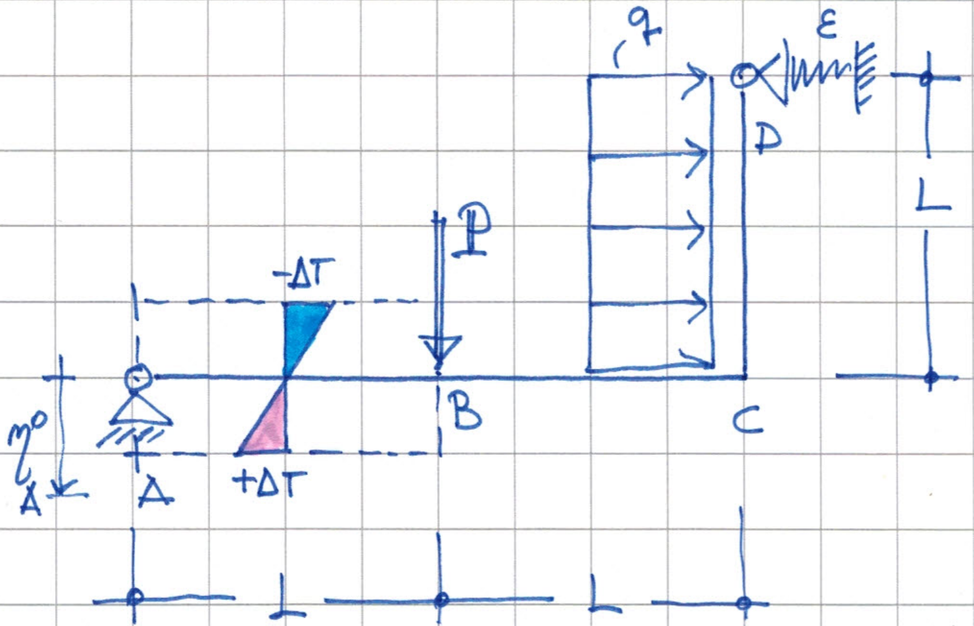
ES. #9 RISOLVERE LA STRUTTURA SEGUENTE 1 VOLTA IPERSTATICA :



Poni:

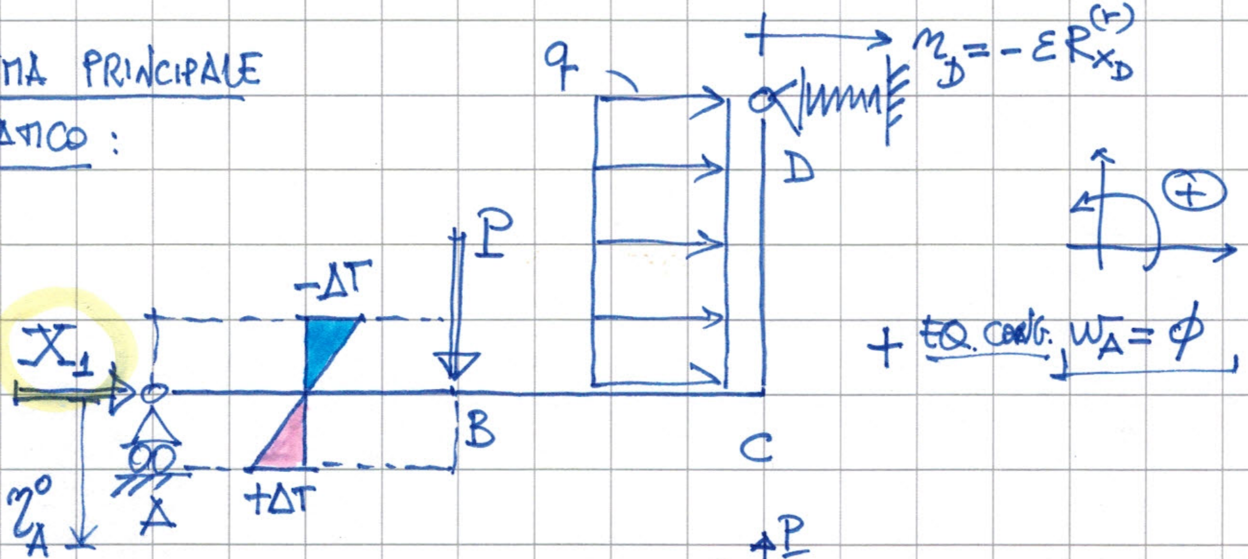
$|P| = qL$

$\left|\dfrac{\alpha \Delta T}{h}\right| = \dfrac{qL^2}{EI}$

$|\eta_A^0| = \dfrac{qL^4}{EI}$

$|\varepsilon| = \dfrac{L^3}{EI}$

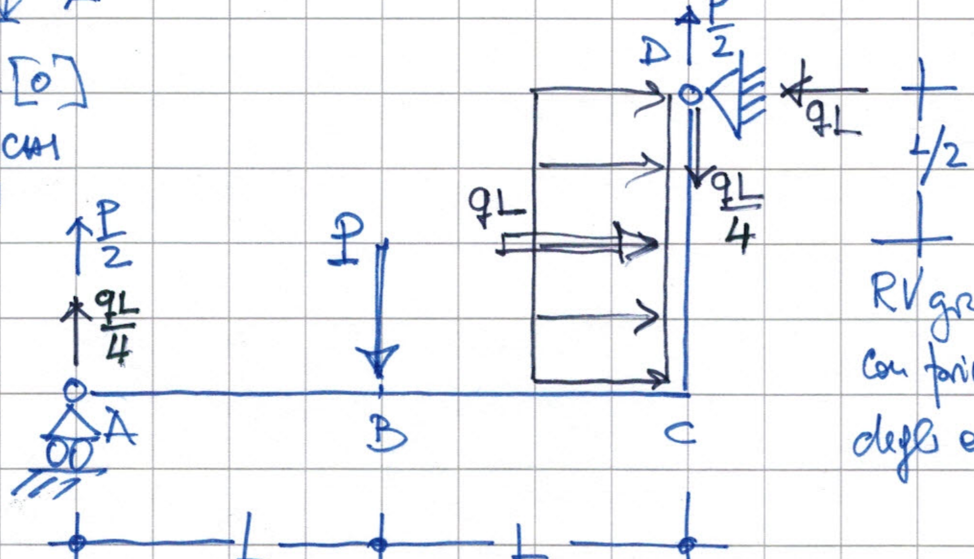
SISTEMA PRINCIPALE ISOSTATICO :



+ EQ. CONG.: $w_A = 0$

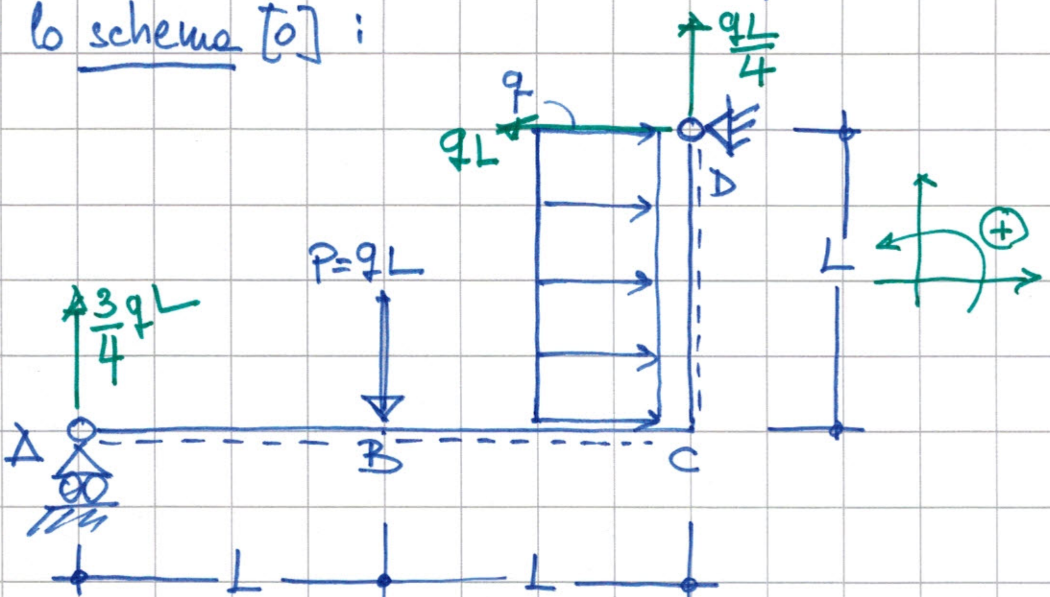
SCHEMA [0]

Solo carichi esterni



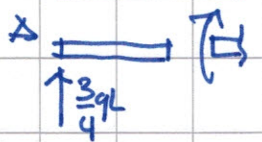
RV grafico con princ. di sovrapp. degli effetti.

