ESAMI di MECCANICA delle STRUTTURE – L17

Corso P. Fuschi – A.A. 2016-17 – prova scritta 28.06.17

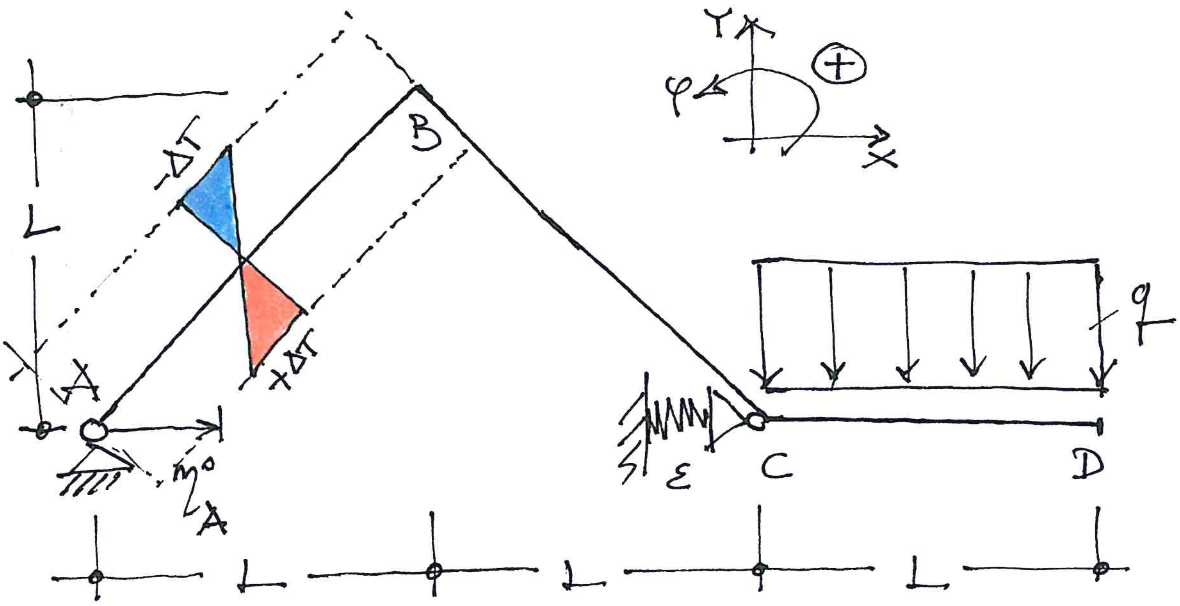
(I)

P. FUSCHI
A. PISANO

SOLUZIONE

Quesito n. 1

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA RIPORTATA IN FIGURA TRACCIANDO IL DIAGRAMMA DEI MOMENTI



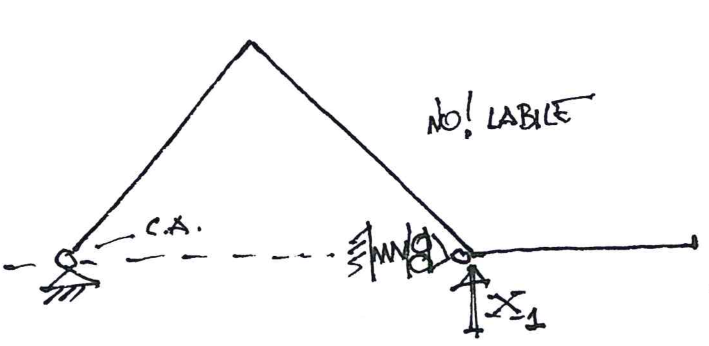
Posizioni:

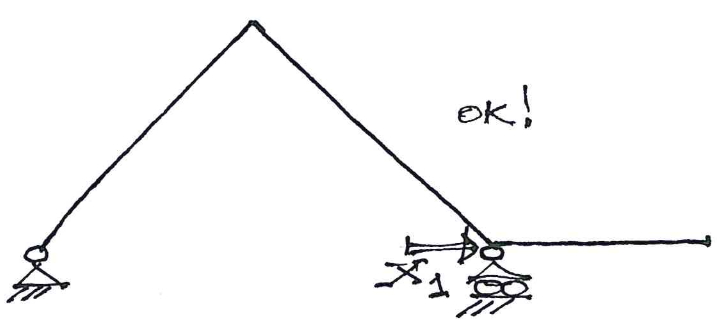
$$|\eta_A^0| = \frac{qL^4\sqrt{2}}{4EI}$$

