ESAMI di MECCANICA delle STRUTTURE - L17

Corso P.FUSCHI - A.A. 2016-2017 - prova scritta 27.09.2017

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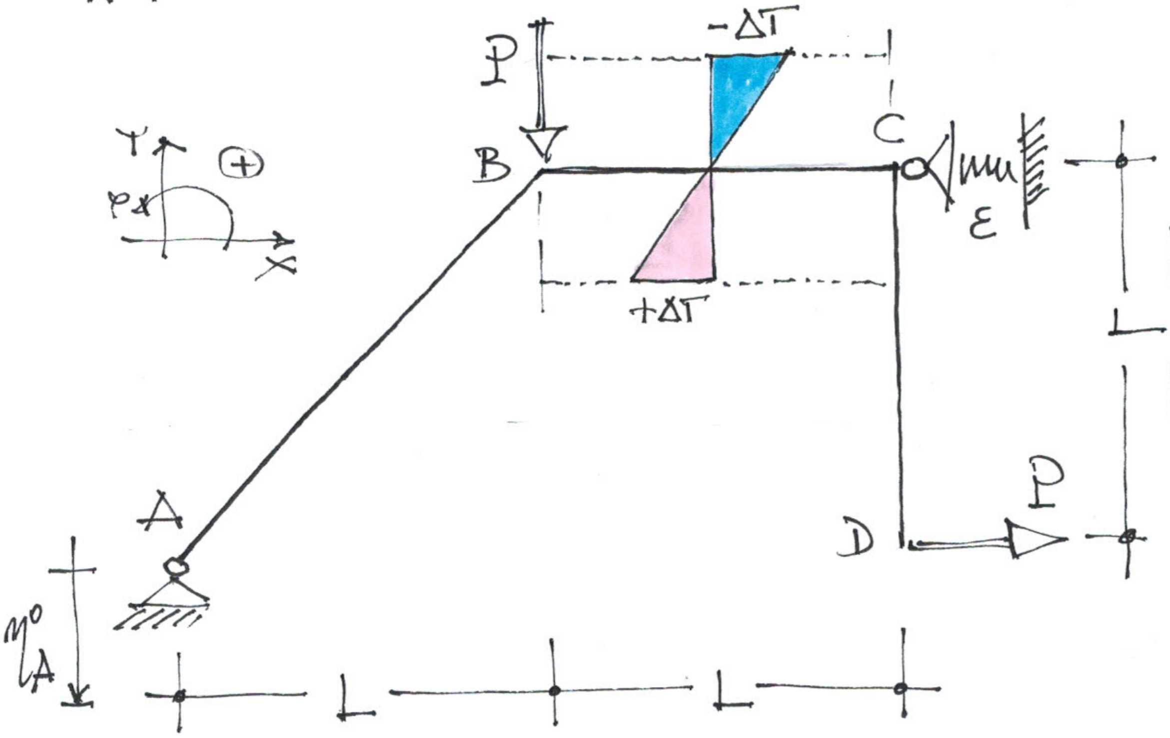
SOLUZIONE

Quesito n. 1

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA RIPORTATA IN FIGURA TRACCIANDO IL DIAGRAMMA DEI MOMENTI



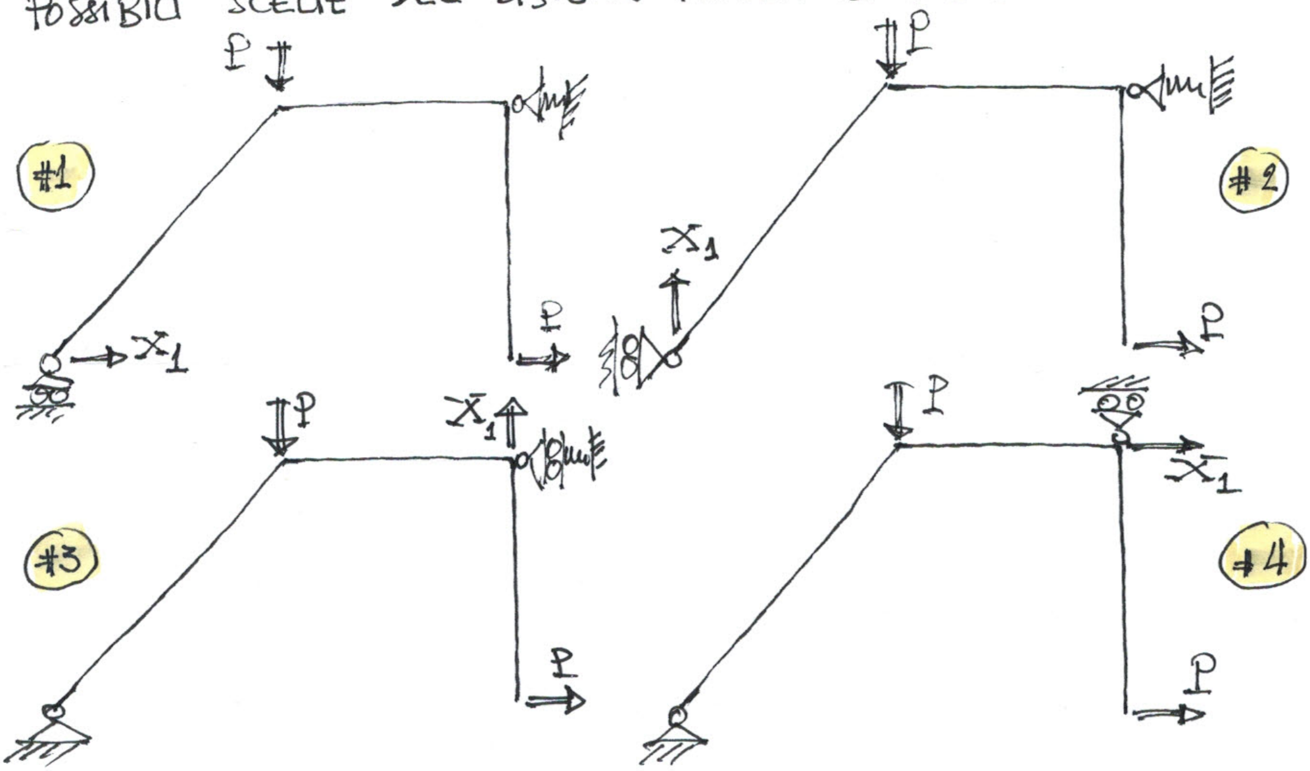
POSIZIONI:

$$|\varepsilon| = \frac{L^3}{12EI}\left[\sqrt{2}+1\right]$$

$$|m_A^0| = \frac{PL^3}{3EI}\left[2\sqrt{2}+3\right]$$

$$\left|\frac{\alpha\,\Delta T}{h}\right| = \frac{PL}{3EI}\left[2\sqrt{2}+3\right]$$

POSSIBILI SCELTE DEL SISTEMA PRINCIPALE ISOSTATICO:



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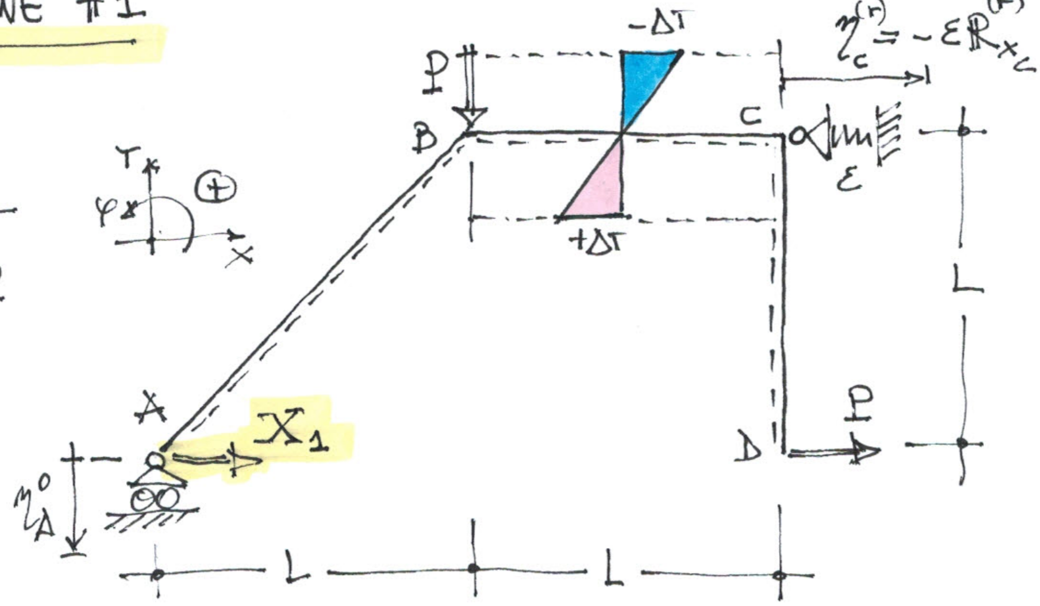
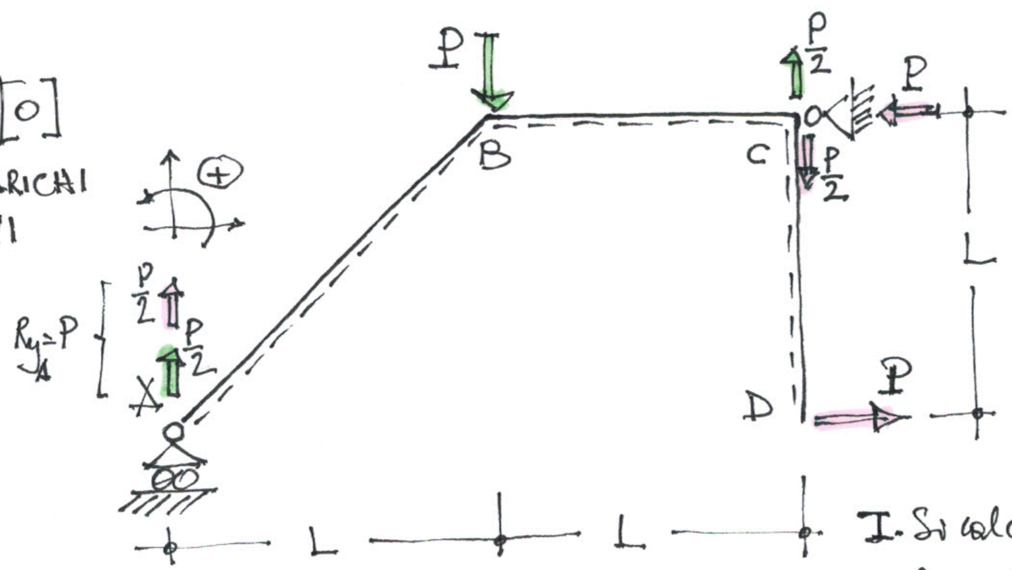
SOLUZIONE #1