Considerando che $|P| = qL$, si ha in definitiva per lo schema [0]:



Calcoliamo $M^{(0)}(z)$, si ha:

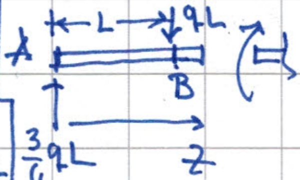
TRATTO AB $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{3}{4} qLz}$$

$$\begin{cases} M_A = M(z)\big|_{z=0} = 0 \\ M_B = M(z)\big|_{z=L} = \frac{3}{4} qL^2 \end{cases}$$

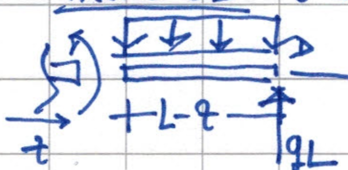
TRATTO BC $\quad L \le z \le 2L$



$$M^{(0)}(z) = \frac{3}{4} qLz - qL(z-L) =$$
$$= \frac{3}{4} qLz - qLz + qL^2 =$$
$$= \boxed{-\frac{qL}{4} z + qL^2}$$

$$\begin{cases} M_B = \frac{3}{4} qL^2 \\ M_C = \frac{qL^2}{2} \end{cases}$$

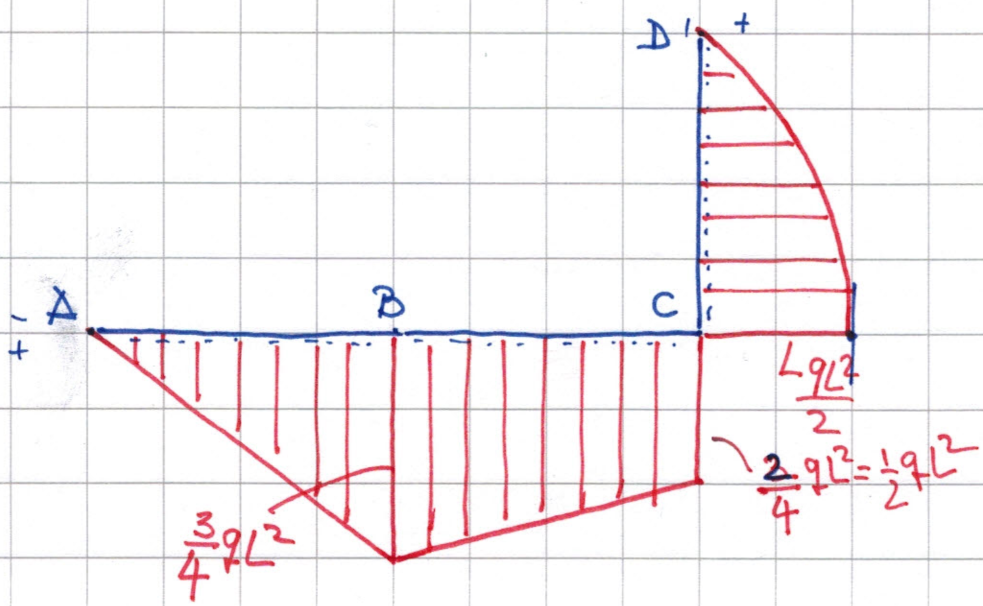
TRATTO CD $\quad 0 \le z \le L$



$$\frac{qL}{4}$$

$$M^{(0)}(z) = qL(L-z) - q\frac{(L-z)^2}{2} = \ldots$$
$$= \boxed{\frac{qL^2}{2} - \frac{q}{2} z^2}$$

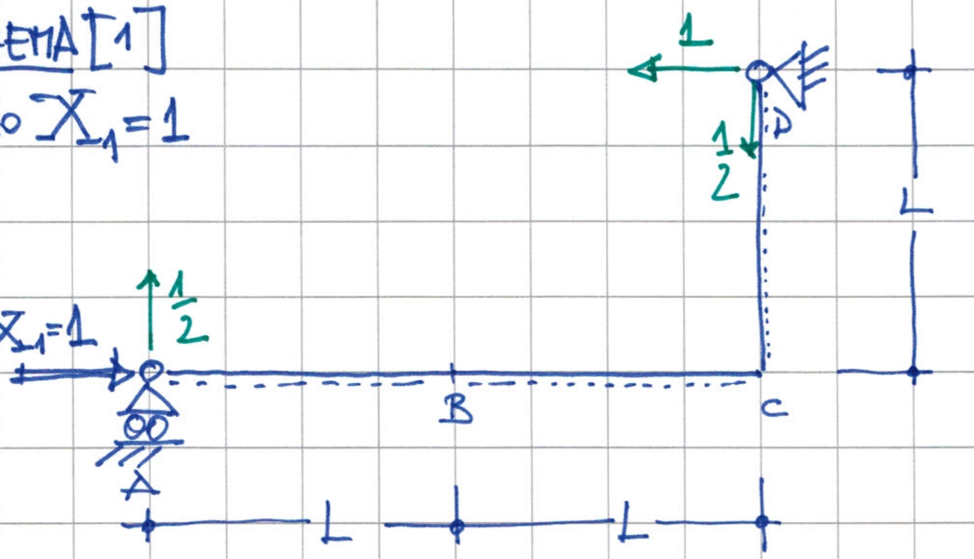
$$\begin{cases} M_C = \frac{qL^2}{2} \\ M_D = \emptyset \end{cases}$$