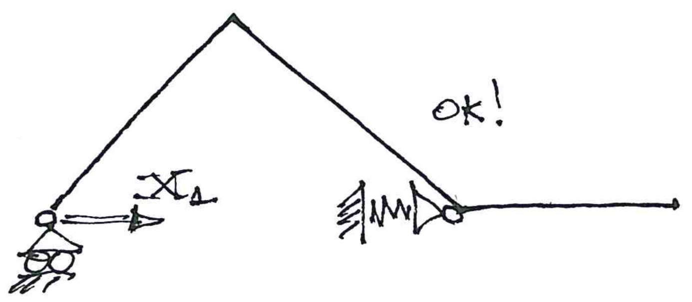
$$|\varepsilon| = \frac{L^3\sqrt{2}}{3EI}$$

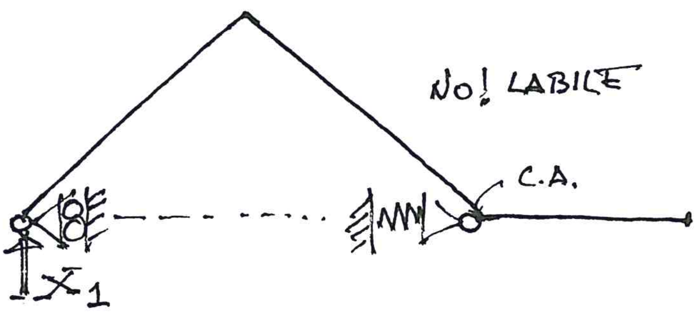
$$\left|\frac{\alpha\,\Delta T}{h}\right| = \frac{qL^2}{3EI}$$

POSSIBILI SCELTE DEL SISTEMA PRINCIPALE ISOSTATICO:









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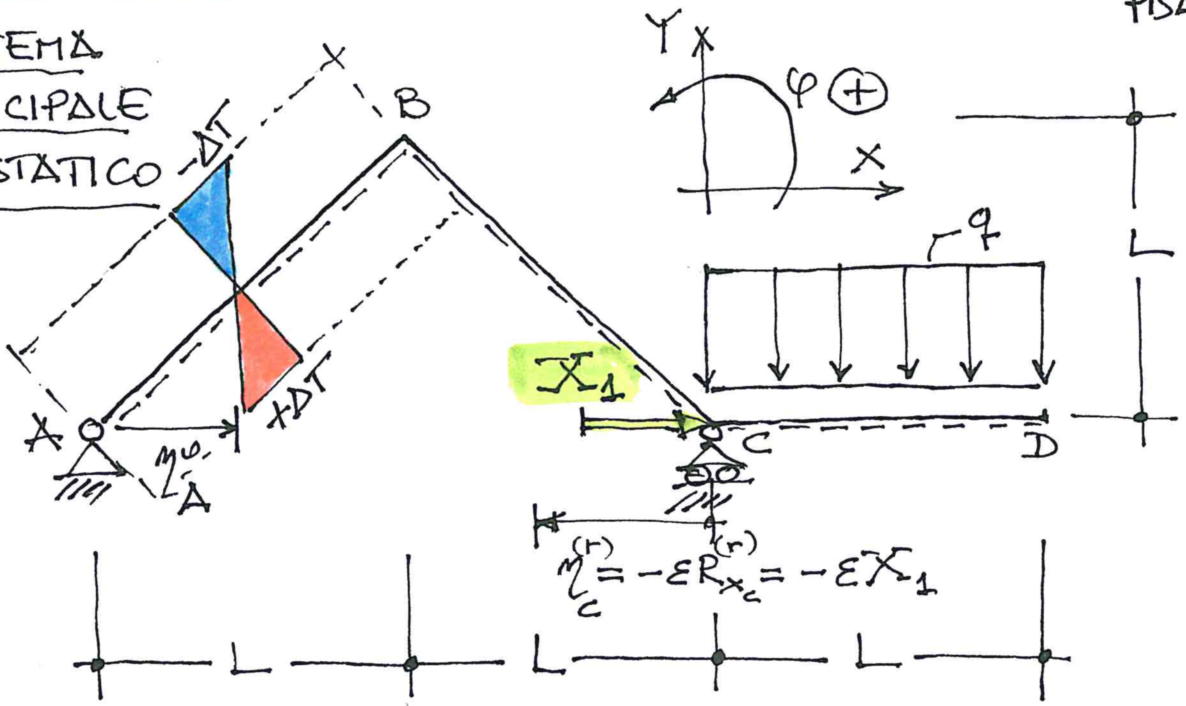
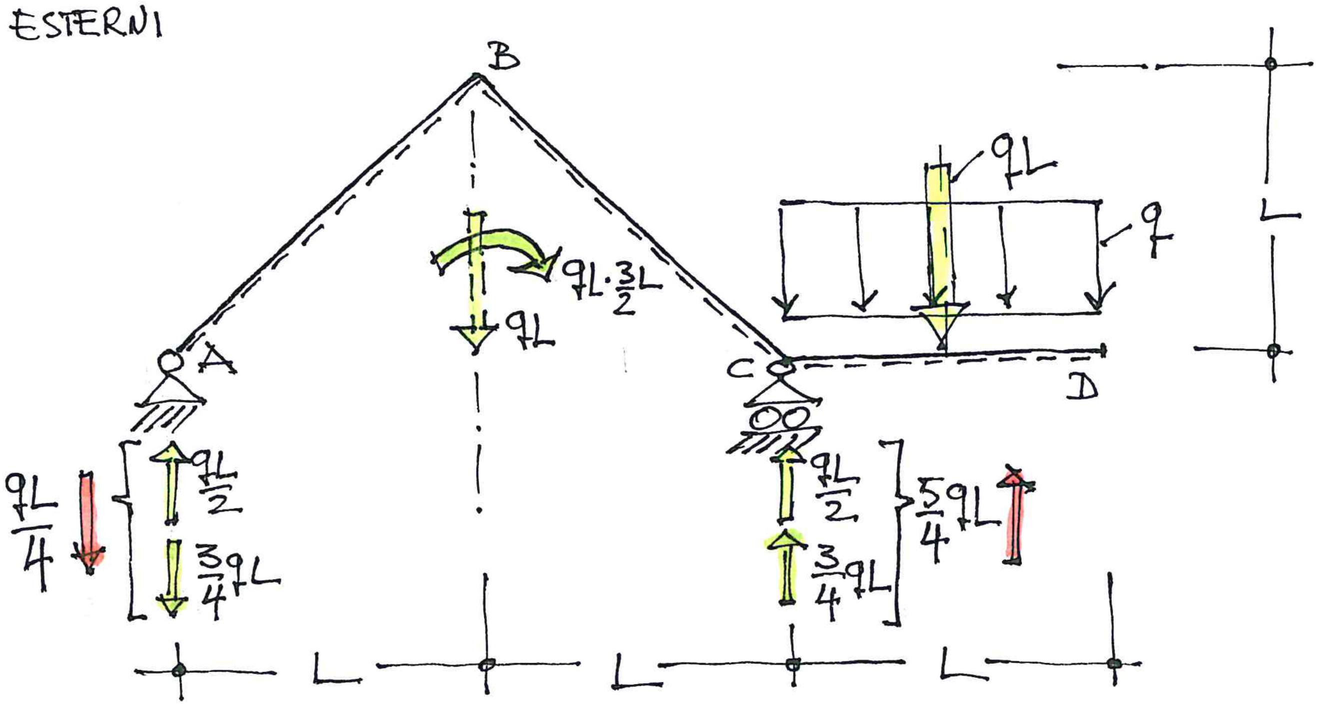
SOLUZIONE 1

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]