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]

SOLO CARICHI ESTERNI



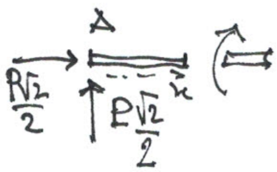
$R_y^C = \phi$

$R_{y_A} = P$

I. Si calcolano le RV con metodo grafico e princ. di sovrapp. d. effetti

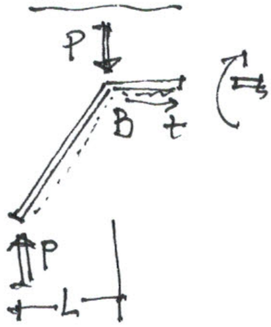
II. Si calcola $M^{(0)}(z)$ sui singoli tratti, si ha:

TRATTO AB $\quad 0 \le z \le L\sqrt{2}$



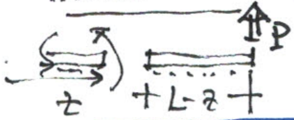
$$\boxed{M^{(0)}(z) = \frac{P\sqrt{2}}{2} z} \quad \begin{cases} M_A = \phi \\ M_B = \frac{P\sqrt{2}}{2} L\sqrt{2} = PL \end{cases}$$

TRATTO BC $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = PL} \text{ costante}$$

TRATTO CD $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = P(L - z)} \quad \begin{cases} M_C = PL \\ M_D = \phi \end{cases}$$

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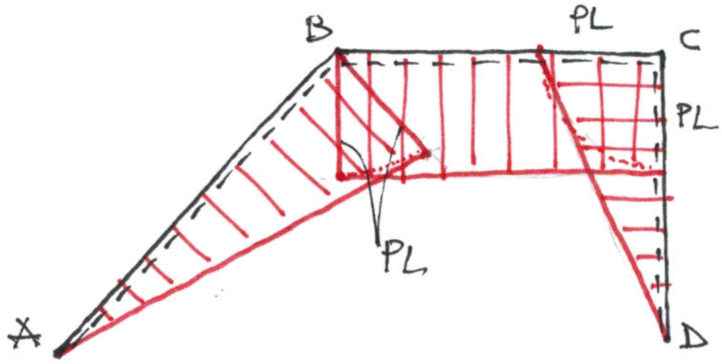
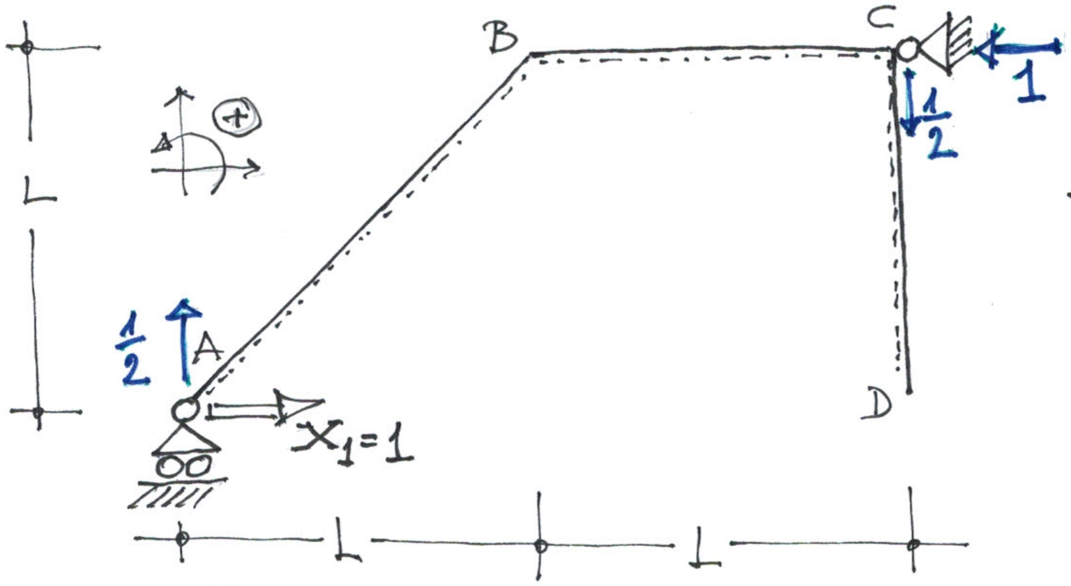
DIAGRAMMA $M^{(0)}(z)$:



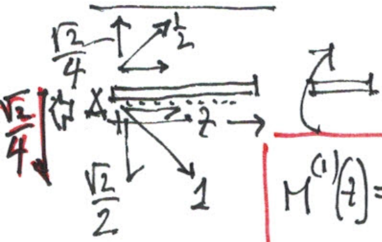
SCHEMA [1]
SOLO $X_1 = 1$



I. RV con metodo grafico !!

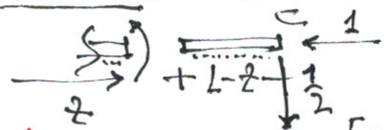
II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(1)}(z) = -\frac{\sqrt{2}}{4} \cdot z} \quad \begin{cases} M_A = 0 \\ M_B = -\frac{\sqrt{2}}{4} \cdot L\sqrt{2} = -\frac{L}{2} \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -\frac{(L-z)}{2}} \quad \begin{cases} M_B = -\frac{L}{2} \\ M_C = \emptyset \end{cases}$$

TRATTO CD $0 \le z \le L$

$$\boxed{M^{(1)}(z) = \emptyset}$$

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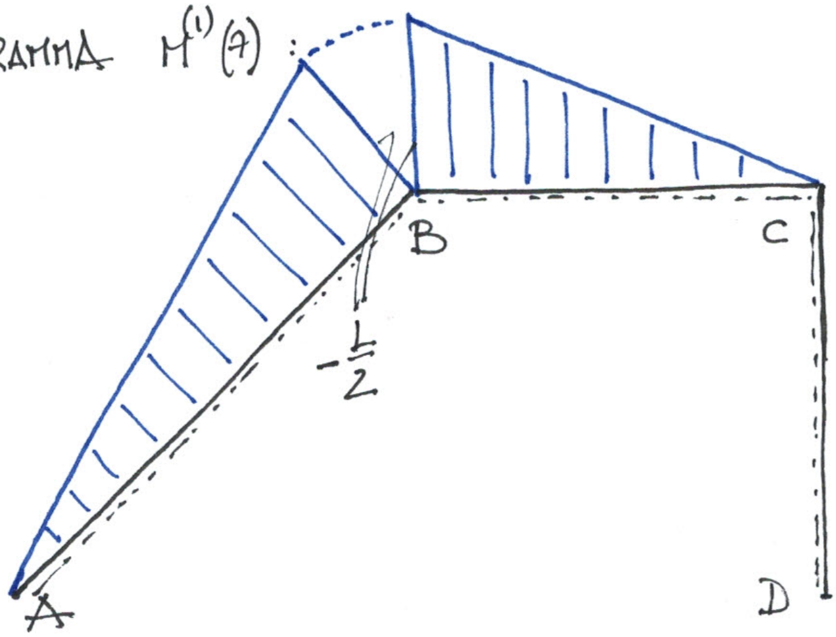
DIAGRAMMA $M^{(1)}(A)$:



L'unica equazione di Müller–Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{f} \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \underset{\overset{\parallel}{\phi}}{\eta_A^{(r)}} + \frac{1}{2} \cdot \left(- \eta_A^{0} \right) + \underbrace{R_{X_C}^{(1)}}_{-1} \cdot \underbrace{\eta_C^{(r)}}_{-\varepsilon \underbrace{R_{X_C}^{(r)}}_{\underbrace{R_{X_C}^{(0)}}_{-P} + \underbrace{R_{X_C}^{(1)}}_{-1} X_1}} - \frac{1}{2} \cdot \phi =$$

$$= -\frac{\eta_A^0}{2} - \varepsilon \left(P + X_1 \right)$$

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$$L_{vi} = \int_{St} \underbrace{M^{(f)}}_{M^{(1)}} \frac{\underbrace{M^{(r)}}_{M^{(0)} + M^{(1)} X_1}}{EI} \, dStr + \int_{Str} M^{(f)} \frac{\alpha \overline{\Delta T}}{h} \, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$= \frac{1}{EI} \left\{ \int_{\overline{AB}} \left(-\frac{\sqrt{2}}{4} z\right) \cdot \left[\frac{P\sqrt{2}}{2} z\right] dz + \int_{\overline{BC}} \left[-\frac{(L-z)}{2}\right] \cdot PL \, dz \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \int_{\overline{AB}} \left[-\frac{\sqrt{2}}{4} \cdot z\right]^2 dz + \int_{\overline{BC}} \left[-\frac{(L-z)}{2}\right]^2 dz \right\} + \int_{\overline{BC}} -\frac{(L-z)}{2} \frac{\alpha \overline{\Delta T}}{h} dz =$$

$$= \frac{1}{EI} \left\{ \int_0^{L\sqrt{2}} -\frac{P}{4} z^2 dz - \frac{1}{2} \int_0^L \left[PL^2 - PLz\right] dz \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \int_0^{L\sqrt{2}} \frac{z^2}{8} dz + \frac{1}{4} \int_0^L \overbrace{(L-z)^2}^{L^2 + z^2 - 2zL} dz \right\} - \frac{\alpha \overline{\Delta T}}{2h} \int_0^L (L-z) \, dz =$$

$$= \frac{1}{EI} \left\{ -\frac{P}{4} \left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} - \frac{PL^2}{2} \left[z\right]_0^L + \frac{PL}{2} \left[\frac{z^2}{2}\right]_0^L \right\} +$$

$-\frac{P L^3 \, 2\sqrt{2}}{\cancel{12} \; 6}$

$$+ \frac{X_1}{EI} \left\{ \frac{1}{8} \left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \frac{L^2}{4} \left[z\right]_0^L + \frac{1}{4} \left[\frac{z^3}{3}\right]_0^L - \frac{L}{2} \left[\frac{z^2}{2}\right]_0^L \right\} +$$

