - \frac{11}{12}qL + \frac{1}{6}qL = X_1\left[\frac{4}{3} + \frac{2}{3}\right]$$

$$-\frac{3}{2}qL = 2X_1 \quad \Rightarrow \quad \boxed{X_1 = -\frac{3}{4}qL}$$

NEGATIVA!
verso opposto
a quello
ipotizzato!
Ok! cfr RV
soluz. 1 e pag 6!

SOLUZIONE #3

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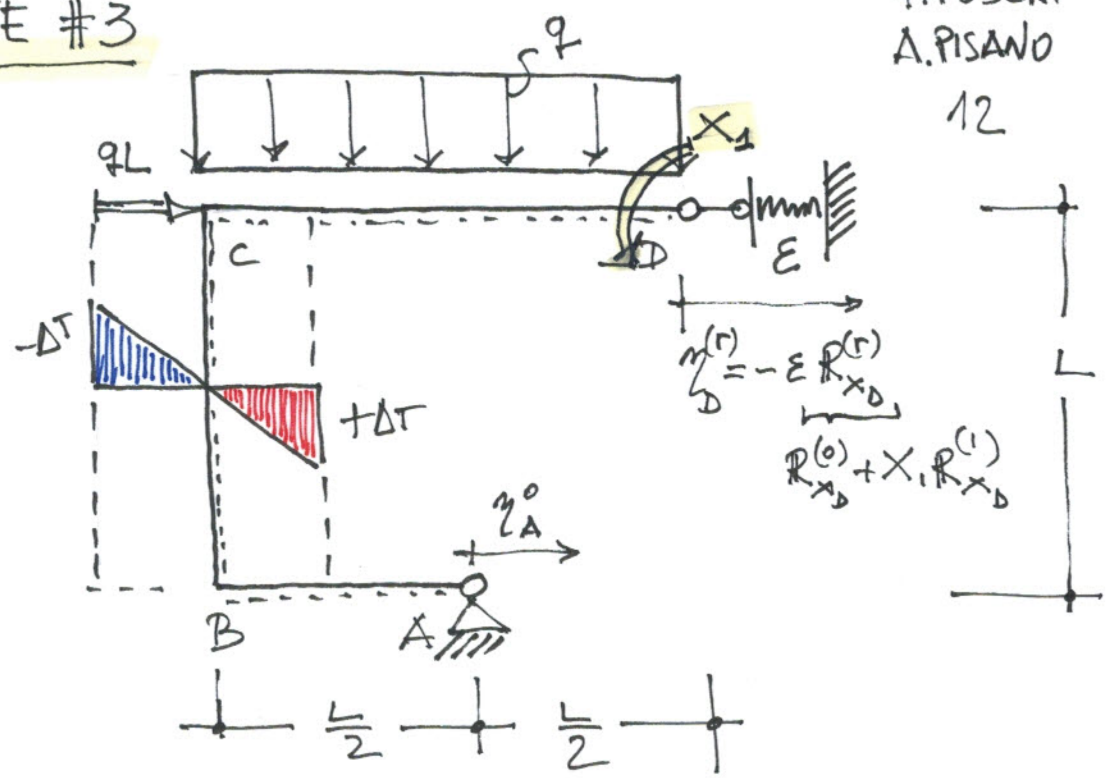
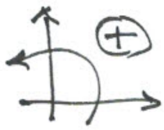
A.PISANO

SISTEMA PRINCIPALE ISOSTATICO

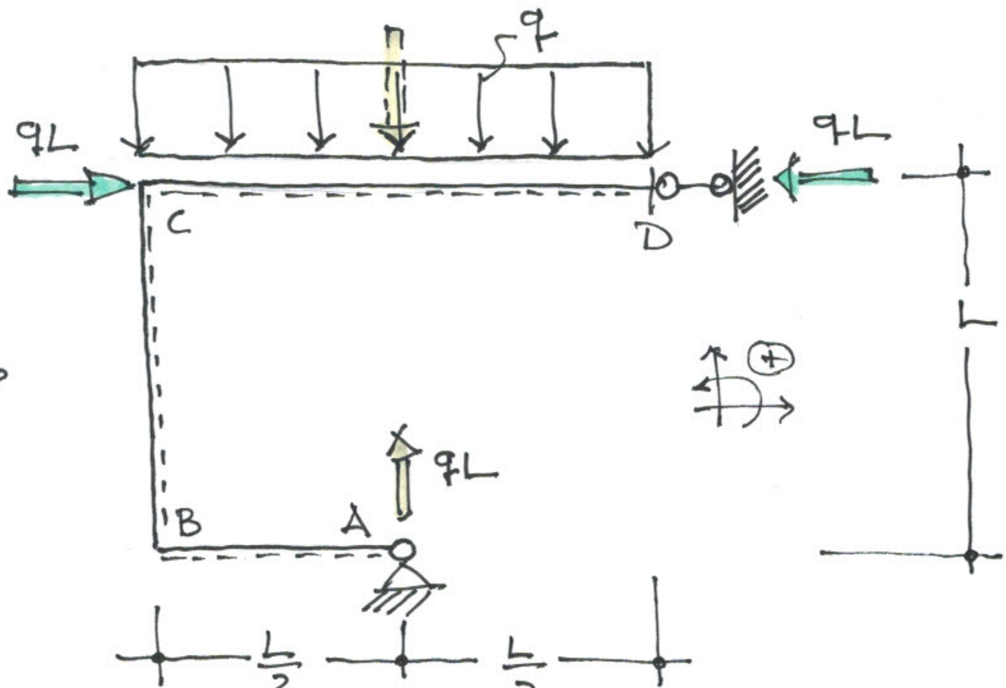




SCHEMA [0]