$M^{(0)}(z)$

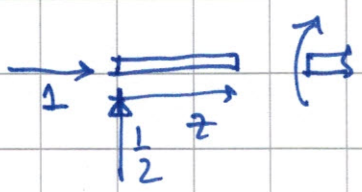
SCHEMA [1]

Solo $X_1 = 1$



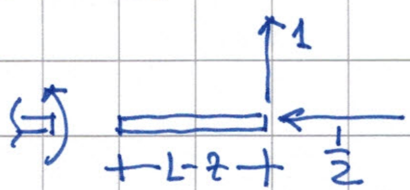
RV metodo grafico

TRATTO AC $\quad 0 \le z \le 2L$

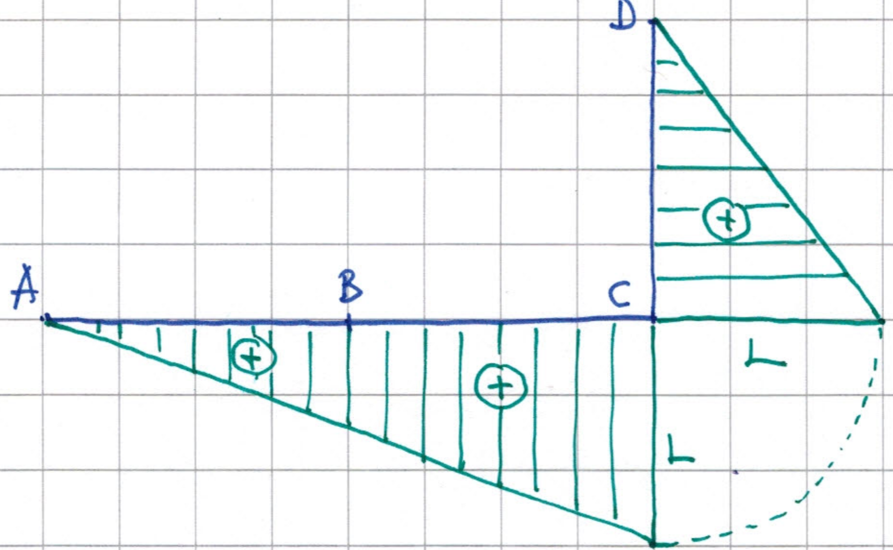


$$\boxed{M^{(1)}(z) = \frac{z}{2}} \quad \begin{cases} M_A = 0 \\ M_C = L \end{cases}$$

TRATTO CD $\quad 0 \le z \le L$

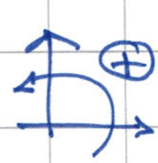


$$\boxed{M^{(1)}(z) = L - z} \quad \begin{cases} M_C = L \\ M_D = \emptyset \end{cases}$$



$M^{(1)}(z)$

L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive assumendo come sistema fittizio o lavorante lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:



$$L_{ve} = \overset{X_i^{(f)}=1}{1} \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \underbrace{w_A^{(r)}}_{=\emptyset} + \underbrace{R_{y_A}^{(1)} \cdot \eta_A^{0}}_{-\frac{\eta_A^0}{2}} + \underbrace{R_{x_D}^{(1)}}_{-1} \cdot \underbrace{\eta_D^{(r)}}_{-\varepsilon R_{x_D}^{(r)}}$$

vincolo perfetto nella direzione della $X_1 = 1$ (orizzontale)

discordi

$$-\varepsilon \underbrace{R_{x_D}^{(r)}}_{R_{x_D}^{(0)} + R_{x_D}^{(1)} \cdot X_1}$$

$$\underbrace{-\varepsilon\left[-qL - X_1\right]}_{-\varepsilon\left[qL + X_1\right]}$$

$$= -\frac{\eta_A^0}{2} - \varepsilon\left[qL + X_1\right]$$

$$L_{vi} = \int_{Str} M^{(f)} \underbrace{\frac{M^{(r)}}{EI}}\, dStr + \int_{Str.} M^{(f)} \underbrace{\frac{\alpha \Delta T}{h}}_{>0} dStr. =$$