SOLO CARICHI ESTERNI



I. Si calcolano le RV con metodo grafico!... vedi figura!... sposta qL!

II. Si calcola $M^{(0)}(z)$ sui singoli tratti, si ha:

III

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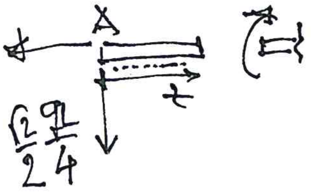
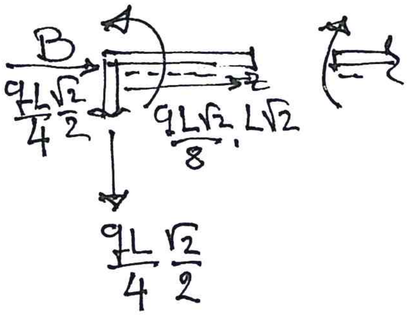
TRATTO AB $\quad 0 \le z \le L\sqrt{2}$



$$\boxed{M^{(0)}(z) = -\frac{qL\sqrt{2}}{8}z}$$

$$\begin{cases} M_A = \phi \\ M_B = -\dfrac{qL^2}{4} \end{cases}$$

TRATTO BC $\quad 0 \le z \le L\sqrt{2}$

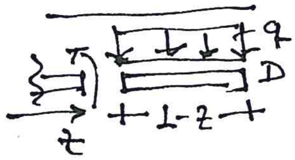


$$\boxed{M^{(0)}(z) = -\frac{qL^2}{4} - \frac{qL\sqrt{2}}{8}\cdot z}$$

$$\begin{cases} M_B = -\dfrac{qL^2}{4} \\ M_C = -\dfrac{qL^2}{4} - \dfrac{qL\sqrt{2}}{8}\cdot L\sqrt{2} = \end{cases}$$

$$= -\frac{qL^2}{4} - \frac{qL^2}{4} = -\frac{qL^2}{2}$$

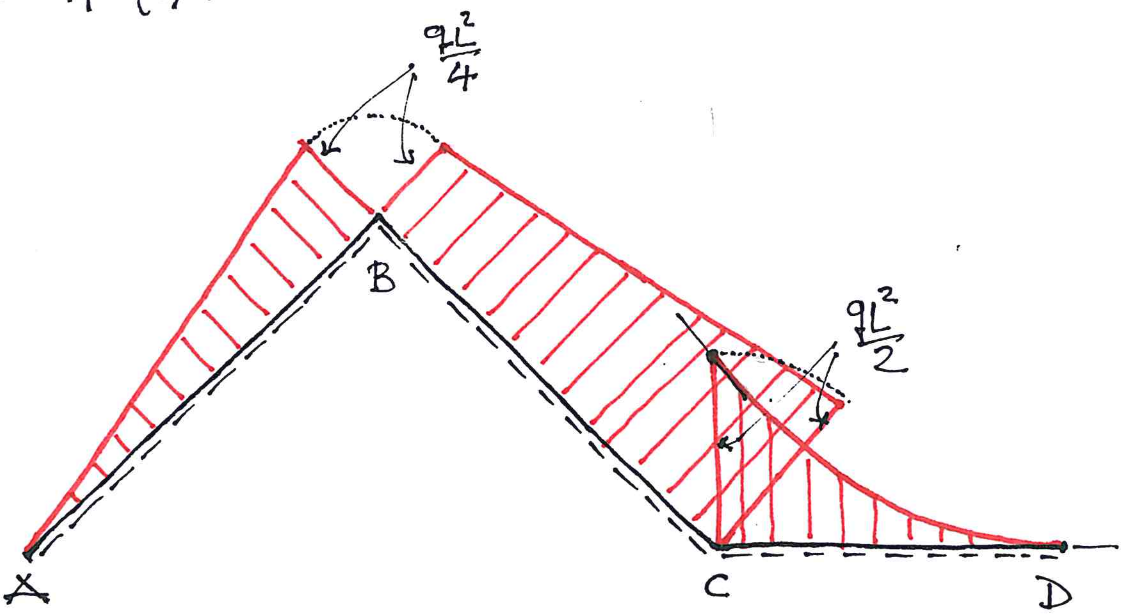
TRATTO CD $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{q(L-z)^2}{2}}$$

$$\begin{cases} M_C = -\dfrac{qL^2}{2} \\ M_D = \phi \end{cases}$$

Diagramma $M^{(0)}(z)$:



IV

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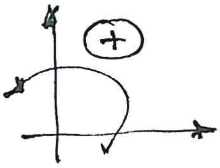
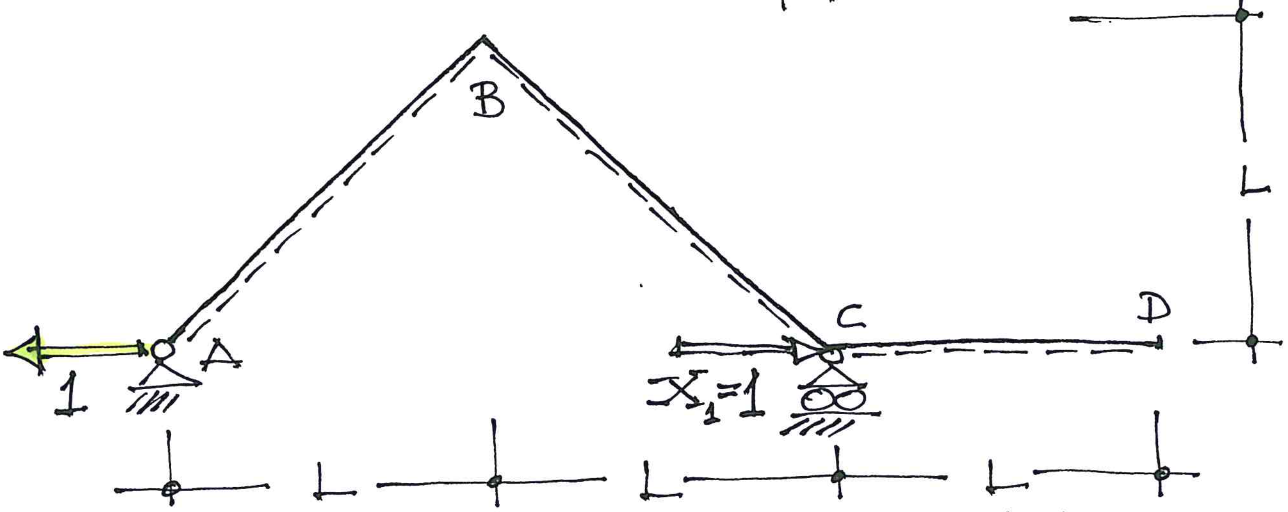
SCHEMA [1]

solo $X_1 = 1$

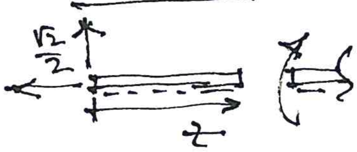




I. Si calcolano le RV con metodo grafico !.... immediate !

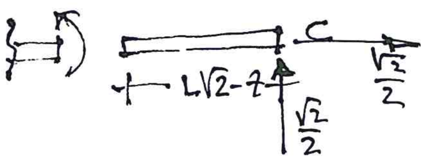
II. Si calcola $M^{(1)}(z)$ sui singoli tratti, si ha:

TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{2} z} \qquad \begin{cases} M_A = \phi \\ M_B = \frac{\sqrt{2}}{2} \cdot L\sqrt{2} = L \end{cases}$$

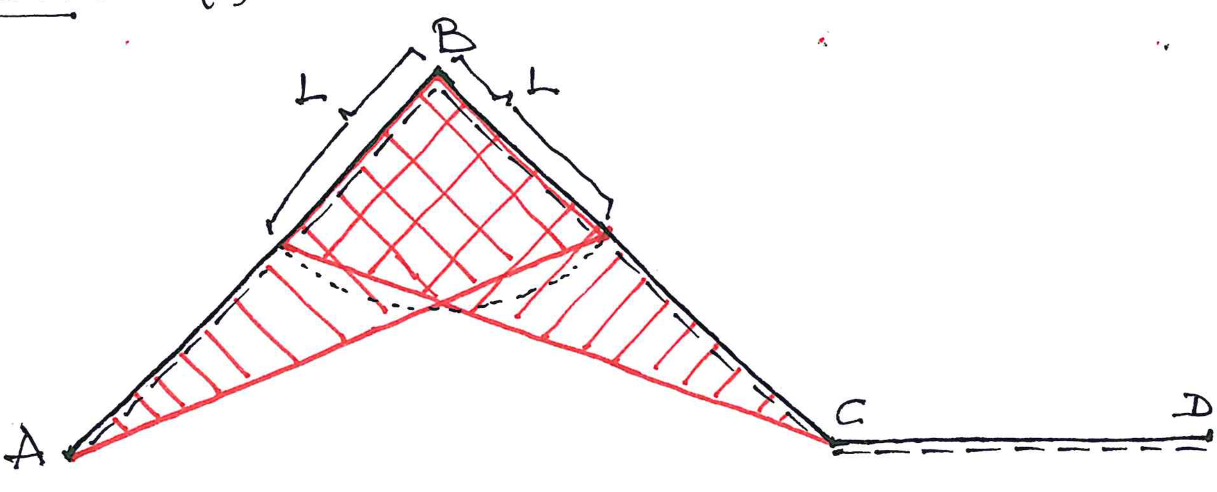
TRATTO BC $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{2}\left[L\sqrt{2} - z\right]} \qquad \begin{cases} M_B = L \\ M_C = \phi \end{cases}$$

