I
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PISANO

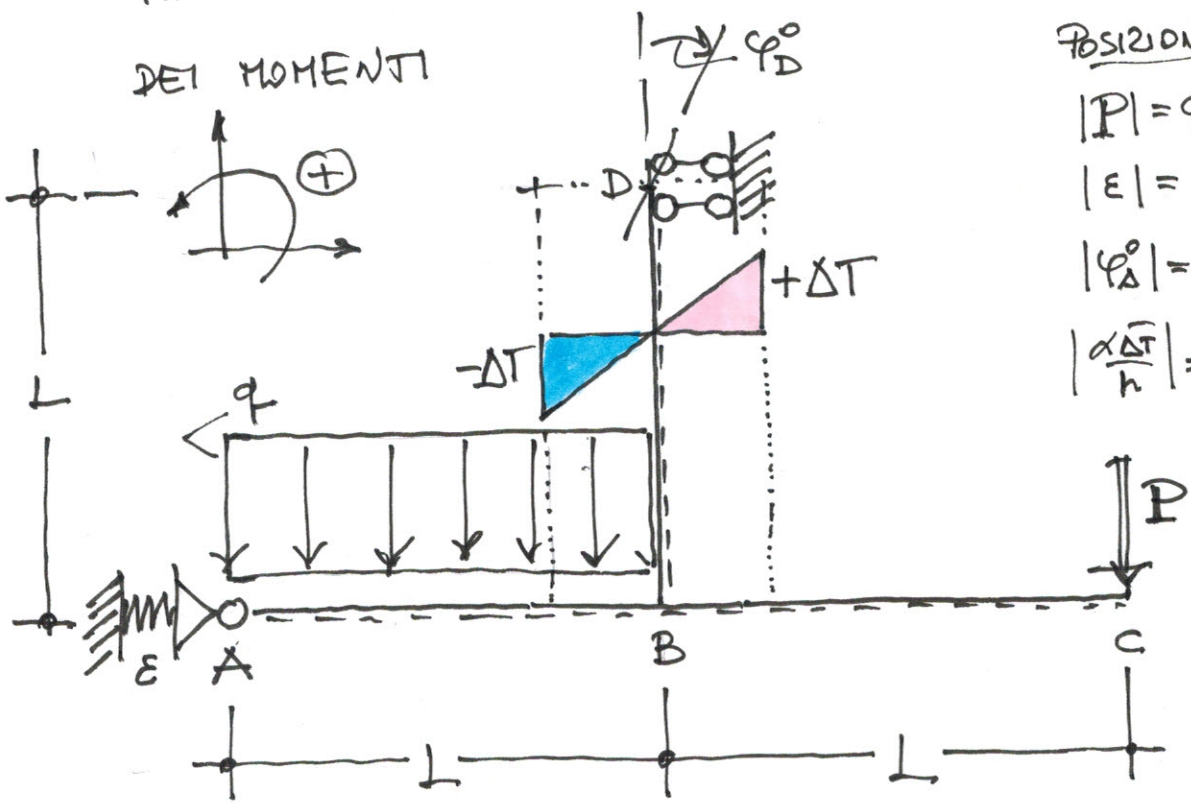
MECCANICA delle STRUTTURE A.A. 2017-18

CORSO P. FUSCHI – TEST in ITINERE 10.01.2018

SOLUZIONE

Quesito (2 CFU)

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA RIPORTATA IN FIGURA E TRACCIARE IL DIAGRAMMA DEI MOMENTI



Posizioni:

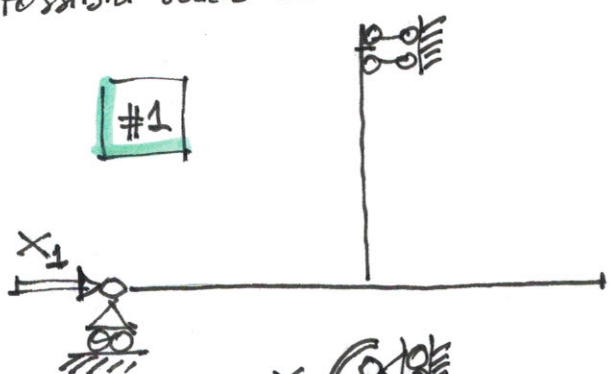
$|P| = qL$

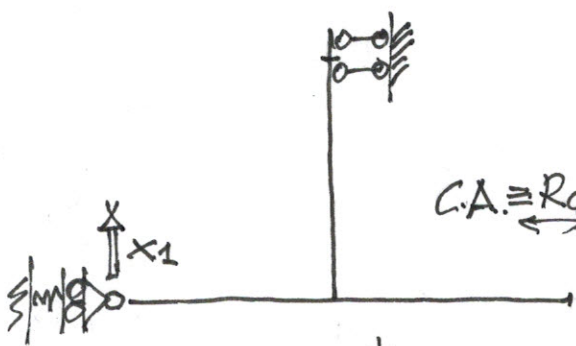
$|\varepsilon| = \frac{2}{3}\frac{L^3}{EI}$

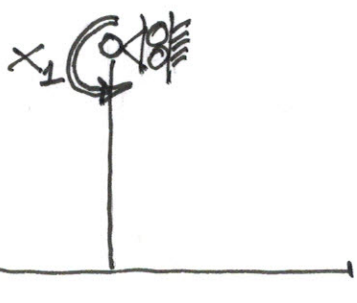
$|\varphi_A^0| = \frac{3}{4}\frac{qL^3}{EI}$

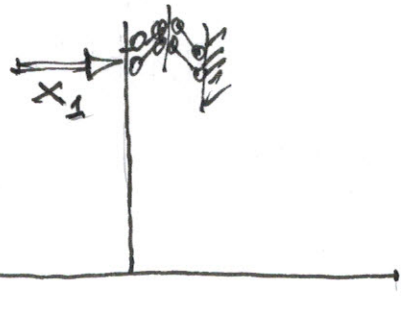
$\left|\alpha\frac{\overline{\Delta T}}{h}\right| = 2\frac{qL^2}{EI}$

➡ POSSIBILI SCELTE DEL SISTEMA PRINCIPALE ISOSTATICO:



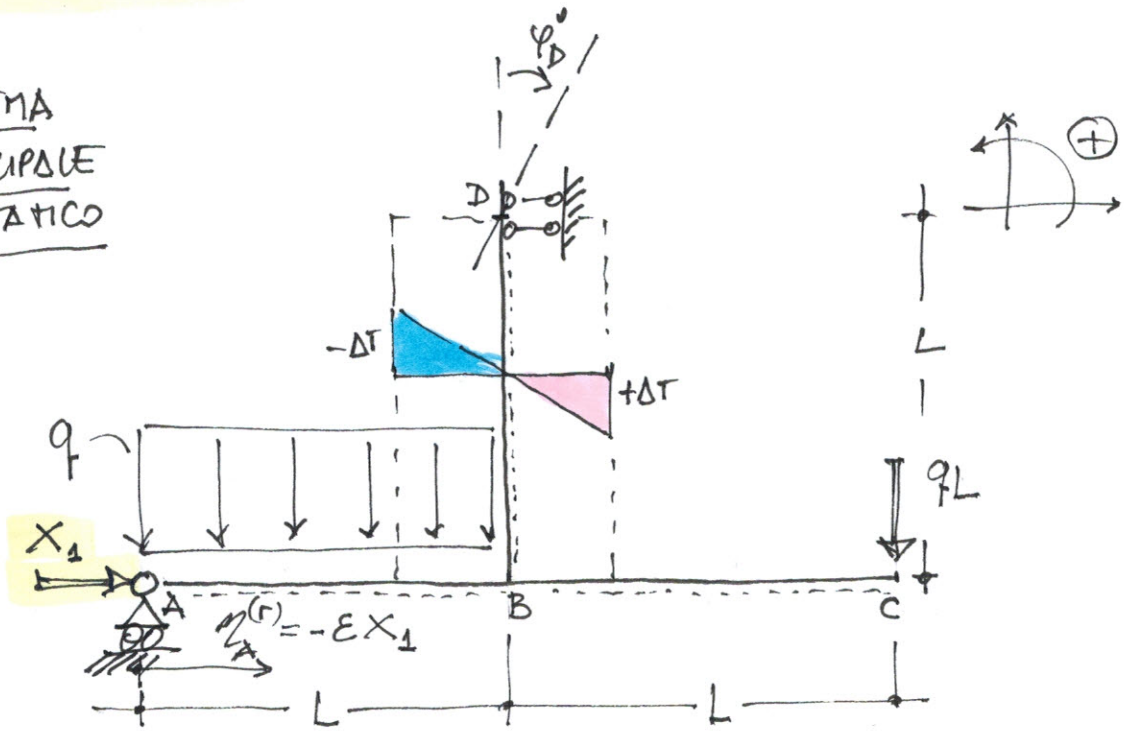




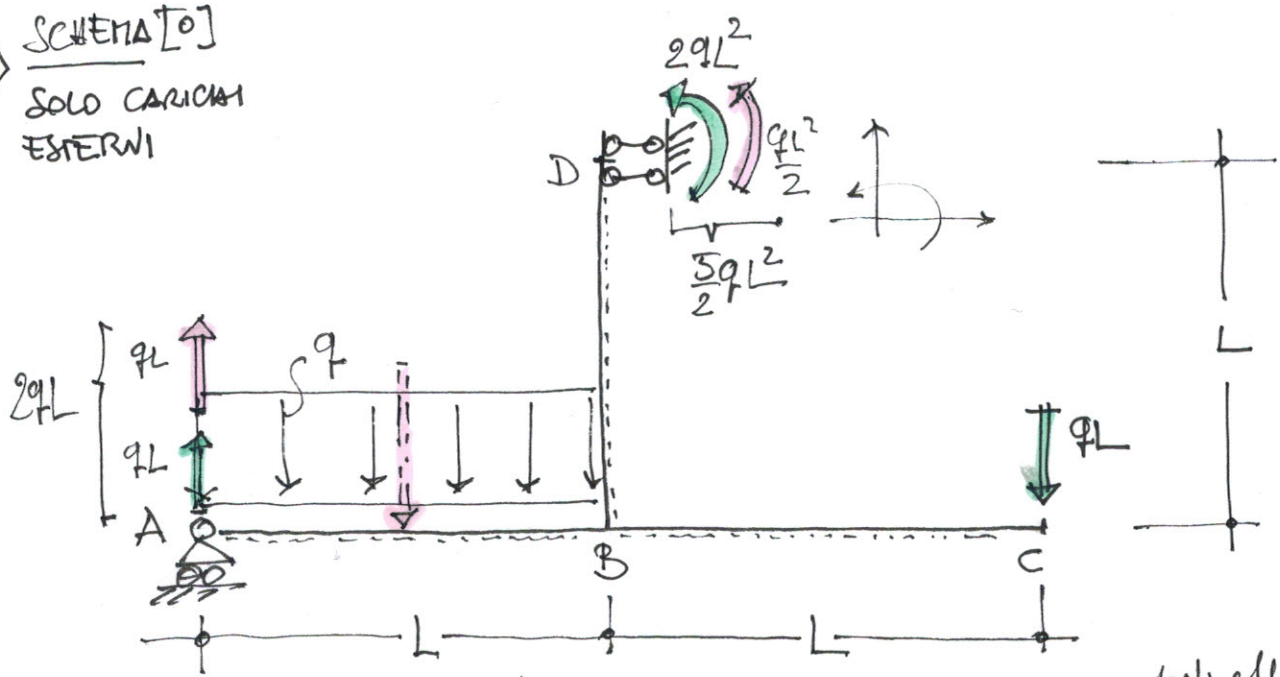


SOLUZIONE #1

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]
SOLO CARICHI ESTERNI



I. Si calcolano le RV con metodo grafico e princ. di sovrapposizione degli effetti

II. Si calcola $\varphi^{(0)}(x)$ nei singoli tratti. Si ha:

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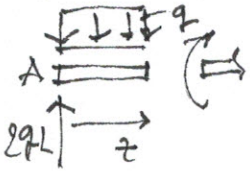
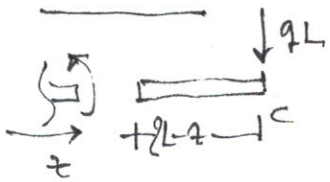
TRATTO AB $0 \le z \le L$



$$M^{(0)}(z) = 2qL \cdot z - \frac{qz^2}{2}$$

$$\begin{cases} M_A = 0 \\ M_B = 2qL^2 - \frac{qL^2}{2} = \frac{3}{2}qL^2 \end{cases}$$

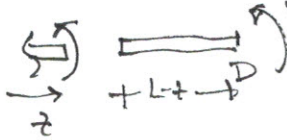
TRATTO BC $L \le z \le 2L$



$$M^{(0)}(z) = -qL(2L - z)$$

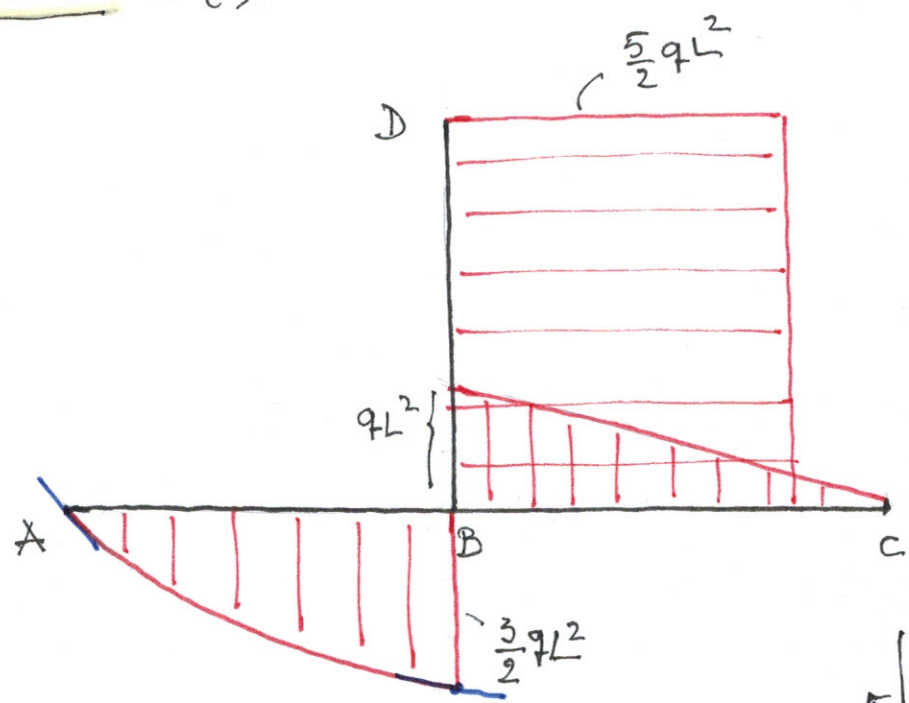
$$\begin{cases} M_B = -qL^2 \\ M_C = 0 \end{cases}$$