$\frac{1}{\cancel{24} \; 12} L^3 \, 2\sqrt{2}$

$$- \frac{\alpha \overline{\Delta T}}{2h} L \left[z\right]_0^L + \frac{\alpha \overline{\Delta T}}{2h} \left[\frac{z^2}{2}\right]_0^L =$$

$$= \frac{1}{EI} \left\{ -\frac{PL^3\sqrt{2}}{6} - \frac{PL^3}{2} + \frac{PL^3}{4} \right\} + \frac{X_1}{EI} \left\{ \frac{L^3\sqrt{2}}{12} + \cancel{\frac{L^3}{4}} + \frac{L^3}{12} - \cancel{\frac{L^3}{4}} \right\} +$$

$$- \frac{\alpha \overline{\Delta T}}{2h} L^2 + \frac{\alpha \overline{\Delta T}}{4h} L^2 =$$

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$$= \frac{PL^3}{EI}\left[-\frac{\sqrt{2}}{6}-\frac{1}{4}\right] + \frac{X_1 L^3}{12EI}\left\{\sqrt{2}+1\right\} - \frac{\alpha\overline{\Delta T}}{4h}L^2$$

$\frac{-4\sqrt{2}-6}{24}$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\frac{\mathcal{M}^0_A}{2} - \varepsilon(P+X_1) = \underbrace{\frac{PL^3}{EI}\left[-\frac{\sqrt{2}}{6}-\frac{1}{4}\right]}_{-\frac{PL^3}{12EI}\left[2\sqrt{2}+3\right]} + \frac{X_1L^3}{12EI}\left[\sqrt{2}+1\right] - \frac{\alpha\overline{\Delta T}}{4h}L^2$$

quest'ultima, tenendo conto delle posizioni iniziali, si scrive:

$$-\frac{\mathcal{M}^0_A}{2} - \varepsilon P - \varepsilon X_1 = -\frac{PL^3}{12EI}\left[2\sqrt{2}+3\right] + \frac{X_1L^3}{12EI}\left[\sqrt{2}+1\right] - \alpha\frac{\overline{\Delta T}}{4h}L^2$$

$$X_1\left\{\frac{L^3}{12EI}\left[\sqrt{2}+1\right] + \underbrace{\varepsilon}_{\frac{L^3}{12EI}\left[\sqrt{2}+1\right]}\right\} = -\frac{\mathcal{M}^0_A}{2} - \underbrace{\varepsilon}_{\frac{L^3}{12EI}\left[\sqrt{2}+1\right]} P + \frac{PL^3}{12EI}\left[\sqrt{2}\cdot 2+3\right] + \underbrace{\frac{\alpha\overline{\Delta T}}{h}}_{\frac{PL}{3EI}\left[2\sqrt{2}+3\right]}\frac{L^2}{4}$$

$$X_1\frac{L^3\left[\sqrt{2}+1\right]}{6EI} = -\underbrace{\frac{\mathcal{M}^0_A}{2}}_{\frac{PL^3}{3EI}\left[2\sqrt{2}+3\right]} - \frac{L^3}{12EI}\left[\sqrt{2}+1\right]P + \frac{PL^3}{6EI}\left[2\sqrt{2}+3\right]$$

$$X_1\frac{\cancel{L^3}\cancel{\left[\sqrt{2}+1\right]}}{6\cancel{EI}} = -\frac{\cancel{L^3}}{12\cancel{EI}}\cancel{\left[\sqrt{2}+1\right]}P$$

$$X_1 = -\frac{6}{12}P = -\frac{P}{2}$$ NEGATIVA! ⇒ Verso contrario a quello ipotizzato!

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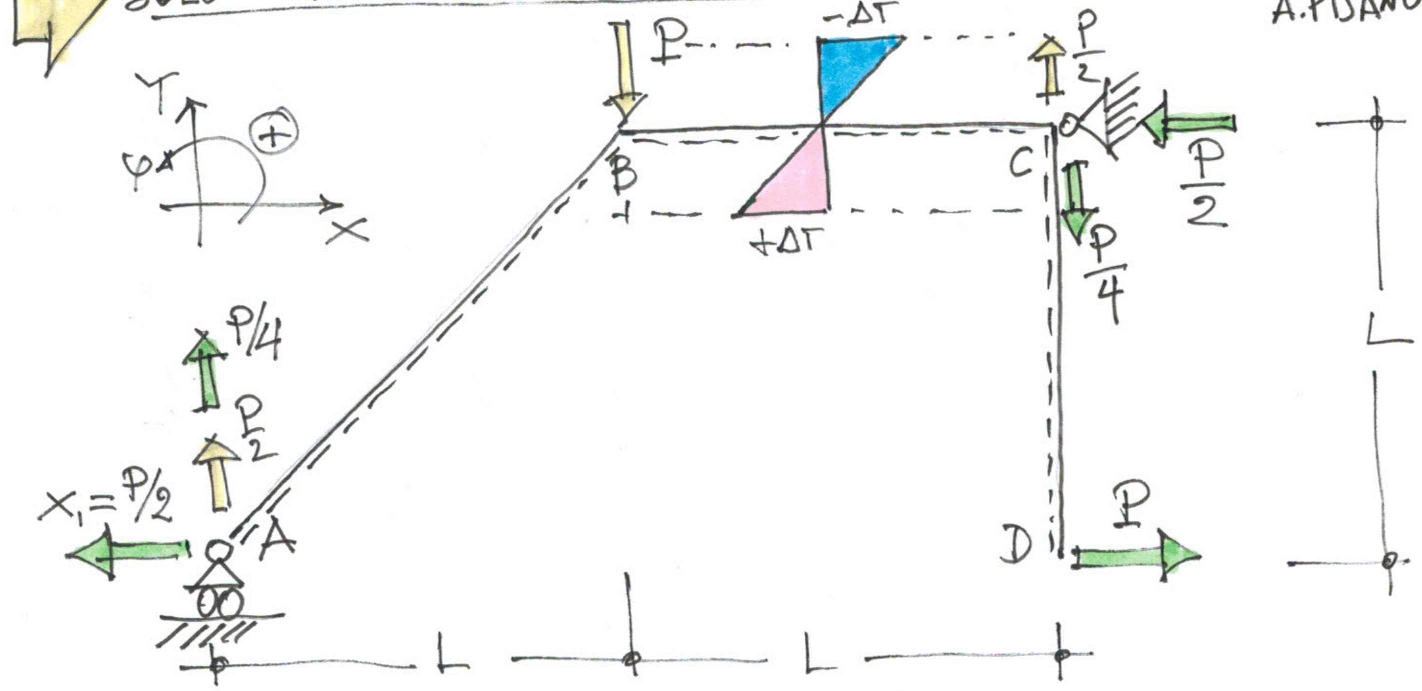
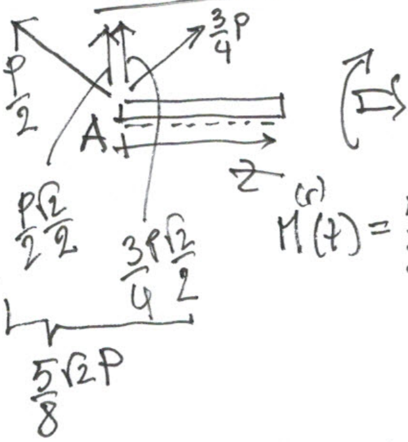
SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO



I. Si calcolano le RV con metodo grafico e principio di sovrapposizione degli effetti! Si noti che $R_{y_A} = \frac{3}{4}P$ ed $R_{y_C} = \frac{P}{4}$, entrambi verso l'alto!

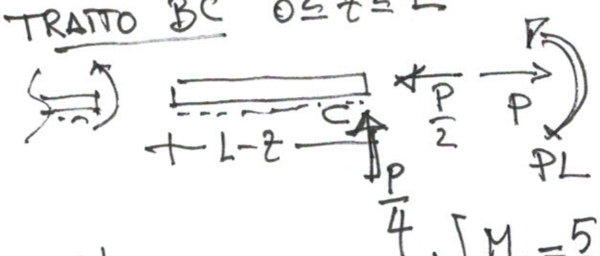
II. Si calcola il $M^{(r)}(z)$ nei singoli tratti. Si ha:

TRATTO AB $\quad 0 \le z \le L\sqrt{2}$



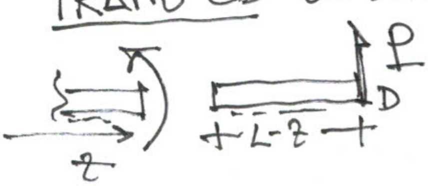
$$M^{(r)}(z) = \frac{5}{8}\sqrt{2}\,P\,z \quad \begin{cases} M_A = 0 \\ M_B = \frac{5}{4}PL \end{cases}$$

TRATTO BC $\quad 0 \le z \le L$



$$M^{(r)}(z) = PL + \frac{P}{4}(L-z) \quad \begin{cases} M_B = \frac{5}{4}PL \\ M_C = PL \end{cases}$$