TRATTO CD $0 \le z \le L \Rightarrow$ SCARICO !!

DIAGRAMMA $M^{(1)}(z)$:



V

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L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = \underset{1''}{X_i^{(f)}} \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \underbrace{\eta_C^{(r)}}_{-\varepsilon X_1} + \underbrace{R_{X_A}^{(1)}}_{-1} \cdot \underbrace{\eta_A^{o}}_{\substack{>0 \\ \text{verso dx!}}} = -\varepsilon X_1 - \eta_A^{o}$$

$$L_{vi} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} \, dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} \, dStr =$$

$$\underbrace{M^{(f)} \equiv M^{(1)}} \qquad \underbrace{M^{(r)} = M^{(0)} + M^{(1)} X_1}$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} \, dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} \, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} \, dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} \left[\frac{\sqrt{2}}{2} z\right]\left[-\frac{qL\sqrt{2}}{8} z\right] dz + \int_{BC} \frac{\sqrt{2}}{2}\left[L\sqrt{2} - z\right]\left[-\frac{qL^2}{4} - \frac{qL\sqrt{2}}{8} z\right] dz \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \int_{AB} \left[\frac{\sqrt{2}}{2} z\right]^2 dz + \int_{BC} \left[\frac{\sqrt{2}}{2}\left(L\sqrt{2} - z\right)\right]^2 dz \right\}$$

$$+ \frac{\alpha \Delta T}{h} \int_{AB} \frac{\sqrt{2}}{2} z \, dz =$$

VI

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$$= \frac{1}{EI}\left[\int_{\overline{AB}} -\frac{qL}{8} z^2\, dz + \frac{\sqrt{2}}{2}\int_{\overline{BC}}\left\{-\frac{qL^3\sqrt{2}}{4} - \cancel{\frac{qL^2 2z}{8}} + \cancel{\frac{qL^2}{4}z} + \frac{qL\sqrt{2}}{8}z^2\right\}dz\right]$$

$$+ \frac{X_1}{EI}\left\{\int_{\overline{AB}} \frac{z^2}{2}\, dz + \frac{1}{2}\int_{\overline{BC}}\left(2L^2 + z^2 - 2L\sqrt{2}\,z\right)dz\right\} + \frac{\alpha\overline{\Delta T}}{h}\int_{\overline{AB}} \frac{\sqrt{2}}{2} z\, dz =$$

$$= \frac{1}{EI}\left[-\frac{qL}{8}\int_0^{L\sqrt{2}} z^2\, dz + \frac{\sqrt{2}}{2}\int_0^{L\sqrt{2}}\left[-\frac{qL^3\sqrt{2}}{4} + \frac{qL\sqrt{2}}{8}z^2\right]dz\right] +$$

$$+ \frac{X_1}{EI}\left\{\int_0^{L\sqrt{2}} \frac{z^2}{2}\, dz + \frac{1}{2}\int_0^{L\sqrt{2}}\left[2L^2 + z^2 - 2L\sqrt{2}\,z\right]dz\right\} + \frac{\alpha\overline{\Delta T}}{h}\int_0^{L\sqrt{2}} \frac{\sqrt{2}}{2} z\, dz =$$

$$= \frac{1}{EI}\left\{-\frac{qL}{8}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \cancel{\frac{\sqrt{2}}{2}}\cdot\left[-\frac{qL^3\cancel{\sqrt{2}}}{4}\left[z\right]_0^{L\sqrt{2}} + \frac{qL\cancel{\sqrt{2}}}{8}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}}\right]\right\} +$$