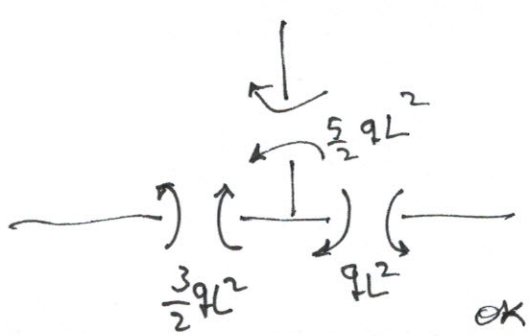
TRATTO BD $0 \le z \le L$



$$M^{(0)}(z) = \frac{5}{2}qL^2$$ costante!

DIAGRAMMA $M^{(0)}(z)$





OK!

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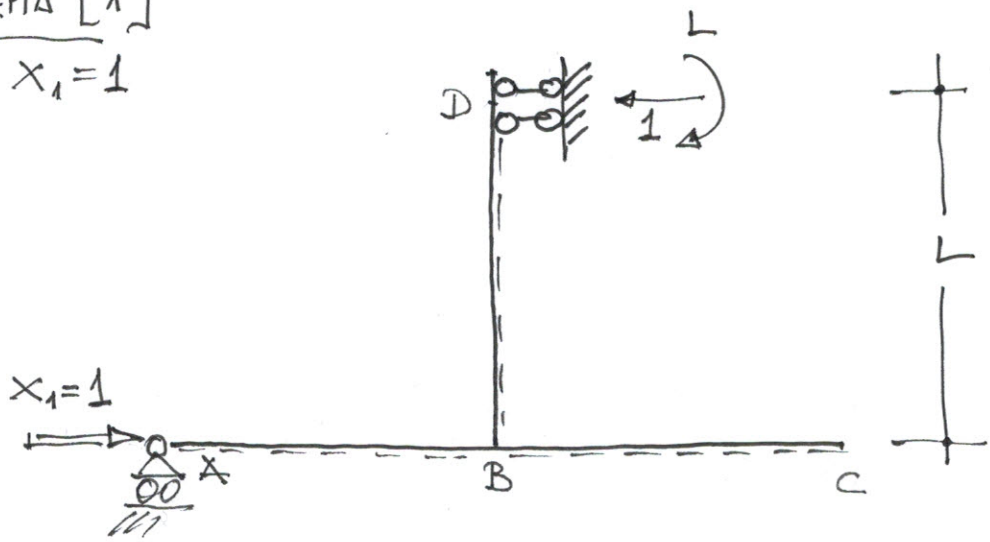
SCHEMA [1]

Solo $X_1 = 1$

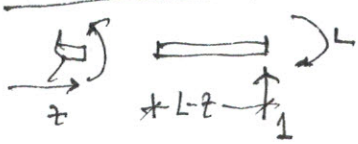


TRATTO AB $0 \le z \le L$

TRATTO BC $L \le z \le L$

$$M^{(1)}(z) = \phi$$

TRATTO BD $0 \le z \le L$

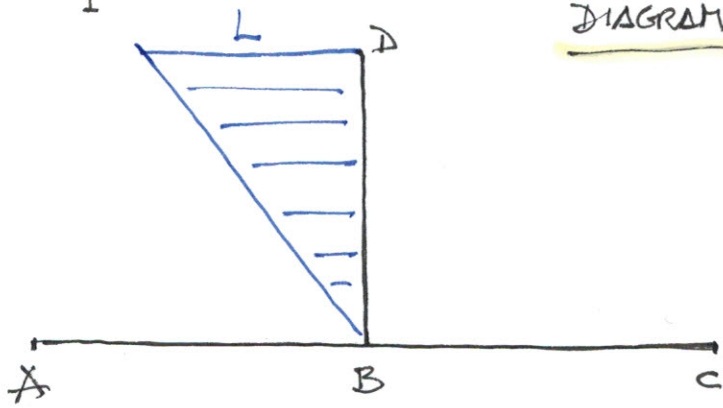


$$M^{(1)}(z) = (L - z) - L = -z$$

$M_B = \phi$

$M_D = -L$

DIAGRAMMA $M^{(1)}(z)$



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} = 1 \cdot [-\varepsilon X_1] + (-L)(-\varphi_D^0) =$$

$$= -\varepsilon X_1 + L\varphi_D^0$$

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$$L_{vi} = \int_{Str} M^{(i)} \frac{M^{(r)}}{EI}\, dStr + \int_{Str} M^{(i)} \alpha \frac{\overline{\Delta T}}{h}\, dStr =$$

($M^{(i)}$ → $M^{(0)} + M^{(1)} X_1$)

$$= \int_{Str} M^{(i)} \frac{M^{(0)}}{EI}\, dStr + X_1 \int_{Str} \frac{[M^{(1)}]^2}{EI}\, dStr + \int_{Str} M^{(i)} \alpha \frac{\overline{\Delta T}}{h}\, dStr =$$

$$= \frac{1}{EI} \left\{ \int_{\overline{BD}} M^{(i)} M^{(0)}\, dz \right\} + \frac{X_1}{EI} \int_{\overline{BD}} [M^{(1)}]^2\, dz + \int_{\overline{BD}} M^{(i)} \alpha \frac{\overline{\Delta T}}{h}\, dz =$$

$$= \frac{1}{EI} \int_0^L [-z] \left[\frac{5}{2} q L^2\right] dz + \frac{X_1}{EI} \int_0^L z^2\, dz + \int_0^L [-z] \alpha \frac{\overline{\Delta T}}{h}\, dz =$$

$$= \frac{1}{EI} \left[-\frac{5}{2} q L^2\right] \left[\frac{z^2}{2}\right]_0^L + \frac{X_1}{EI} \left[\frac{z^3}{3}\right]_0^L - \alpha \frac{\overline{\Delta T}}{h} \left[\frac{z^2}{2}\right]_0^L =$$

$$= -\frac{5 q L^2}{2EI} \cdot \frac{L^2}{2} + \frac{X_1}{3EI} L^3 - \alpha \frac{\overline{\Delta T}}{h} \frac{L^2}{2} = -\frac{5}{4} \frac{q L^4}{EI} + \frac{X_1}{3EI} L^3 - \alpha \frac{\overline{\Delta T}}{h} \frac{L^2}{2}$$