TRATTO CD $\quad 0 \le z \le L$



$$M^{(r)}(z) = P(L-z) \quad \begin{cases} M_C = PL \\ M_D = 0 \end{cases}$$

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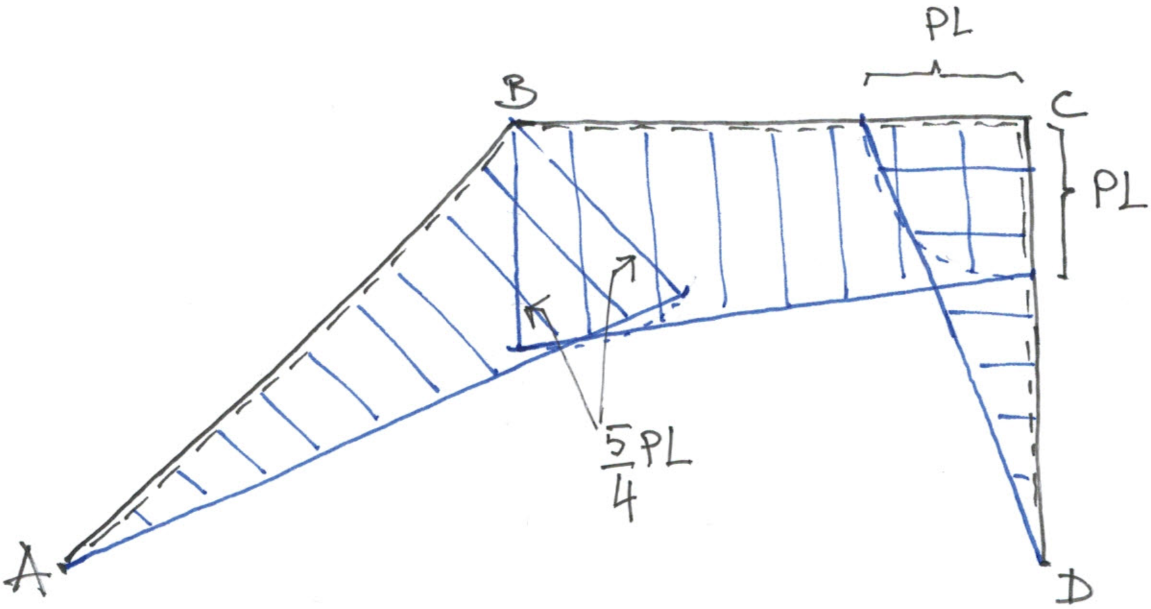
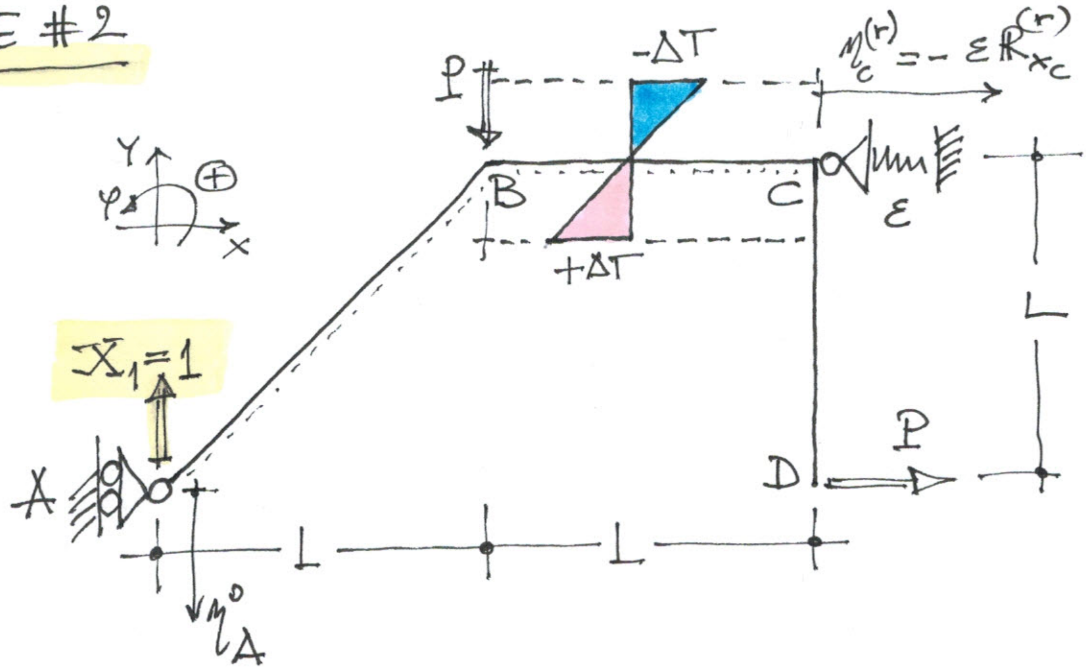
DIAGRAMMA DEL MOMENTO SULLA STRUTTURA IPERSTATICA ANALIZZATA



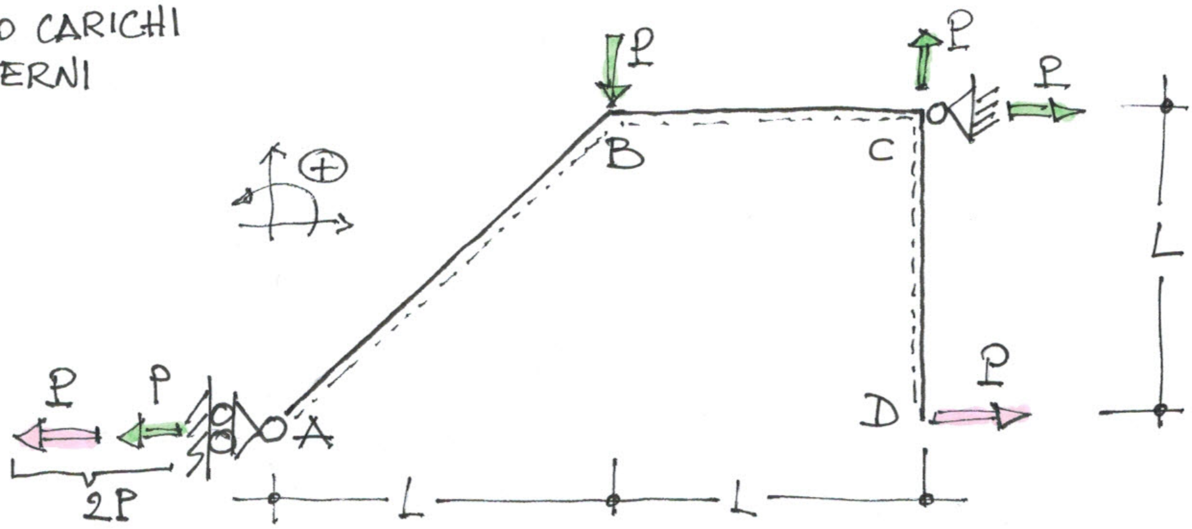
SOLUZIONE #2

⇨ SISTEMA PRINCIPALE ISOSTATICO



⇨ SCHEMA [0]

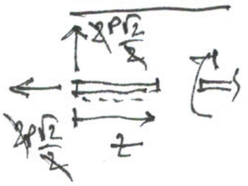
SOLO CARICHI ESTERNI



I. Si calcolano le RV con metodo grafico e principio di sovrapp. degli effetti!

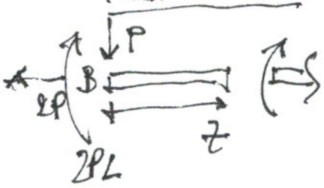
II. Si calcola $M^{(0)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $\quad 0 \le z \le L\sqrt{2}$



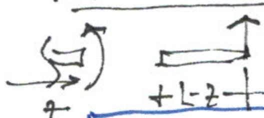
$$\boxed{M^{(0)}(z) = P\sqrt{2} \cdot z} \quad \begin{cases} M_A = \phi \\ M_B = P\sqrt{2} \cdot L\sqrt{2} = \\ = 2PL \end{cases}$$

TRATTO BC $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = 2PL - Pz} \quad \begin{cases} M_B = 2PL \\ M_C = PL \end{cases}$$

TRATTO CD $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = P(L - z)} \quad \begin{cases} M_C = PL \\ M_D = \phi \end{cases}$$

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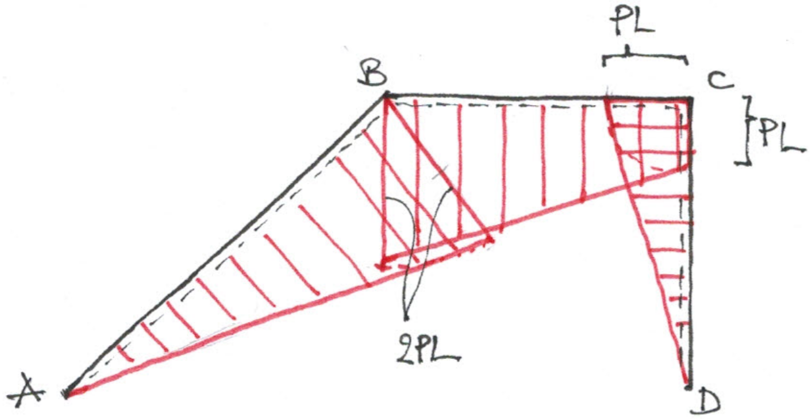
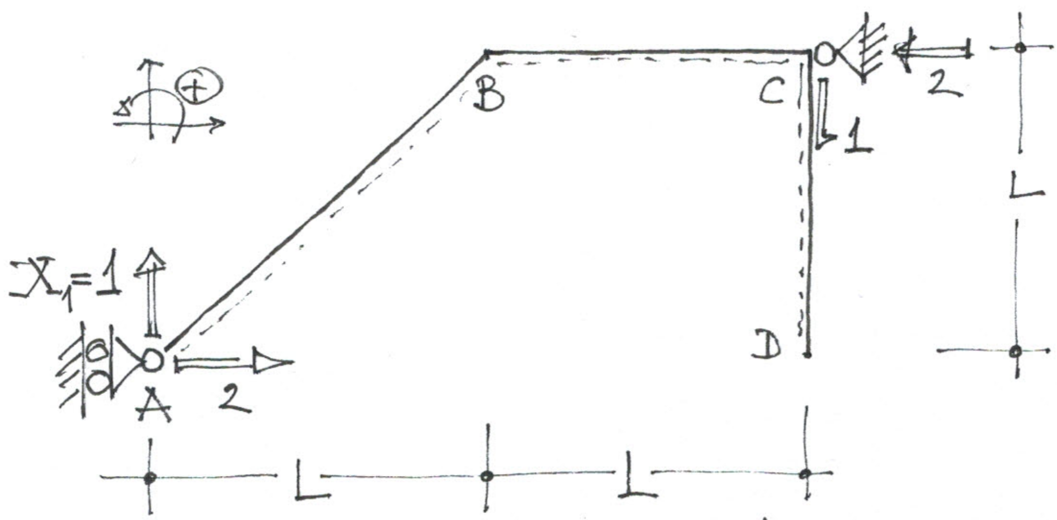
DIAGRAMMA $M^{(0)}(z)$:



SCHEMA [1]
SOLO $X_1 = 1$



I. Si calcolano le RV con metodo grafico. Immediate!

II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $0 \le z \le L\sqrt{2}$

$$\boxed{M^{(1)}(z) = -\frac{\sqrt{2}}{2}z} \quad \begin{cases} M_A = \phi \\ M_B = -\frac{\sqrt{2}}{2} \cdot L\sqrt{2} = -L \end{cases}$$