$\hookrightarrow$ con: $M^{(r)} = M^{(0)} + M^{(1)} X_1$

$$= \int_{Str} \frac{M^{(1)} M^{(0)}}{EI} dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} dStr. + \int_{Str} \underbrace{M^{(1)}}_{>0} \alpha \underbrace{\frac{\Delta T}{h}}_{>0} dStr =$$

$$= \frac{1}{EI}\left\{ \int_{\overline{AB}} \overbrace{\frac{z}{2}\left[\frac{3}{4}qLz\right]}^{\frac{3}{8}qLz^2} dz + \int_{\overline{BC}} \overbrace{\frac{z}{2}\left[-\frac{qL}{4}z+qL^2\right]}^{-\frac{qL}{8}z^2+\frac{qL^2}{2}z} dz + \int_{\overline{CD}} \overbrace{(L-z)\left[\frac{qL^2}{2}-\frac{q}{2}z^2\right]}^{\frac{qL^3}{2}-\frac{qL}{2}z^2-\frac{qL^2}{2}z+\frac{q}{2}z^3} dz \right\} +$$

$$+\frac{X_1}{EI}\left\{\int_{\overline{AC}}\frac{z^2}{4}dz + \int_{\overline{CD}} \overbrace{(L-z)^2}^{L^2+z^2-2Lz} dz\right\} + \frac{\alpha\overline{\Delta T}}{h}\int_{\overline{AB}}\frac{z}{2}dz =$$

$$=\frac{1}{EI}\left\{\int_0^L \frac{3}{8}qLz^2dz + \int_L^{2L}\left[-\frac{qL}{8}z^2+\frac{qL^2}{2}z\right]dz + \int_0^L\left[\frac{qL^3}{2}-\frac{qL}{2}z^2-\frac{qL^2}{2}z+\frac{q}{2}z^3\right]dz\right\}+$$

$$+\frac{X_1}{EI}\left\{\int_0^{2L}\frac{z^2}{4}dz+\int_0^L\left[L^2+z^2-2Lz\right]dz\right\}+\frac{\alpha\overline{\Delta T}}{h}\int_0^L\frac{z}{2}dz =$$

$$=\frac{1}{EI}\left\{ \overbrace{\frac{\cancel{3}}{8}qL\left[\frac{z^3}{\cancel{3}}\right]_0^L}^{\frac{qL^4}{8}} \overbrace{-\frac{qL}{8}\left[\frac{z^3}{3}\right]_L^{2L}}^{-\frac{7}{24}qL^4} + \overbrace{\frac{qL^2}{2}\left[\frac{z^2}{2}\right]_L^{2L}}^{\frac{3}{4}qL^4} + \overbrace{\frac{qL^3}{2}\left[z\right]_0^L}^{\frac{qL^4}{2}} \overbrace{-\frac{qL}{2}\left[\frac{z^3}{3}\right]_0^L}^{-\frac{qL^4}{6}} + \right.$$

$$\left. \overbrace{-\frac{qL^2}{2}\left[\frac{z^2}{2}\right]_0^L}^{-\frac{qL^4}{4}} + \overbrace{\frac{q}{2}\left[\frac{z^4}{4}\right]_0^L}^{qL^4/8}\right\} + \frac{X_1}{EI}\left\{\overbrace{\frac{1}{4}\left[\frac{z^3}{3}\right]_0^{2L}}^{\frac{8}{12}L^3} + \overbrace{L^2\left[z\right]_0^L}^{\cancel{L^3}} + \overbrace{\left[\frac{z^3}{3}\right]_0^L}^{L^3/3} \overbrace{-\cancel{2}L\left[\frac{z^2}{\cancel{2}}\right]_0^L}^{\cancel{L^3}}\right\}$$

$$+\frac{\alpha\overline{\Delta T}}{2h}\underbrace{\left[\frac{z^2}{2}\right]_0^L}_{\frac{L^2}{2}} =$$

$$=\frac{qL^4}{EI}\underbrace{\left[\frac{1}{8}-\frac{7}{24}+\frac{3}{4}+\frac{1}{2}-\frac{1}{6}-\frac{1}{4}+\frac{1}{8}\right]}_{\frac{19}{24}}+\frac{X_1L^3}{EI}\left[\overbrace{\frac{8}{12}+\frac{1}{3}}^{1}\right]+\frac{\alpha\overline{\Delta T}L^2}{4h}=$$

$$=\frac{19}{24}\frac{qL^4}{EI}+\frac{X_1L^3}{EI}+\frac{\alpha\overline{\Delta T}L^2}{4h}$$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\frac{\eta_A^0}{2} - \varepsilon q L - \varepsilon X_1 = \frac{19}{24}\frac{qL^4}{EI} + X_1\frac{L^3}{EI} + \frac{\alpha\Delta T L^2}{4h}$$

$$X_1\left[\frac{L^3}{EI} + \varepsilon\right] = -\left[\frac{\eta_A^0}{2} + \varepsilon q L + \frac{19}{24}\frac{qL^4}{EI} + \frac{\alpha\Delta T L^2}{4h}\right]$$

da cui X NEGATIVA! ⇨ verso corretto opposto a quello ipotizzato!