⇒ In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\varepsilon X_1 + L \varphi_D^0 = -\frac{5}{4} \frac{q L^4}{EI} + \frac{X_1}{3EI} L^3 - \alpha \frac{\overline{\Delta T}}{h} \frac{L^2}{2}$$

tenendo conto delle posizioni iniziali si ha:

$$X_1 \left[\frac{L^3}{3EI} + \frac{2}{3} \frac{L^3}{EI}\right] = \frac{3}{4} \frac{q L^4}{EI} + \frac{5}{4} \frac{q L^4}{EI} + \frac{q L^4}{EI}$$

$$X_1 \frac{L^3}{EI} = \frac{12}{4} \frac{q L^4}{EI} \Rightarrow X_1 = 3 q L$$

positiva! verso ipotizzato corretto

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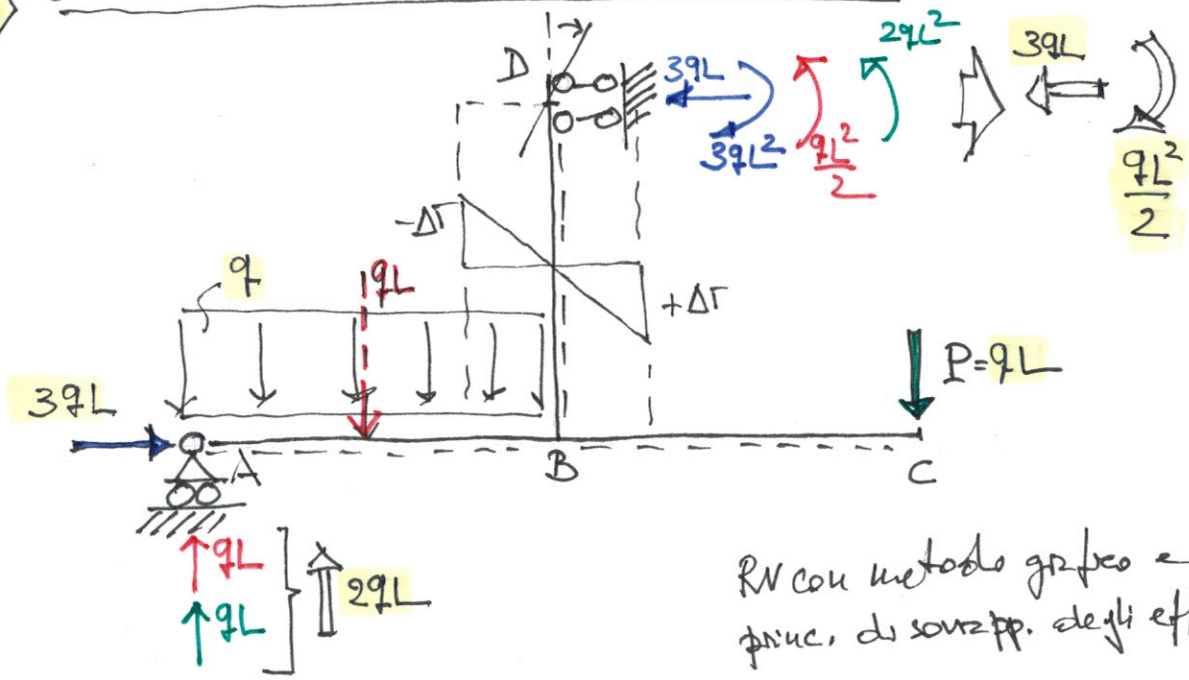
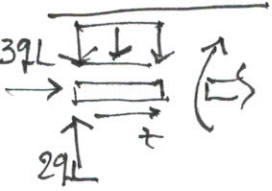
SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



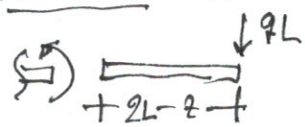
RV con metodo grafico ← princ. di sovrapp. degli effetti !

TRATTO AB $0 \le z \le L$



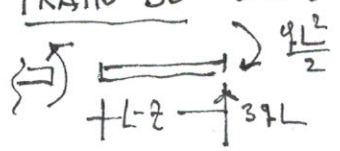
$$M^{(r)}(z) = 2qL \cdot z - q\frac{z^2}{2} \begin{cases} M_A = 0 \\ M_B = \frac{3}{2}qL^2 \end{cases}$$

TRATTO BC $L \le z \le 2L$

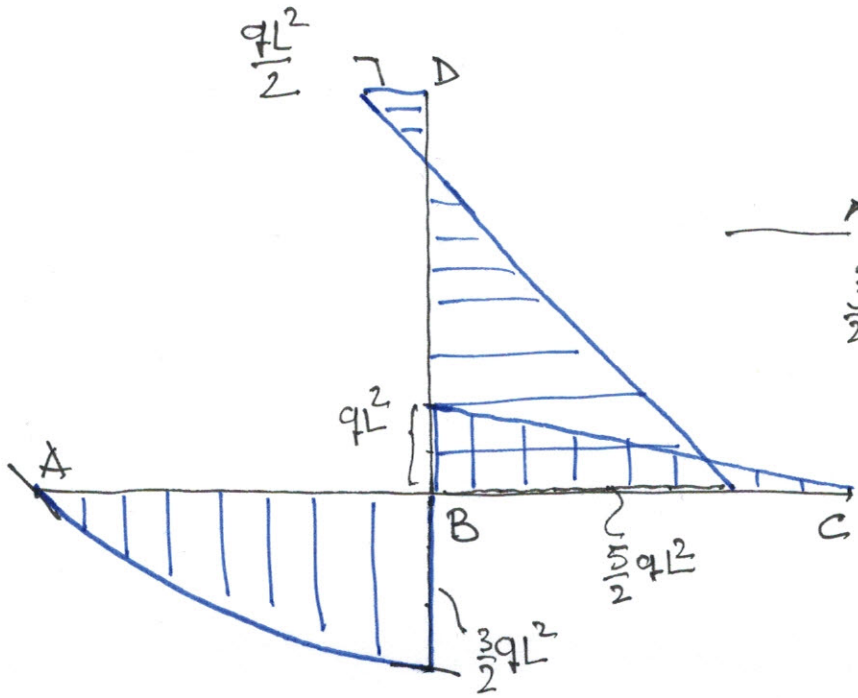


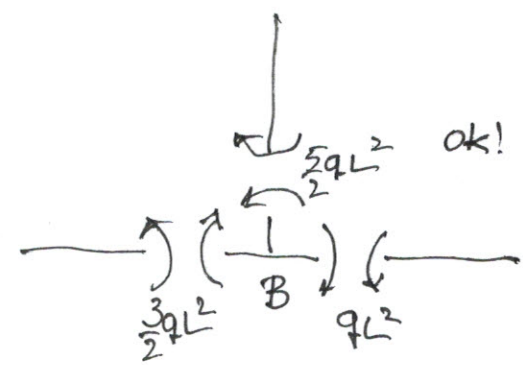
$$M^{(r)}(z) = -qL(2L-z) \begin{cases} M_B = -qL^2 \\ M_C = 0 \end{cases}$$

TRATTO BD $0 \le z \le L$



$$M^{(r)}(z) = -\frac{qL^2}{2} + 3qL(L-z) \begin{cases} M_B = \frac{5}{2}qL^2 \\ M_D = -\frac{qL^2}{2} \end{cases}$$