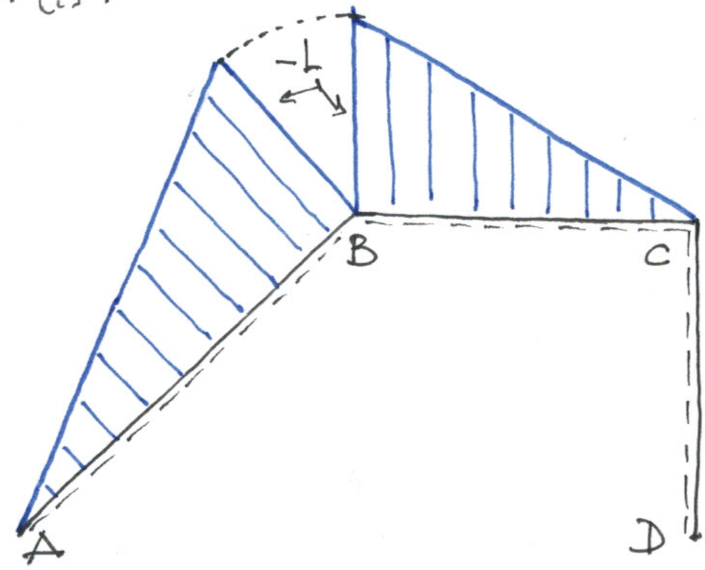
TRATTO BC $0 \le z \le L$

$$\boxed{M^{(1)}(z) = -(L-z)} \quad \begin{cases} M_B = -L \\ M_C = \phi \end{cases}$$

TRATTO CD

$$\boxed{M^{(1)}(z) = \phi}$$

DIAGRAMMA $M^{(1)}(t)$:



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve}=L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \left(-\eta_A^o\right) + \underbrace{R_{x_c}^{(1)}}_{-2} \cdot \underbrace{\eta_c^{(r)}}_{-\varepsilon \underbrace{R_{x_c}^{(r)}}_{\underbrace{R_{x_c}^{(o)}}_{=+P} + \underbrace{R_{x_c}^{(1)}}_{=-2} X_1}} = -\eta_A^o + 2\varepsilon\left[P - 2X_1\right]$$

$$L_{vi} = \int_{Str} \underbrace{M^{(f)}}_{M^{(1)}} \frac{\overbrace{M^{(r)}}^{M^{(o)}+M^{(1)}X_1}}{EI} dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(o)}}{EI} dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr =$$

XII
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$$= \frac{1}{EI}\left[\int_{\overline{AB}} \left[-\frac{\cancel{\sqrt{2}}}{\cancel{2}} z\right]\left[P\cancel{\sqrt{2}}\, z\right] dz + \int_{\overline{BC}} \overbrace{(z-L)\left[2PL - Pz\right]}^{3PLz - Pz^2 - 2PL^2} dz\right] +$$

$$+ \frac{X_1}{EI}\left[\int_{\overline{AB}} \left[-\frac{\sqrt{2}}{2} z\right]^2 + \int_{\overline{BC}} (L-z)^2 dz\right] + \int_{\overline{BC}} -(L-z)\,\alpha\frac{\Delta T}{h}\, dz =$$

$$= \frac{1}{EI}\left[\int_0^{L\sqrt{2}} \left[-Pz^2\right] dz + \int_0^L \left[3PLz - Pz^2 - 2PL^2\right]\right] +$$

$$+ \frac{X_1}{EI}\left[\int_0^{L\sqrt{2}} \frac{z^2}{2}\, dz + \int_0^L \left[L^2 + z^2 - 2Lz\right]\right] + \frac{\alpha\Delta T}{h}\int_0^L (z-L)\, dz =$$

$$= \frac{1}{EI}\left[-P\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + 3PL\left[\frac{z^2}{2}\right]_0^L - P\left[\frac{z^3}{3}\right]_0^L - 2PL^2\left[z\right]_0^L\right] +$$

$$+ \frac{X_1}{EI}\left[\frac{1}{2}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L\right] + \frac{\alpha\Delta T}{h}\left[\left[\frac{z^2}{2}\right]_0^L - L\left[z\right]_0^L\right] =$$

$$= \frac{1}{EI}\left[-\frac{P}{3}L^3 2\sqrt{2} + \underbrace{3\frac{PL^3}{2} - \frac{PL^3}{3} - 2PL^3}_{-\frac{5}{6}PL^3}\right] + \frac{X_1}{EI}\left[\frac{1}{\cancel{2}\,3}L^3\cancel{2}\sqrt{2} + \cancel{L^3} + \frac{L^3}{3} - \cancel{2\frac{L^2}{2}}\right]$$

$$+ \alpha\frac{\Delta T}{h}\left[\frac{L^2}{2} - L^2\right] =$$

$$= \frac{PL^3}{EI}\underbrace{\left[-\frac{2\sqrt{2}}{3} - \frac{5}{6}\right]}_{-\frac{4\sqrt{2}+5}{6}} + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] - \alpha\frac{\Delta T}{h}\frac{L^2}{2}$$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\eta_A^0 + 2\varepsilon\left[P - 2X_1\right] = -\frac{PL^3}{6EI}\left[4\sqrt{2}+5\right] + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] - \alpha\frac{\Delta T}{h}\frac{L^2}{2}$$

$$-\eta_A^0 + 2\varepsilon P - 4\varepsilon X_1 = -\frac{PL^3}{6EI}\cdot 4\sqrt{2} - \frac{5}{6}\frac{PL^3}{EI} + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] - \alpha\frac{\Delta T}{h}\frac{L^2}{2}$$

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quest'ultime, tenendo conto delle posizioni iniziali, si scrive:

$$-\frac{P\cancel{L}^3}{3\cancel{EI}}\left[2\sqrt{2}+3\right]+2P\frac{\cancel{L}^3}{12\cancel{EI}}\left[\sqrt{2}+1\right]-\frac{\cancel{L}^3}{3\cancel{EI}}\left[\sqrt{2}+1\right]X_1=$$

$$=-\frac{2}{3}\frac{P\cancel{L}^3}{\cancel{EI}}\sqrt{2}-\frac{5}{6}\frac{P\cancel{L}^3}{\cancel{EI}}+\frac{X_1\cancel{L}^3}{3\cancel{EI}}\left[\sqrt{2}+1\right]-\frac{P\cancel{L}}{3\cancel{EI}}\left[2\sqrt{2}+3\right]\frac{\cancel{L}^2}{2}$$

$$X_1\cdot\frac{2}{3}\left[\sqrt{2}+1\right]=-P\frac{2}{3}\sqrt{2}-P+\frac{P\sqrt{2}}{6}+\frac{P}{6}+\frac{2}{3}P\sqrt{2}+\frac{5}{6}P+\frac{P}{3}\sqrt{2}+\frac{P}{2}$$

$$X_1\,\frac{2}{3}\left[\sqrt{2}+1\right]=\frac{P}{2}\left[\sqrt{2}+1\right]$$

$X_1=\frac{3}{4}P$ POSITIVA! ok! $\Rightarrow$ confronta con la RV trovata con la Soluz. #1 a pag. VII!

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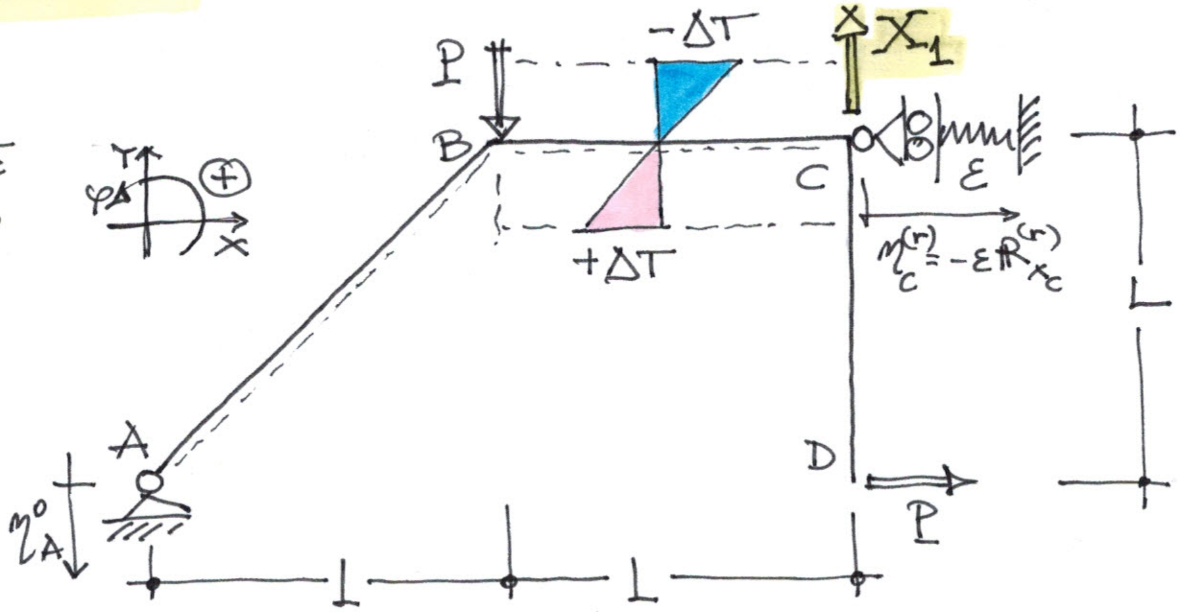
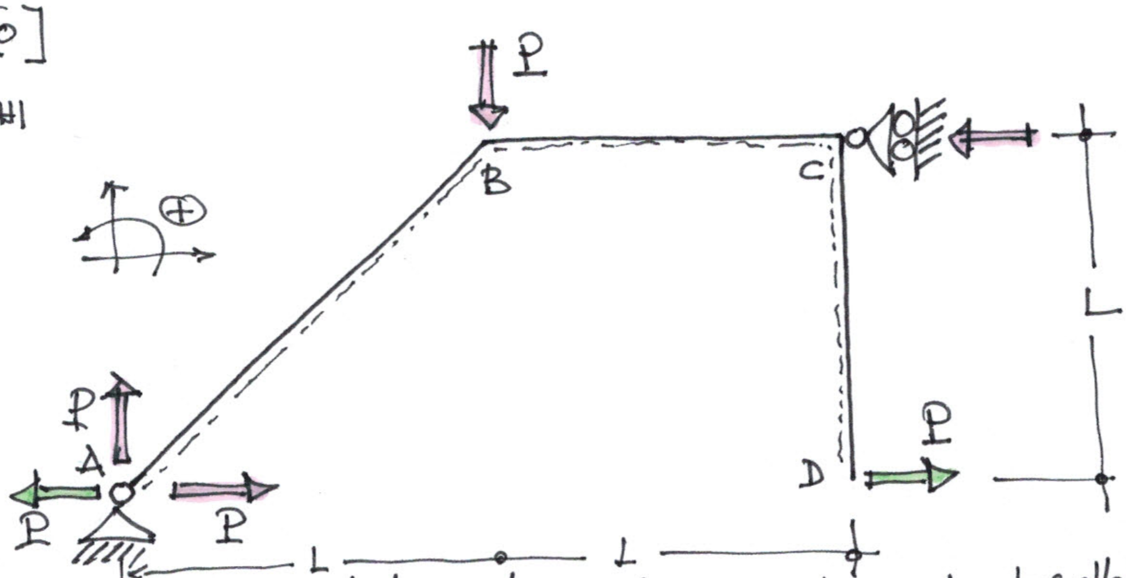
SOLUZIONE # 3