⇨ Se teniamo conto delle posizioni iniziali che permettono di esprimere tutto in funzione di q, L ed EI e cioè

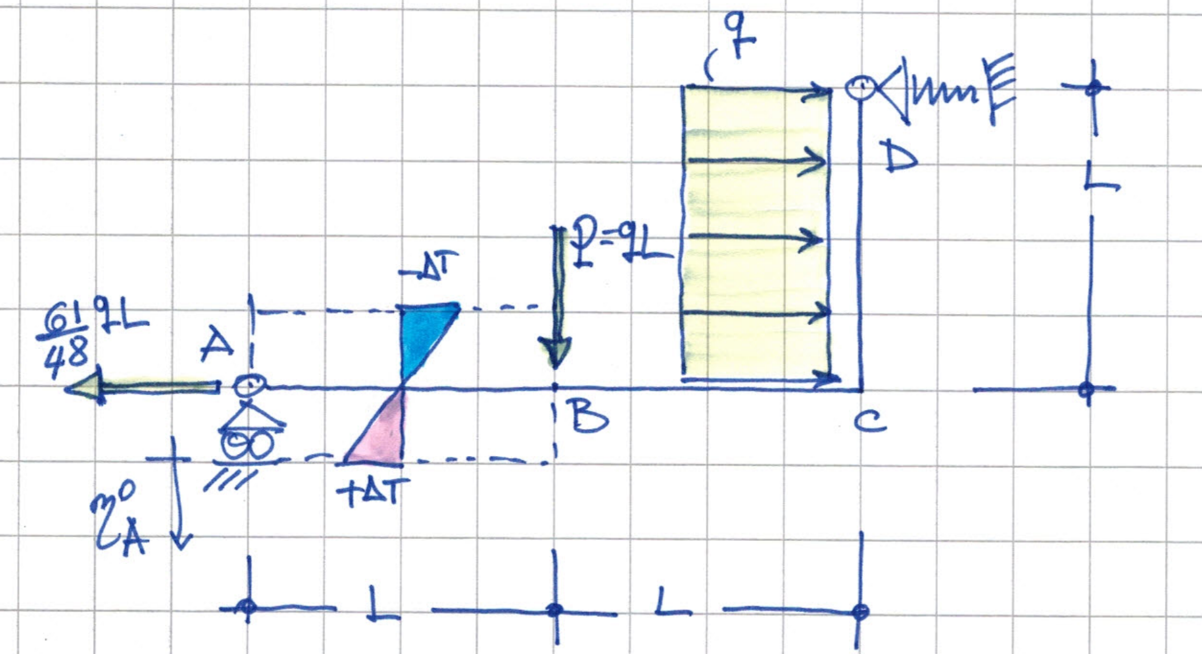
$$|\varepsilon| = \frac{L^3}{EI}\ ;\quad |\eta_A^0| = \frac{qL^4}{EI}\ ;\quad \left|\frac{\alpha\Delta T}{h}\right| = \frac{qL^2}{EI}$$

la precedente può scriversi:

$$X_1\underbrace{\left\{\frac{\cancel{L^3}}{\cancel{EI}} + \frac{\cancel{L^3}}{\cancel{EI}}\right\}}_{2} = \underbrace{-\left[\frac{qL^{\cancel{4}}}{2\cancel{EI}} + \frac{qL^{\cancel{4}}}{\cancel{EI}} + \frac{19}{24}\frac{qL^{\cancel{4}}}{\cancel{EI}} + \frac{qL^{\cancel{4}}}{4\cancel{EI}}\right]}_{-qL\underbrace{\left[\frac{1}{2}+1+\frac{19}{24}+\frac{1}{4}\right]}_{\frac{61}{24}}}$$