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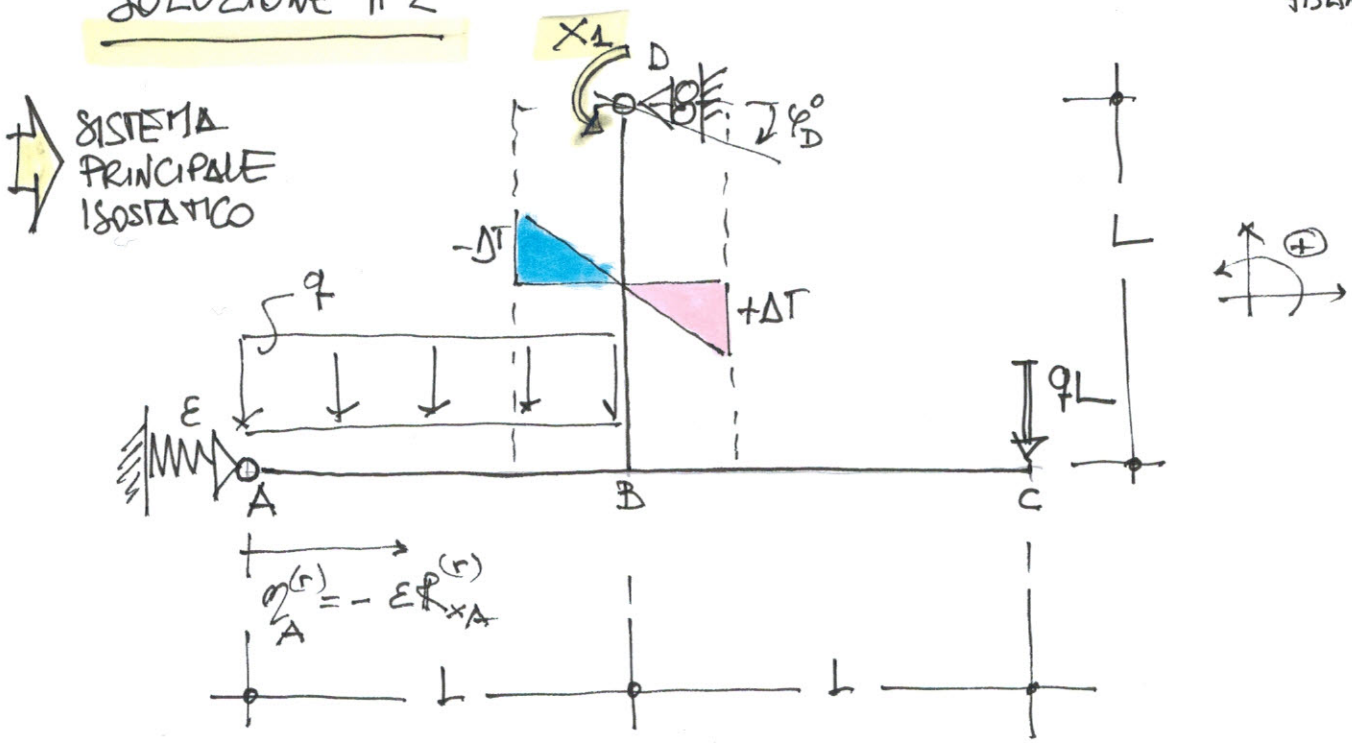
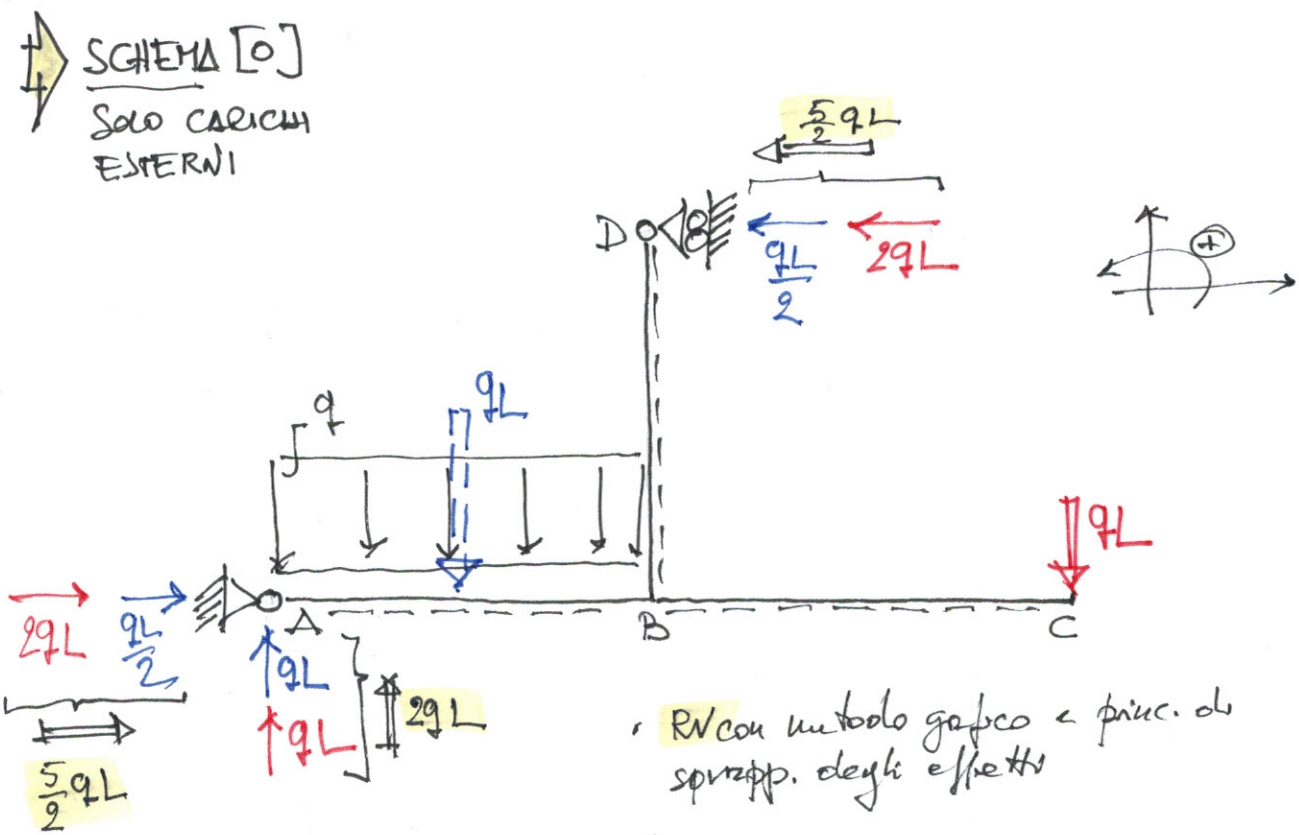
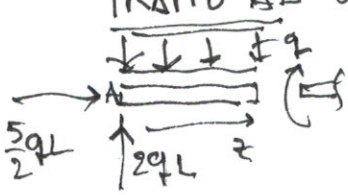
SOLUZIONE #2