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]

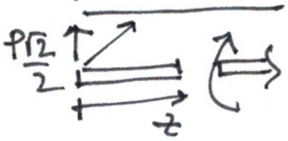
SOLO CARICHI ESTERNI



I. Si calcolano le RV con metodo grafico e principio di sovrapp. degli effetti!

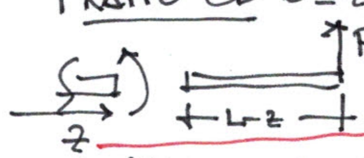
II. Si calcola $M^{(0)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $0 \le z \le L\sqrt{2}$



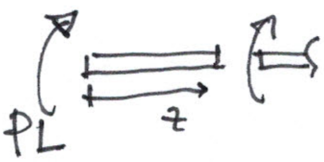
$$\boxed{M^{(0)}(z) = \frac{P\sqrt{2}}{2} \cdot z} \quad \begin{cases} M_A = \emptyset \\ M_B = PL \end{cases}$$

TRATTO CD $0 \le z \le L$



$$\boxed{M^{(0)}(z) = P(L-z)} \quad \begin{cases} M_C = PL \\ M_D = \emptyset \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = PL}$$ costante

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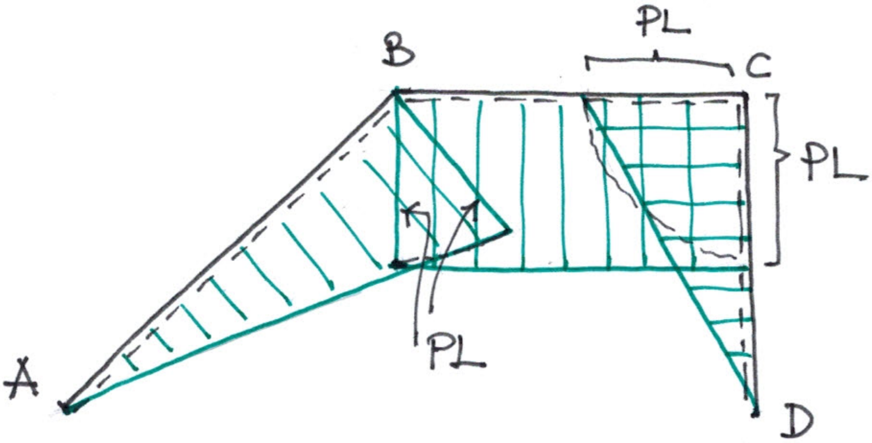
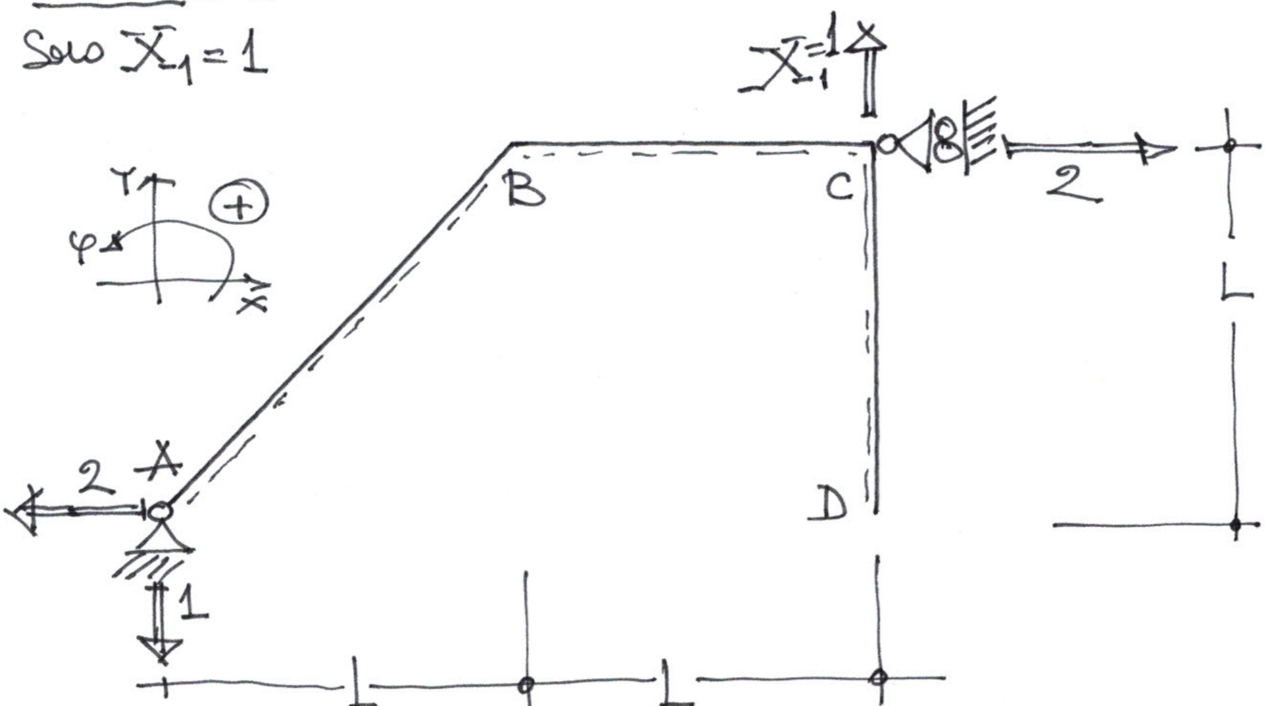
DIAGRAMMA $M^{(0)}(z)$:



SCHEMA [1]

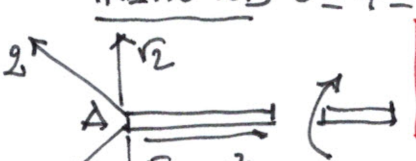
Solo $X_1 = 1$



I. Si calcolano RV con metodo grafico!

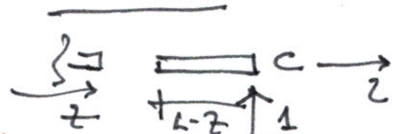
II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{2} z} \qquad \begin{cases} M_A = 0 \\ M_B = L \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = L - z} \qquad \begin{cases} M_B = L \\ M_C = 0 \end{cases}$$

TRATTO CD $0 \le z \le L$

$$\boxed{M^{(1)}(z) = 0}$$

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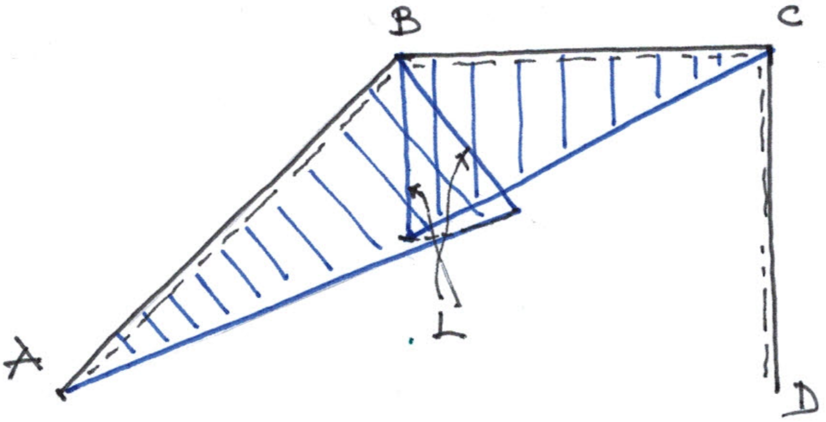
DIAGRAMMA $M^{(1)}(f)$:



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \cdot \eta_j^{(r)} =$$

$$= 1 \cdot \phi + \underbrace{(-1)}_{R_{y_A}^{(1)}} \left(-\eta_A^0\right) + \underbrace{R_{x_c}^{(1)}}_{2} \cdot \underbrace{\eta_c^{(r)}}_{-\varepsilon \underbrace{R_{x_c}^{(r)}}_{\underbrace{R_{x_c}^{(0)}}_{-P} + \underbrace{R_{x_c}^{(1)}}_{2} X_1}} = \eta_A^0 - 2\varepsilon\left[-P + 2X_1\right]$$

$$L_{vi} = \int_{Str} \underbrace{M^{(f)}}_{M^{(1)}} \frac{\overbrace{M^{(r)}}^{M^{(0)} + M^{(1)} X_1}}{EI} \, dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} \, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} \, dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} \, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} \, dStr =$$

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$$= \frac{1}{EI}\left\{\int_{\overline{AB}}\left[\frac{\sqrt{2}}{2}z\right]\left[\frac{P\sqrt{2}}{2}z\right]ds_{tr} + \int_{\overline{BC}}[L-z][PL]\,dz\right\} +$$

$$+ \frac{X_1}{EI}\left\{\int_{\overline{AB}}\left[\frac{\sqrt{2}}{2}z\right]^2 dz + \int_{\overline{BC}}(L-z)^2\,dz\right\} + \int_{\overline{BC}}(L-z)\frac{\alpha\overline{\Delta T}}{h}\,dz =$$

$$= \frac{1}{EI}\left\{\int_0^{L\sqrt{2}}\frac{P}{2}z^2\,dz + \int_0^L\left[PL^2 - PLz\right]dz\right\} +$$

$$+ \frac{X_1}{EI}\left\{\int_0^{L\sqrt{2}}\frac{z^2}{2}\,dz + \int_0^L\left(L^2 + z^2 - 2Lz\right)dz\right\} +$$

$$+ \int_0^L L\frac{\alpha\overline{\Delta T}}{h}\,dz - \int_0^L z\frac{\alpha\overline{\Delta T}}{h}\,dz =$$

$$= \frac{1}{EI}\left\{\frac{P}{2}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + PL^2\left[z\right]_0^L - PL\left[\frac{z^2}{2}\right]_0^L\right\} +$$

$$+ \frac{X_1}{EI}\left\{\frac{1}{2}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L\right\} +$$

$$+ \frac{\alpha\overline{\Delta T}}{h}L\left[z\right]_0^L - \frac{\alpha\overline{\Delta T}}{h}\left[\frac{z^2}{2}\right]_0^L =$$

$$= \frac{1}{EI}\left\{\frac{PL^3\cancel{2}\sqrt{2}}{\cancel{6}\,3} + PL^3 - \frac{PL^3}{2}\right\} +$$

$$+ \frac{X_1}{EI}\left\{\frac{1}{6}L^3 2\sqrt{2} + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right\} + \frac{\alpha\overline{\Delta T}}{h}L^2 - \alpha\frac{\overline{\Delta T}}{h}\frac{L^2}{2} =$$

$$= \frac{PL^3}{EI}\left[\frac{\sqrt{2}}{3} + \frac{1}{2}\right] + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] + \alpha\frac{\overline{\Delta T}}{h}\frac{L^2}{2}$$