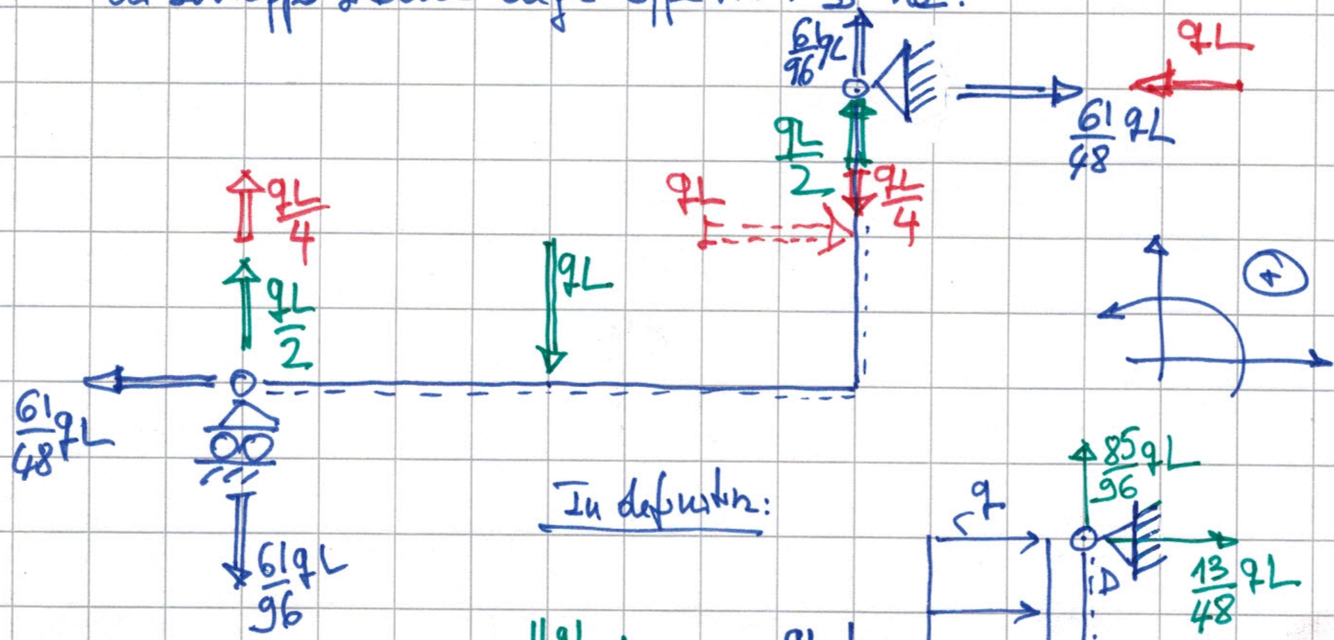
$$X_1 = -\frac{61}{48}qL$$

Le CS, ovvero il solo diagramma del momento (per brevità), sulla struttura iperstatica di partenza possono facilmente determinarsi lavorando sul sistema principale isostatico soggetto ai carichi assegnati e alla X_1 ormai nota! Si ha:

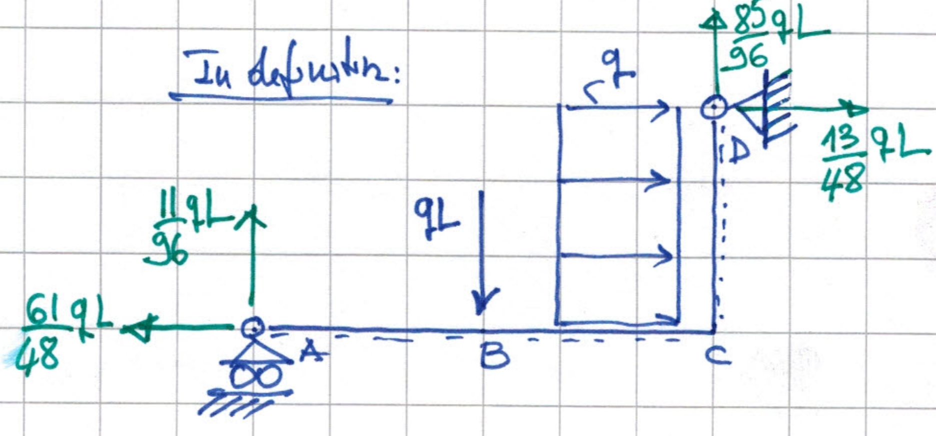


REM. CEDIMENTI & DISTORSIONI POSSONO MANIFESTARSI LIBERAMENTE! in questo schema ma la X_1 determinata ne ha tenuto conto!!