• RV con metodo grafico e princ. di sovrapp. degli effetti

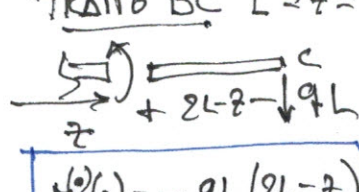
TRATTO AB $0 \le z \le L$



$$M^{(0)}(z) = 2qL \cdot z - q\frac{z^2}{2}$$

$$\begin{cases} M_A = 0 \\ M_B = \frac{3}{2}qL^2 \end{cases}$$

TRATTO BC $L \le z \le 2L$



$$M^{(0)}(z) = -qL(2L - z)$$

$$\begin{cases} M_B = -qL^2 \\ M_C = 0 \end{cases}$$

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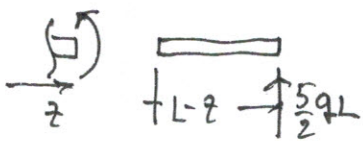
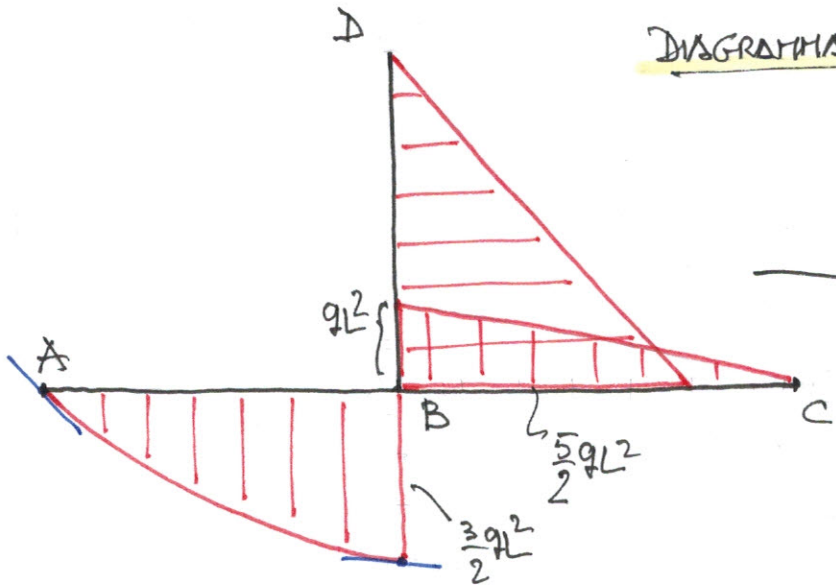
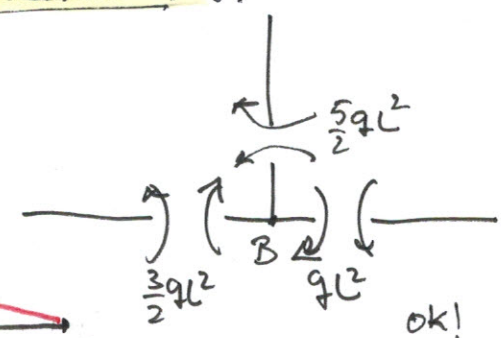
TRATTO BD $0 \le z \le L$



$$M^{(0)}(z) = \frac{5}{2}qL(L-z) \quad \begin{cases} M_B = \frac{5}{2}qL^2 \\ M_D = 0 \end{cases}$$

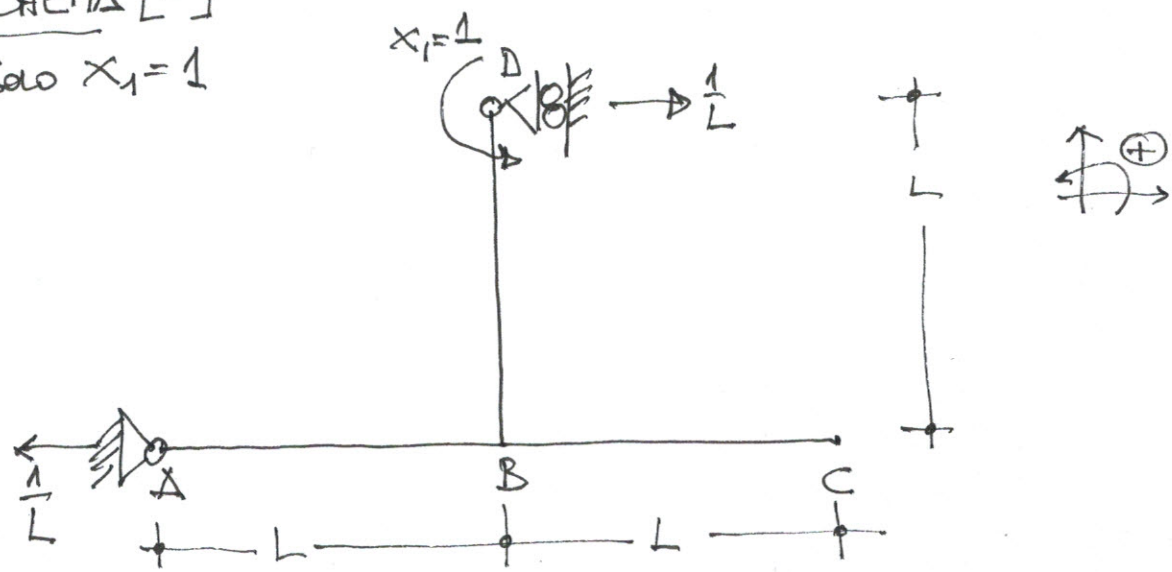
DIAGRAMMA $M^{(0)}(z)$





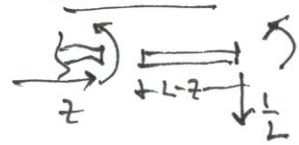
SCHEMA [1]

Solo $X_1 = 1$



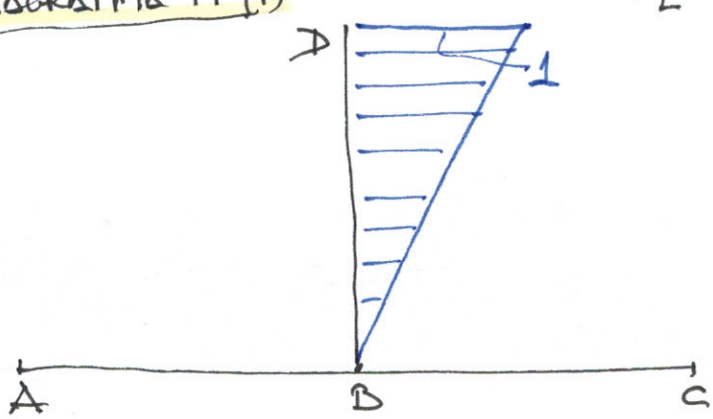
TRATTO AB
TRATTO BC
$\} \; M^{(1)}(z) = 0$

TRATTO BD $0 \le z \le L$



$$M^{(1)}(z) = 1 - \frac{(L-z)}{L} = \frac{z}{L} \quad \begin{cases} M_B = 0 \\ M_D = 1 \end{cases}$$