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In definitiva $L_{ve} = L_{vi}$ fornisce:

$$\eta_A^0 - 2\varepsilon\left[-P+2X_1\right] = \frac{PL^3}{EI}\left[\frac{\sqrt{2}}{3}+\frac{1}{2}\right] + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] + \frac{\alpha\Delta T}{h}\frac{L^2}{2}$$

$$X_1\left\{\frac{L^3}{3EI}\left(\sqrt{2}+1\right)+4\varepsilon\right\} = -\frac{PL^3}{EI}\left[\frac{\sqrt{2}}{3}+\frac{1}{2}\right] - \frac{\alpha\Delta T}{h}\frac{L^2}{2} + \eta_A^0 + 2\varepsilon P$$

tenendo conto delle posizioni iniziali quest'ultima si può scrivere:

$$X_1\,\frac{2}{3}\frac{\cancel{L^3}}{\cancel{EI}}\cancel{\left[\sqrt{2}+1\right]} = \frac{P\cancel{L^3}}{6\cancel{EI}}\cancel{\left[\sqrt{2}+1\right]}$$

$X_1 = \dfrac{P}{4}$ positiva! verso l'alto! ok! confronta con RV di pag. VII

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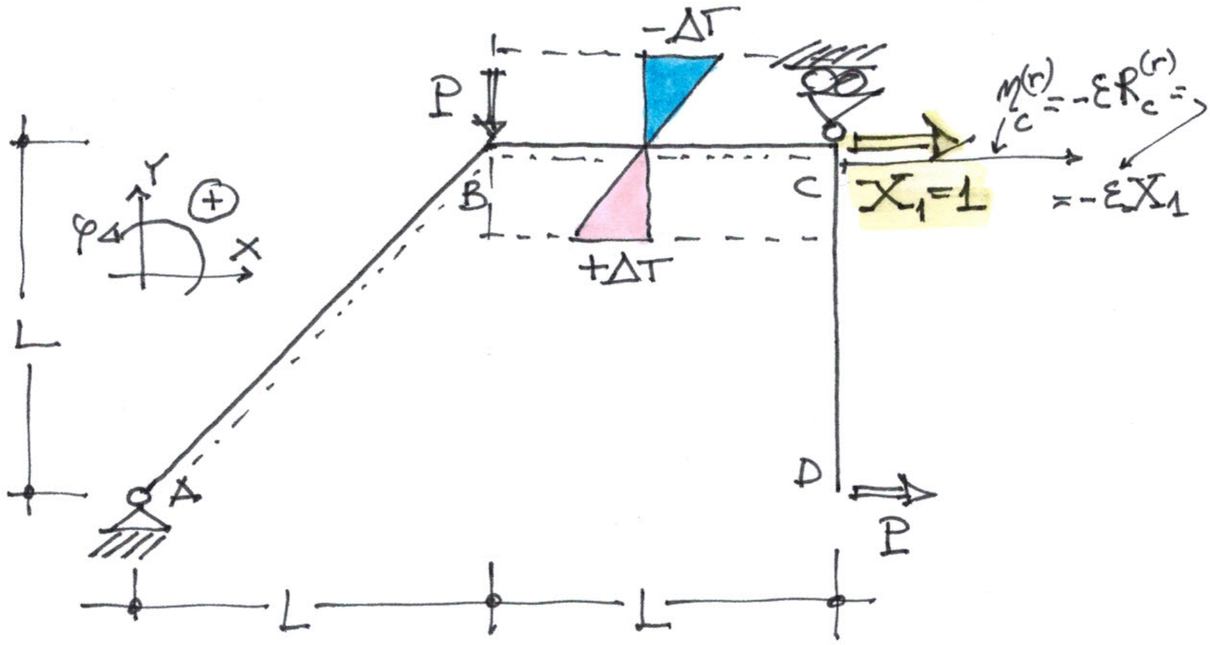
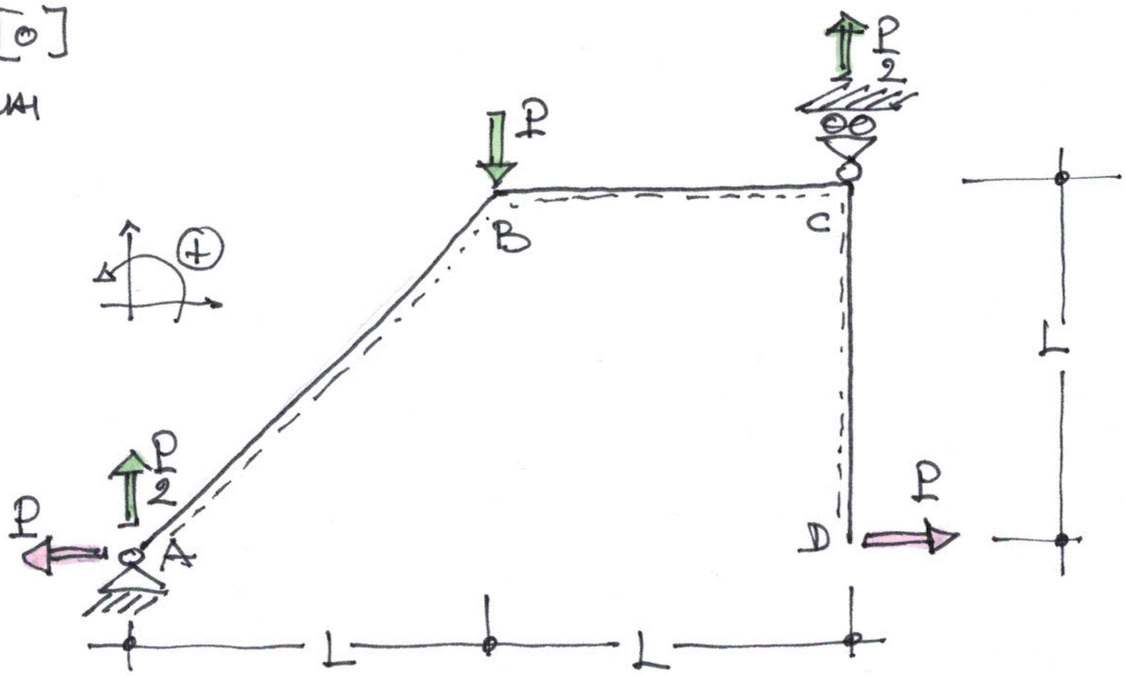
SOLUZIONE # 4

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]

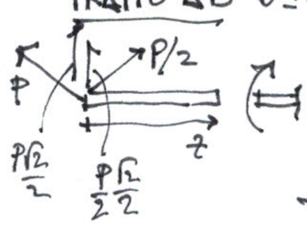
SOLO CARICHI ESTERNI



I. Si calcolano le RV con metodo grafico e princ. di sovrapp. degli effetti.

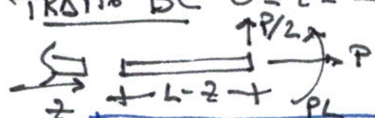
II. Si calcola $M^{(0)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $0 \le z \le L\sqrt{2}$



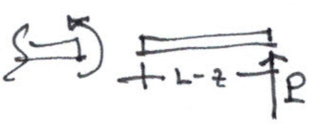
$$\boxed{M^{(0)}(z) = \frac{3}{4}P\sqrt{2}\cdot z} \quad \begin{cases} M_A = \phi \\ M_B = \frac{3}{2}PL \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = PL + \frac{P}{2}(L-z)} \quad \begin{cases} M_B = \frac{3}{2}PL \\ M_C = PL \end{cases}$$

TRATTO CD $0 \le z \le L$



$$\boxed{M^{(0)}(z) = P(L-z)} \quad \begin{cases} M_C = PL \\ M_D = \phi \end{cases}$$