SOLO CARICHI ESTERNI

I. Si calcolano le RV con metodo grafico e principio di sovrapposizione degli effetti



N.B. Lo schema [0] coincide con quello già esaminato nella soluzione #1 → pag. 2 cui si rimanda per le leggi di $M^{(0)}(t)$ ed i relativi diagrammi.

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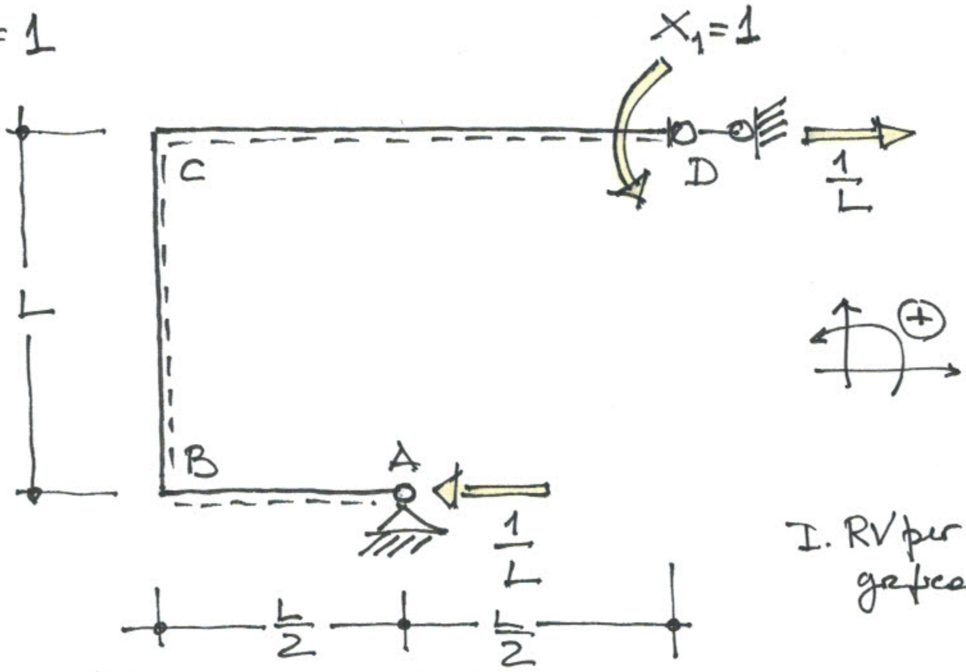
A. PISANO

SCHEMA [1]

solo $X_1 = 1$



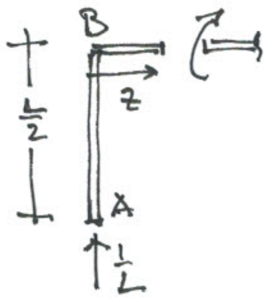
I. RV per via grafica!

II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

TRATTO BA $0 \le z \le \frac{L}{2}$

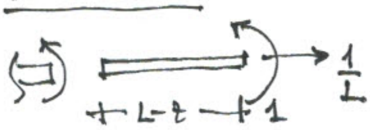
$$\boxed{M^{(1)}(z) = \phi}$$

TRATTO BC $0 \le z \le L$



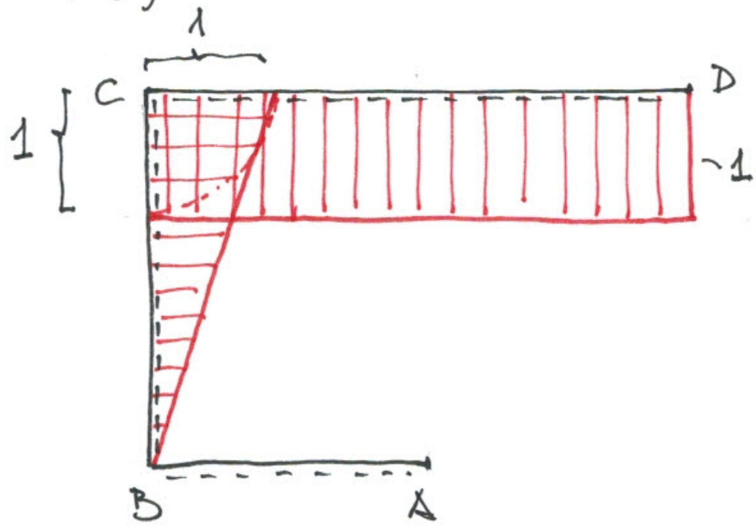
$$\boxed{M^{(1)}(z) = \frac{z}{L}} \quad \begin{cases} M_B = \phi \\ M_C = 1 \end{cases}$$