DIAGRAMMA $M^{(1)}(z)$



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L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante e fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1\left(-\varphi_D^0\right) + \underbrace{R_{x_A}^{(1)}}_{-\frac{1}{L}} \cdot \underbrace{\eta_A^{(r)}}_{-\varepsilon R_{x_A}^{(r)}} = -\varphi_D^0 + \frac{\varepsilon}{L}\left[\frac{5}{2}qL - \frac{X_1}{L}\right]$$

$$R_{x_A}^{(r)} = \underbrace{R_{x_A}^{(0)}}_{\frac{5}{2}qL} + X_1 \underbrace{R_{x_A}^{(1)}}_{-\frac{1}{L}}$$

$$L_{vi} = \int_{Str} \underbrace{M^{(f)}}_{M^{(1)}} \frac{\overbrace{M^{(r)}}^{M^{(0)}+M^{(1)}X_1}}{EI} dStr + \int_{Str} M^{(f)} \alpha \frac{\overline{\Delta T}}{h} dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} dStr + \int_{Str} M^{(1)} \alpha \frac{\overline{\Delta T}}{h} dStr =$$

$$= \frac{1}{EI}\left\{\int_{\overline{BD}} M^{(1)} M^{(0)} dz\right\} + \frac{X_1}{EI}\int_{\overline{BD}} \left[M^{(1)}\right]^2 dz + \int_{\overline{BD}} M^{(1)} \alpha \frac{\overline{\Delta T}}{h} dz =$$

$$= \frac{1}{EI}\int_0^L \frac{z}{L} \cdot \frac{5}{2} qL (L-z)\, dz + \frac{X_1}{EI}\int_0^L \left[\frac{z}{L}\right]^2 dz + \alpha\frac{\overline{\Delta T}}{h}\int_0^L \frac{z}{L} dz =$$

$$= \frac{1}{EI}\int_0^L \left[\frac{5}{2}qLz - \frac{5}{2}qz^2\right] dz + \frac{X_1}{EI L^2}\left[\frac{z^3}{3}\right]_0^L + \alpha\frac{\overline{\Delta T}}{h}\left[\frac{z^2}{2}\right]_0^L =$$

$$= \frac{1}{EI}\left\{\frac{5}{2}qL\left[\frac{z^2}{2}\right]_0^L - \frac{5}{2}q\left[\frac{z^3}{3}\right]_0^L\right\} + \frac{X_1}{EI}\frac{L^3}{3} + \alpha\frac{\overline{\Delta T}}{h}\frac{L}{2} =$$

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$$= \frac{1}{EI}\left\{\frac{5}{2}qL\cdot\frac{L^2}{2} - \frac{5}{2}q\frac{L^3}{3}\right\} + X_1\frac{L}{3EI} + \alpha\frac{\overline{\Delta T}}{h}\cdot\frac{L}{2} =$$

$$= \frac{5}{12}\frac{qL^3}{EI} + \frac{X_1 L}{3EI} + \alpha\frac{\overline{\Delta T}}{h}\cdot\frac{L}{2}$$

⇒ In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\varphi_D^0 + \frac{\varepsilon}{L}\left[\frac{5}{2}q - \frac{X_1}{L^2}\right] = \frac{5}{12}\frac{qL^3}{EI} + \frac{X_1 L}{3EI} + \alpha\frac{\overline{\Delta T}}{h}\cdot\frac{L}{2}$$

quest'ultima, tenendo conto della posizione iniziale, si scrive:

$$-\frac{3}{4}\frac{qL^3}{EI} + \frac{2}{3}\frac{L^3}{EI}\left[\frac{5}{2}q - \frac{X_1}{L^2}\right] = \frac{5}{12}\frac{qL^3}{EI} + \frac{X_1 L}{3EI} + \frac{2qL^2}{EI}\frac{L}{2}$$

$$\frac{11}{12}qL^3 - \frac{5}{12}qL^3 - qL^3 = X_1\left\{\frac{L}{3} + \frac{2}{3}L\right\}$$

$$-\frac{qL^3}{2} = X_1 L \quad \Rightarrow \quad X_1 = -\frac{qL^2}{2}$$

Negativa!
Verso opposto a quello ipotizzato!
OK! cfr con RV calcolate a pag. VI

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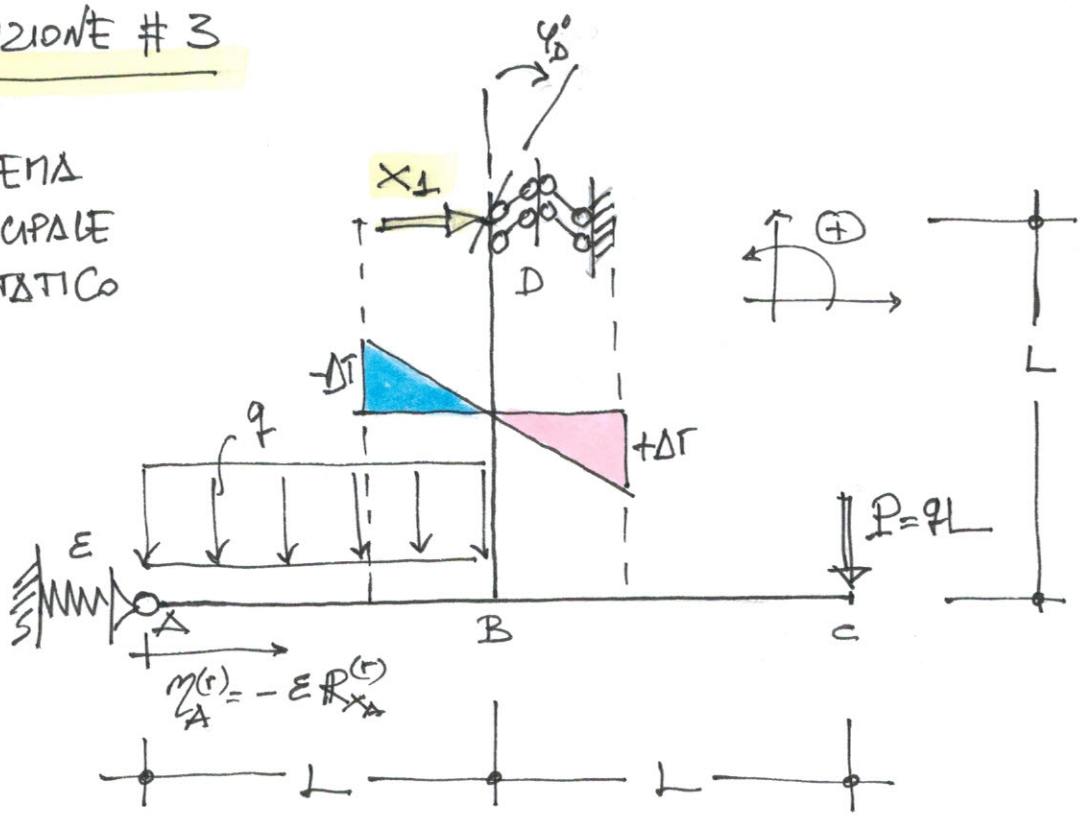
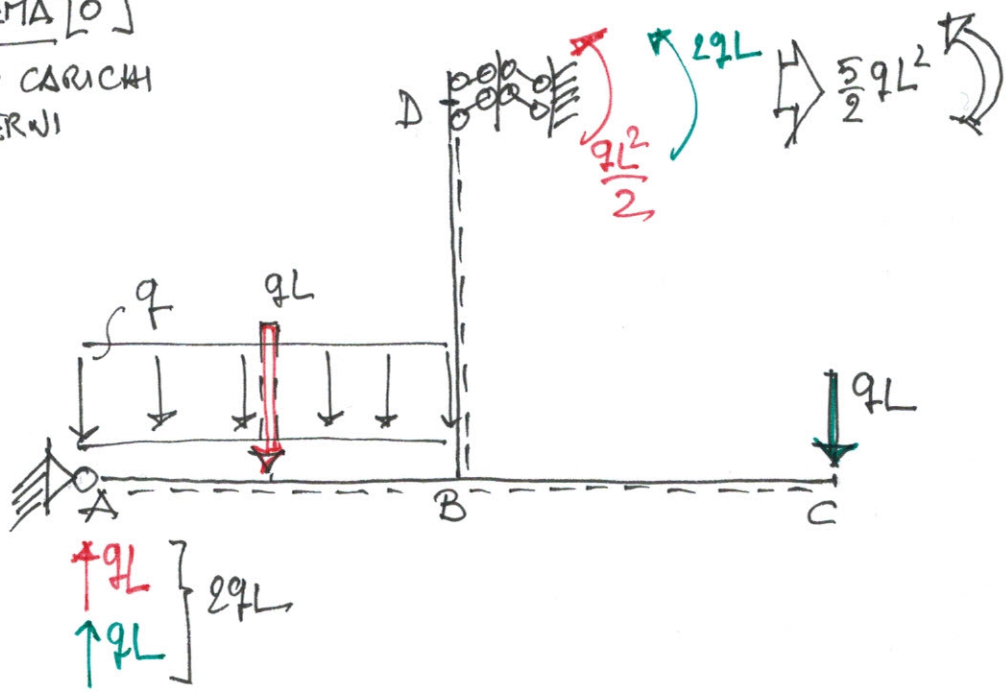
SOLUZIONE #3