Le RV possono determinarsi per via grafica con il principio di sovrapposizione degli effetti! si ha:

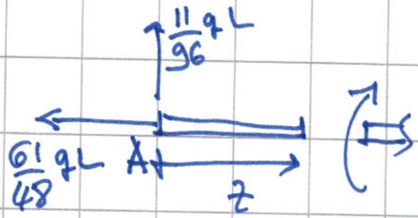


In definitiva:



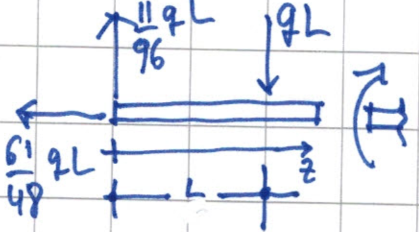
La funzione $M^{(r)}(z)$ può quindi valutarsi tratto per tratto. Si ha:

TRATTO AB $\quad 0 \le z \le L$



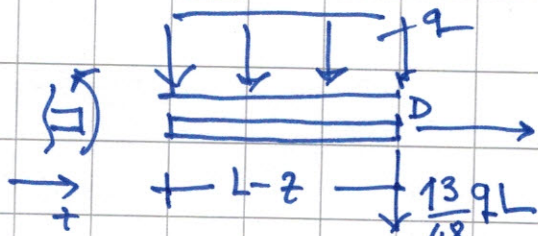
$$M^{(r)}(z) = \underbrace{\frac{11}{96} qL \cdot z}_{\text{LINEARE}} \qquad \begin{cases} M_A = 0 \\ M_B = \frac{11}{96} qL^2 \end{cases}$$

TRATTO BC $\quad L \le z \le 2L$



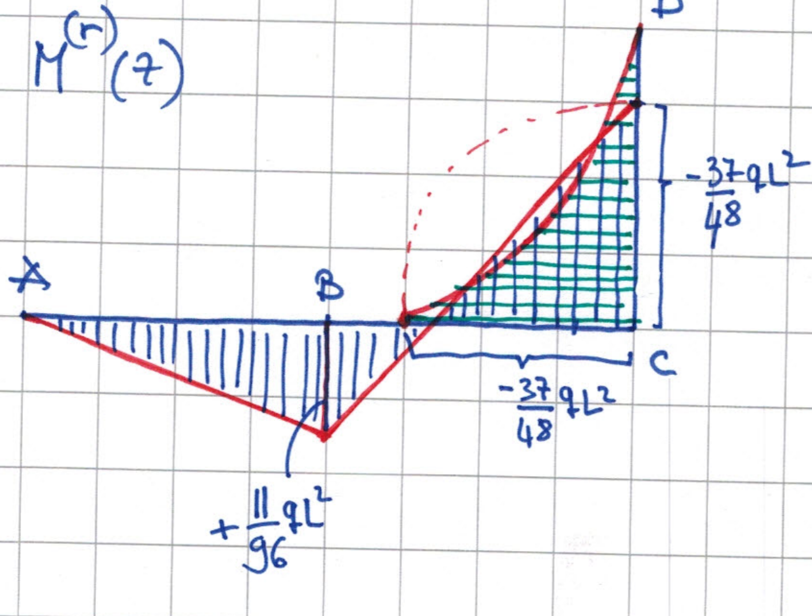
$$M^{(r)}(z) = \underbrace{\frac{11}{96} qL \cdot z - qL(z-L)}_{\text{LINEARE}} \qquad \begin{cases} M_B = \frac{11}{96} qL^2 \\ M_C = \dots = -\frac{37}{48} qL^2 \end{cases}$$

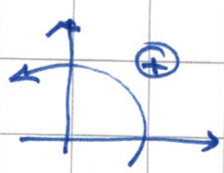
TRATTO CD $\quad 0 \le z \le L$



$$M^{(r)}(z) = \underbrace{-\frac{13}{48} qL \cdot (L-z) - \frac{q (L-z)^2}{2}}_{\text{QUADRATICO}} \qquad \begin{cases} M_C = \dots = -\frac{37}{48} qL^2 \\ M_D = \phi \end{cases}$$

Si ha in definitiva:





È facile verificare che:

$$R_{y_A}^{(r)} = R_{y_A}^{(0)} + R_{y_A}^{(1)} \cdot X_1 =$$

$$= \frac{3}{4} qL + \frac{1}{2}\left[-\frac{61}{48} qL\right] =$$

$$= qL\left[\frac{3}{4} - \frac{61}{96}\right] = qL\left[\frac{72-61}{96}\right] = \frac{11}{96} qL > 0$$ verso l'alto!

Lo stesso dicasi per le CS reali ovviamente, per esempio

$$M_B^{(r)} = M_B^{(0)} + M_B^{(1)} \cdot X_1 =$$

$$= \frac{3}{4} qL^2 + \frac{L}{2}\left(-\frac{61}{48} qL\right) = qL^2\left[\frac{3}{4} - \frac{61}{96}\right] = \frac{11}{96} qL^2 > 0$$