TRATTO CD $0 \le z \le L$



$$\boxed{M^{(1)}(z) = 1} = \text{costante}$$

diagramma $M^{(1)}(z)$



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L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} = 1 \cdot \phi + \underbrace{R_{x_A}^{(1)}}_{-\frac{1}{L}} \cdot \eta_A^{0} + \underbrace{R_{x_D}^{(1)}}_{\frac{1}{L}} \cdot \underbrace{\eta_D^{(r)}}_{-\varepsilon\left[R_{x_D}^{(0)} + X_1 R_{x_D}^{(1)}\right]} =$$

$$R_{x_D}^{(0)} = -qL \qquad R_{x_D}^{(1)} = \frac{1}{L}$$

$$= -\frac{\eta_A^{0}}{L} - \frac{\varepsilon}{L}\left[-qL + \frac{X_1}{L}\right]$$

$$L_{vi} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(f)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$= \int_{Str} M^{(1)} \frac{\overbrace{M^{(r)}}^{M^{(0)} + X_1 M^{(1)}}}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$= \frac{1}{EI} \int_{Str} M^{(1)} M^{(0)} dStr + \frac{X_1}{EI} \int_{Str} \left[M^{(1)}\right]^2 dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$= \frac{1}{EI} \left\{ \int_{\overline{BC}} \frac{z}{L} \left[-\frac{qL^2}{2}\right] dz + \int_{\overline{CD}} 1 \cdot \left[-\frac{q(L-z)^2}{2}\right] dz \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \int_{\overline{BC}} \left[\frac{z}{L}\right]^2 dz + \int_{\overline{CD}} [1]^2 dz \right\} + \int_{\overline{BC}} \frac{z}{L} \frac{\alpha \overline{\Delta T}}{h} dz =$$

$$= \frac{1}{EI} \left\{ \int_0^L -\frac{qL}{2} \cdot z \, dz - \frac{q}{2} \int_0^L \left(L^2 + z^2 - 2Lz\right) dz \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \int_0^L \frac{z^2}{L^2} dz + \int_0^L dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \cdot \frac{1}{L} \int_0^L z \, dz =$$

$$= \frac{1}{EI}\left\{ -\frac{qL}{2}\left[\frac{z^2}{2}\right]_0^L - \frac{qL^2}{2}\left[z\right]_0^L - \frac{q}{2}\left[\frac{z^3}{3}\right]_0^L + qL\left[\frac{z^2}{2}\right]_0^L \right\} +$$

$$+ \frac{X_1}{EI}\left\{ \frac{1}{L^2}\left[\frac{z^3}{3}\right]_0^L + \left[z\right]_0^L \right\} + \frac{\alpha\overline{\Delta T}}{h}\frac{1}{L}\left[\frac{z^2}{2}\right]_0^L =$$

$$= \frac{1}{EI}\left\{ -\frac{qL^3}{4} - \cancel{\frac{qL^3}{2}} - \frac{qL^3}{6} + \cancel{\frac{qL^3}{2}} \right\} +$$

$$+ \frac{X_1}{EI}\left\{ \frac{L}{3} + L \right\} + \frac{\alpha\overline{\Delta T}}{h}\cdot\frac{L}{2} =$$

$$= -\frac{5}{12EI}qL^3 + \frac{4}{3}\frac{L}{EI}X_1 + \frac{\alpha\overline{\Delta T}}{h}\cdot\frac{L}{2}$$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\frac{\eta_A^0}{L} - \frac{\varepsilon}{L}\left[-qL + \frac{X_1}{L}\right] = -\frac{5}{12EI}qL^3 + \frac{4L}{3EI}X_1 + \frac{\alpha\overline{\Delta T}}{h}\cdot\frac{L}{2}$$

tenendo conto delle posizioni:

$$\cancel{-\frac{5}{12}\frac{qL^3}{EI}} - \frac{2}{3}\frac{L^2}{\cancel{EI}}\left[-qL + \frac{X_1}{L}\right] = \cancel{-\frac{5}{12EI}qL^3} + \frac{4L}{3\cancel{EI}}X_1 + \frac{L}{2}\frac{qL^2}{3\cancel{EI}}$$

$$\frac{2}{3}qL^{\cancel{3}\,2} - \frac{2}{3}X_1\cancel{L} = \frac{4}{3}X_1\cancel{L} + \frac{1}{6}qL^{\cancel{3}\,2}$$

$$\left\{\frac{2}{3} - \frac{1}{6}\right\}qL^2 = \frac{\cancel{6}^{\,2}}{\cancel{3}}X_1 \quad \Rightarrow \quad X_1 = \frac{qL^2}{4}$$

$\frac{3}{6}$

positivo! verso ipotizzato corretto. Ok cfr. con RV della soluz. #1 a pag 6

FINE