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]
SOLO CARICHI ESTERNI



Si ottiene lo stesso schema [0] della Soluzione #1.

Valgono funzioni e diagrammi già calcolati a pp. II e III.

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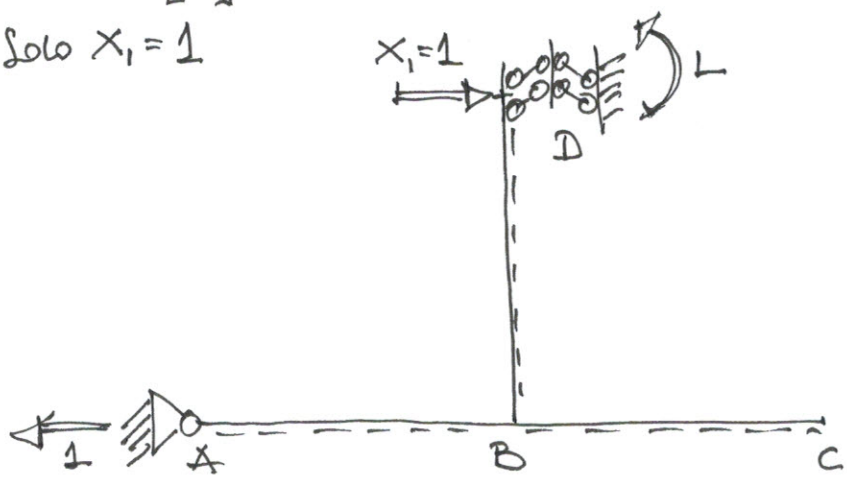
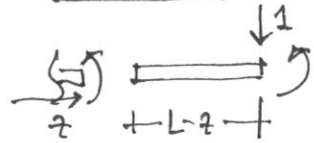
SCHEMA [1]
Solo $X_1 = 1$



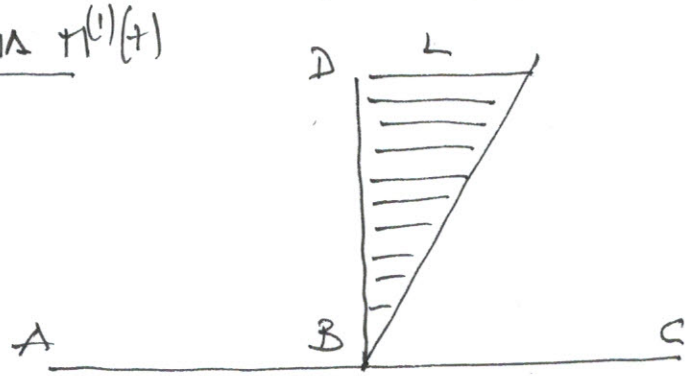
TRATTO AB
TRATTO BC
$\Big\}\ M^{(1)}(z) = \phi$

TRATTO BD $0 \le z \le L$



$M^0(z) = L - (L-z) = z \begin{cases} M_B = \phi \\ M_D = L \end{cases}$

DIAGRAMMA $M^{(1)}(z)$



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{ri}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \phi + R_{x_A}^{(1)} \cdot \eta_A^{(r)} + M_D^{(1)} \varphi_D^0 =$$

$$= -1 \cdot \underbrace{\eta_A^{(r)}}_{-\varepsilon R_{x_A}^{(r)}} + L\left(-\varphi_D^0\right) = -\varepsilon X_1 - L\varphi_D^0$$

$$R_{x_A}^{(r)} = \underbrace{R_{x_A}^{(0)}}_{\phi''} + X_1 \underbrace{R_{x_A}^{(1)}}_{=-1}$$

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$$L_{vi} = \int_{Str} \underbrace{M^{(f)}}_{M^{(1)}} \underbrace{\frac{M^{(r)}}{EI}}_{M^{(0)}+M^{(1)}X_1} dStr + \int_{Str} M^{(f)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + \int_{Str} \frac{[M^{(1)}]^2}{EI} X_1 \, dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$= \frac{1}{EI} \int_{\overline{BD}} M^{(1)} M^{(0)} dz + \frac{X_1}{EI} \int_{\overline{BD}} [M^{(1)}]^2 dz + \int_{\overline{BD}} M^{(1)} \cdot \frac{\alpha \overline{\Delta T}}{h} dz =$$

$$= \frac{1}{EI} \int_0^L z \cdot \frac{5}{2} q L^2 dz + \frac{X_1}{EI} \int_0^L z^2 dz + \frac{\alpha \overline{\Delta T}}{h} \int_0^L z \, dz =$$

$$= \frac{1}{EI} \cdot \frac{5}{2} q L^2 \left[\frac{z^2}{2}\right]_0^L + \frac{X_1}{EI} \left[\frac{z^3}{3}\right]_0^L + \frac{\alpha \overline{\Delta T}}{h} \left[\frac{z^2}{2}\right]_0^L =$$

$$= \frac{1}{EI} \frac{5}{4} q L^4 + \frac{X_1}{3EI} L^3 + \frac{\alpha \overline{\Delta T}}{h} \frac{L^2}{2}$$

$\Rightarrow$ $L_{ve} = L_{vi}$ fornisce quindi:

$$-\varepsilon X_1 - L\varphi_D^0 = \frac{1}{EI} \frac{5}{4} q L^4 + \frac{1}{3EI} X_1 L^3 + \frac{\alpha \overline{\Delta T}}{h} \frac{L^2}{2}$$

tenendo conto delle posizioni iniziali si ha infine:

$$-\frac{2}{3} \frac{L^3}{EI} X_1 - \frac{3}{4} \frac{q L^4}{EI} = \frac{1}{EI} \frac{5}{4} q L^4 + \frac{L^3}{3EI} X_1 + \frac{2 q L^2}{EI} \frac{L^2}{2}$$

da cui:

$$X_1 = -3qL$$

verso opposto a quello ipotizzato!

Ok! cfr. con RV calcolati a p. VI.