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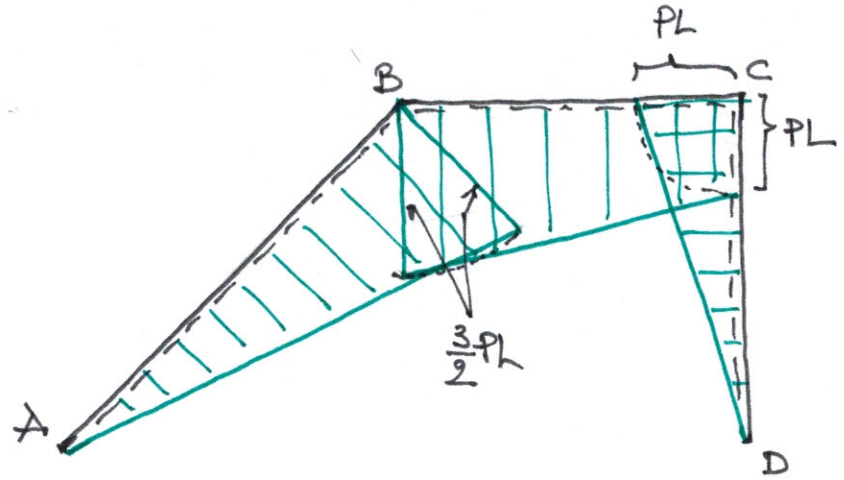
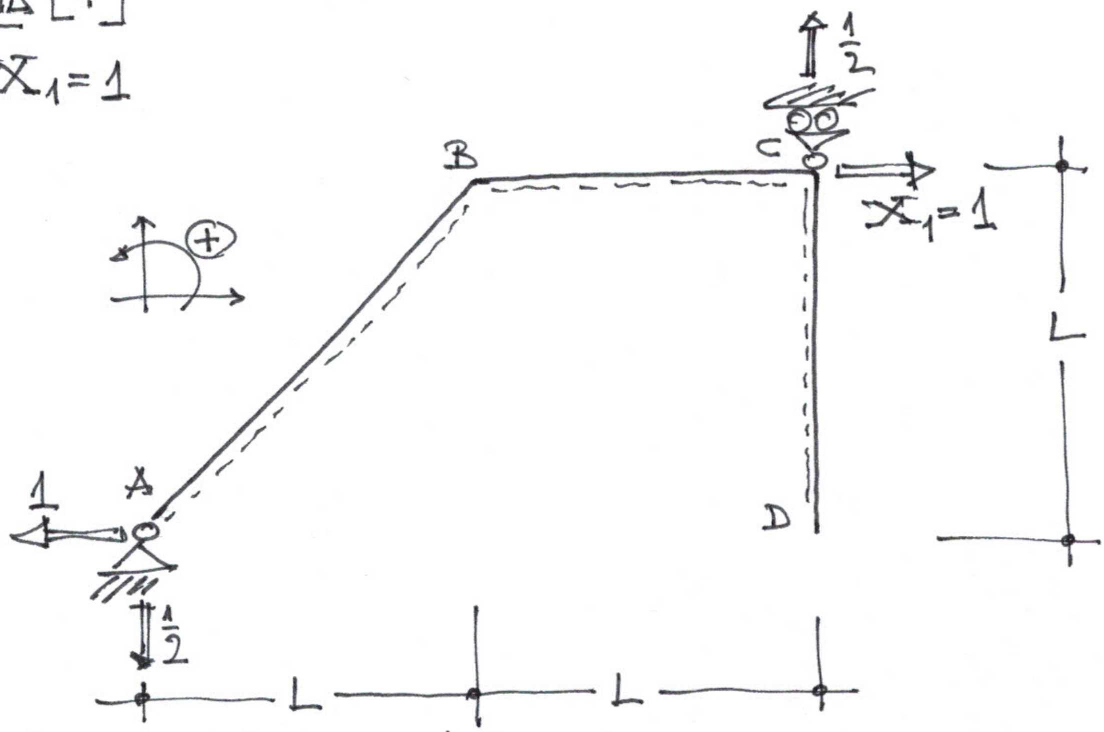
DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1]

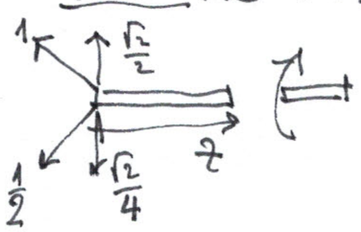
SOLO $X_1 = 1$



I. Si calcolano le RV con metodo grafico.

II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $0 \le z \le L\sqrt{2}$

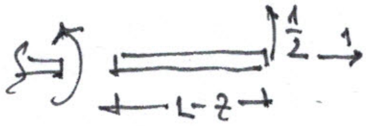


$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{4} z} \quad \begin{cases} M_A = \phi \\ M_B = \frac{\sqrt{2}}{4} \cdot L\sqrt{2} = \frac{L}{2} \end{cases}$$

TRATTO CD $0 \le z \le L$

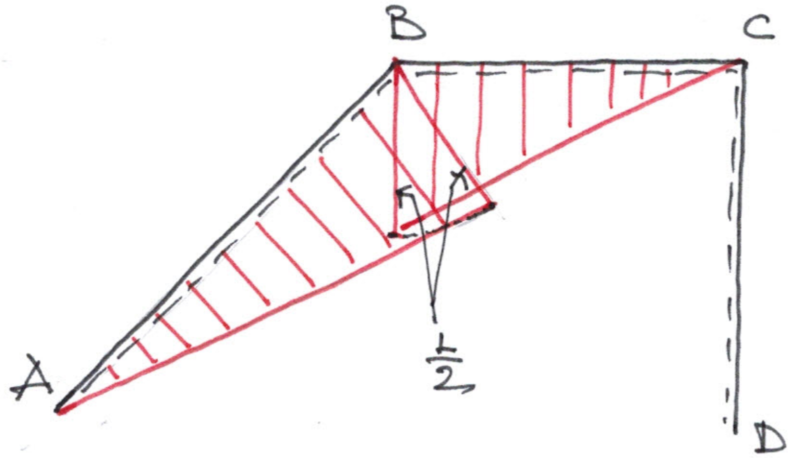
$$\boxed{M^{(1)}(z) = \phi}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = (L - z) \cdot \frac{1}{2}} \quad \begin{cases} M_B = \frac{L}{2} \\ M_C = \phi \end{cases}$$

DIAGRAMMA $M^{(1)}(z)$:



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \underbrace{\eta_C^{(r)}}_{-\varepsilon X_1} - \frac{1}{2}\left(-\eta_A^{0}\right) = -\varepsilon X_1 + \frac{\eta_A^{0}}{2}$$

$$L_{vi} = \int_{Str} \underbrace{M^{(f)}}_{M^{(1)}} \frac{\underbrace{M^{(r)}}_{M^{(0)} + M^{(1)} X_1}}{EI}\, dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI}\, dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI}\, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h}\, dStr =$$

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$$= \frac{1}{EI}\left[\int_{\overline{AB}} \overbrace{\left[\frac{\sqrt{2}}{4}z\right]\left[\frac{3}{4}P\sqrt{2}\,z\right]}^{\frac{3Pz^2\cancel{2}}{\cancel{16}\,8}} dz + \int_{\overline{BC}} \overbrace{\frac{(L-z)}{2}\left[PL+\frac{P}{2}(L-z)\right]}^{\frac{P}{4}\left[3L^2-4zL+z^2\right]}\right] +$$

$$+ \frac{X_1}{EI}\left\{\int_{\overline{AB}} \underbrace{\left[\frac{\sqrt{2}}{4}z\right]^2}_{\frac{\cancel{2}}{8\,\cancel{16}}z^2} dz + \int_{\overline{BC}} \underbrace{\left[\frac{L-z}{2}\right]^2}_{\frac{1}{4}\left[L^2+z^2-2Lz\right]} dz\right\} + \int_{\overline{BC}} \frac{L-z}{2}\cdot\frac{\alpha\overline{\Delta T}}{h}\,dz =$$

$$= \frac{1}{EI}\left\{\int_0^{L\sqrt{2}} \frac{3}{8}Pz^2\,dz + \int_0^{L}\frac{P}{4}\left[3L^2-4zL+z^2\right]dz\right\} +$$

$$+ \frac{X_1}{EI}\left\{\int_0^{L\sqrt{2}} \frac{z^2}{8}\,dz + \int_0^{L}\frac{1}{4}\left[L^2+z^2-2Lz\right]dz\right\} + \frac{\alpha\overline{\Delta T}}{h}\int_0^{L}(L-z)\,dz =$$

$$= \frac{1}{EI}\left\{\frac{3}{8}P\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \frac{3}{4}PL^2\left[z\right]_0^{L} - PL\left[\frac{z^2}{2}\right]_0^{L} + \frac{P}{4}\left[\frac{z^3}{3}\right]_0^{L}\right\} +$$

$$+ \frac{X_1}{EI}\left\{\frac{1}{8}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \frac{L^2}{4}\left[z\right]_0^{L} + \frac{1}{4}\left[\frac{z^3}{3}\right]_0^{L} - \frac{L}{2}\left[\frac{z^2}{2}\right]_0^{L}\right\} +$$

$$+ \frac{\alpha\overline{\Delta T}}{2h}\left\{L\left[z\right]_0^{L} - \left[\frac{z^2}{2}\right]_0^{L}\right\} =$$

$$= \frac{1}{EI}\left\{\frac{1}{\cancel{8}\,4}PL^3\cancel{2}\sqrt{2} + \frac{3}{4}PL^3 - \frac{1}{2}PL^3 + \frac{PL^3}{12}\right\} +$$

$$+ \frac{X_1}{EI}\left\{\frac{1}{\cancel{24}\,12}L^3\sqrt{2}\,\cancel{2} + \cancel{\frac{L^3}{4}} + \frac{1}{12}L^3 - \cancel{\frac{L^3}{4}}\right\} + \frac{\alpha\overline{\Delta T}}{2h}\left[L^2 - \frac{L^2}{2}\right] =$$

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$$= \frac{1}{EI}\left[\frac{PL^3}{4}\sqrt{2} + \frac{PL^3}{3}\right] + \frac{X_1}{EI}\left[\frac{L^3\sqrt{2}}{12} + \frac{L^3}{12}\right] + \frac{\alpha\Delta T}{2h}\frac{L^2}{2} =$$

$$= \frac{PL^3}{EI}\left\{\frac{\sqrt{2}}{4} + \frac{1}{3}\right\} + \frac{X_1 L^3}{12EI}\left[\sqrt{2}+1\right] + \frac{\alpha\Delta T}{4h}L^2$$

Imponendo l'uguaglianza $L_{ve} = L_{vi}$ e utilizzando le posizioni iniziali si ha:

$$\underbrace{-\varepsilon X_1}_{\frac{L^3}{12EI}[\sqrt{2}+1]} + \underbrace{\frac{\varphi_A^0}{2}}_{\frac{PL^3}{3EI}[2\sqrt{2}+3]} = \frac{PL^3}{EI}\left\{\frac{\sqrt{2}}{4}+\frac{1}{3}\right\} + \frac{X_1 L^3}{12EI}\left[\sqrt{2}+1\right] + \overbrace{\frac{\alpha\Delta T}{h}}^{\frac{PL}{3EI}[2\sqrt{2}+3]}\frac{L^2}{4}$$

$$X_1 \frac{\cancel{2}L^3}{\cancel{12}_{6}EI}\left[\sqrt{2}+1\right] = \frac{PL^3}{6EI}\left[2\sqrt{2}+3\right] - \frac{PL^3}{EI}\left\{\frac{\sqrt{2}}{4}+\frac{1}{3}\right\} - \frac{PL^3}{12EI}\left[2\sqrt{2}+3\right]$$

$$X_1 \frac{\cancel{L^3}}{6\cancel{EI}}\cancel{\left[\sqrt{2}+1\right]} = -\frac{P\cancel{L^3}}{12\cancel{EI}}\cancel{\left[\sqrt{2}+1\right]}$$

$X_1 = -\frac{6}{12}P = -\frac{P}{2}$ negativa! cioè verso sx! ⇒ ok! cfr. RV di pag. VII!