$$+ \frac{X_1}{EI}\left\{\frac{1}{2}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + L^2\left[z\right]_0^{L\sqrt{2}} + \frac{1}{2}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} - L\sqrt{2}\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}}\right\} + \frac{\alpha\overline{\Delta T}\sqrt{2}}{2h}\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}} =$$

$$= \frac{1}{EI}\left[-\cancel{\frac{qL^4}{12}\sqrt{2}} - \frac{qL^4}{4}\sqrt{2} + \cancel{\frac{qL^4}{12}\sqrt{2}}\right] +$$

$$+ \frac{X_1}{EI}\left\{\frac{L^3}{3}\sqrt{2} + \cancel{L^3\sqrt{2}} + \frac{L^3}{3}\sqrt{2} - \cancel{L^3\sqrt{2}}\right\} + \frac{\alpha\overline{\Delta T}}{h}\frac{\sqrt{2}}{2}L^2 =$$

$$= -\frac{qL^4}{4EI}\sqrt{2} + \frac{X_1}{EI}\frac{2}{3}L^3\sqrt{2} + \frac{\alpha\overline{\Delta T}}{h}\frac{\sqrt{2}}{2}L^2$$

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In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\varepsilon X_1 - \eta_A^0 = -\frac{qL^4}{4EI}\sqrt{2} + \frac{X_1}{EI}\frac{2}{3}L^3\sqrt{2} + \frac{\alpha\Delta T}{h}\frac{\sqrt{2}}{2}L^2$$

quest'ultima, tenendo conto delle posizioni iniziali, si scrive:

$$-\frac{L^3\sqrt{2}}{3\cancel{EI}}X_1 - \cancel{\frac{qL^4\sqrt{2}}{4EI}} = -\cancel{\frac{qL^4\sqrt{2}}{4EI}} + \frac{X_1}{\cancel{EI}}\frac{2}{3}L^3\sqrt{2} + \frac{qL^2}{3\cancel{EI}}\cdot\frac{\sqrt{2}}{2}L^2$$

$$-L^3\cancel{\sqrt{2}}\,X_1 = \frac{qL^4\cancel{\sqrt{2}}}{6} \Rightarrow X_1 = -\frac{qL}{6} \Rightarrow$$ NEGATIVA!

Verso effettivo opposto a quello ipotizzato!

SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO

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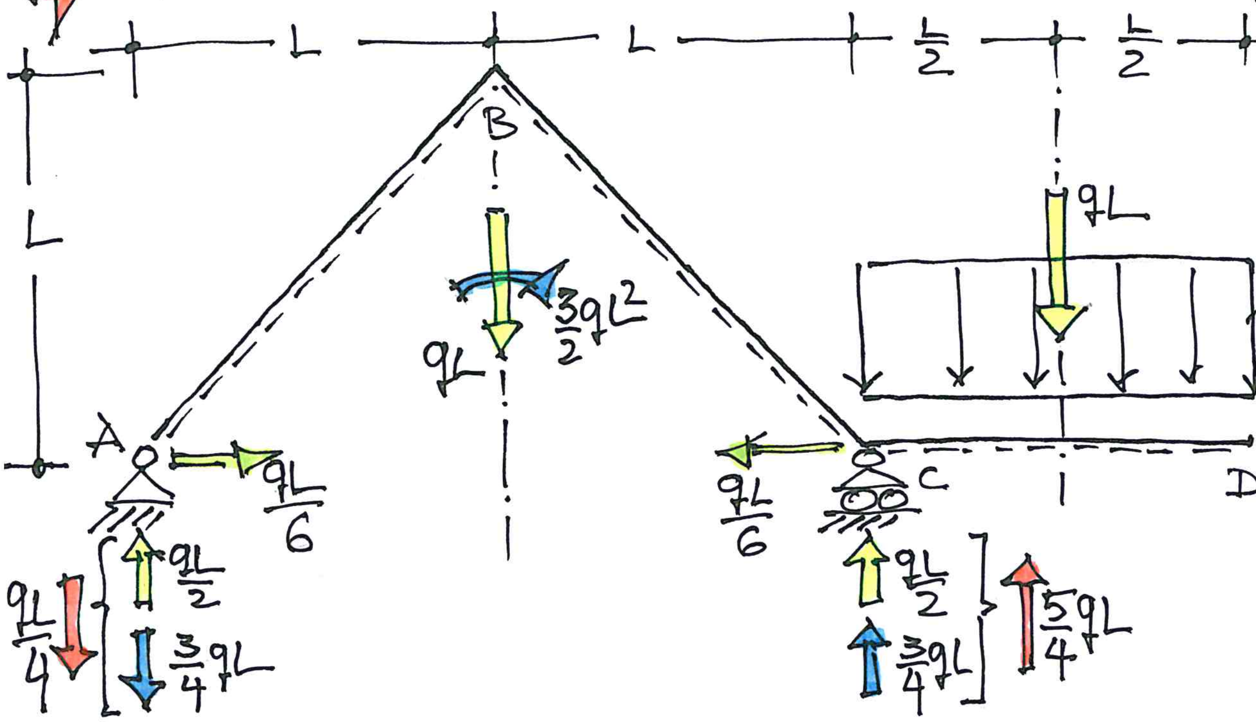
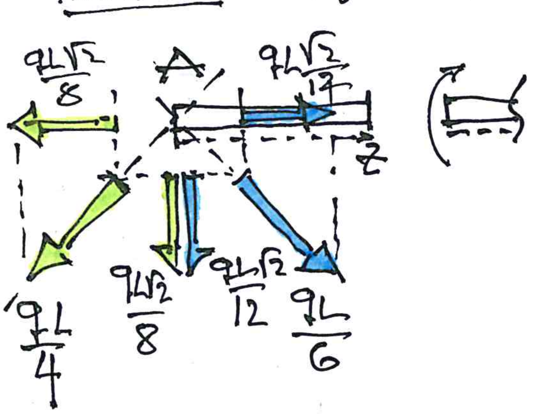
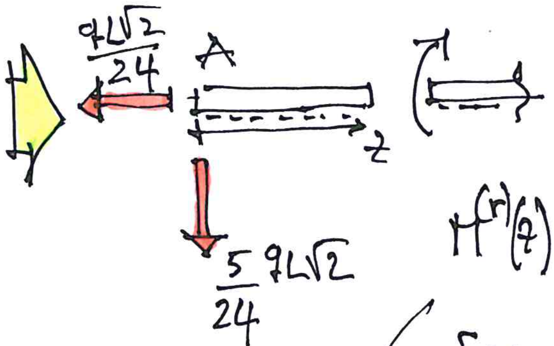


I. Si calcolano le RV con metodo grafico e principio di sovrapposiz. degli effetti! ... vedi figura.

II. Si calcola $M^{(r)}(z)$ sui singoli tratti, si ha:

TRATTO AB $0 \le z \le L\sqrt{2}$

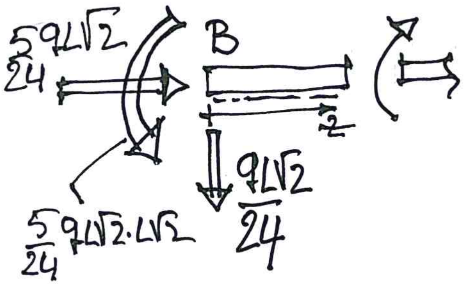




$$M^{(r)}(z) = -\frac{5}{24}\, qL\sqrt{2}\cdot z$$

$$\begin{cases} M_A = \phi \\ M_B = -\frac{5}{12}\, qL^2 \end{cases}$$

TRATTO BC $0 \le z \le L\sqrt{2}$ (trasporta le azioni da A)



$$M^{(r)}(z) = -\frac{5}{12}\, qL^2 - \frac{qL\sqrt{2}}{24}\cdot z \quad \begin{cases} M_B = -\frac{5}{12}\, qL^2 \\ M_C = -\frac{qL^2}{2} \end{cases}$$

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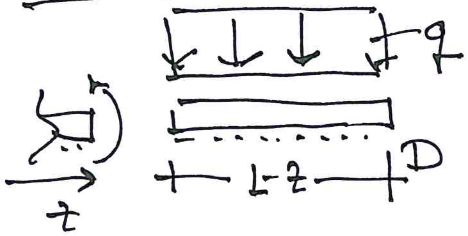
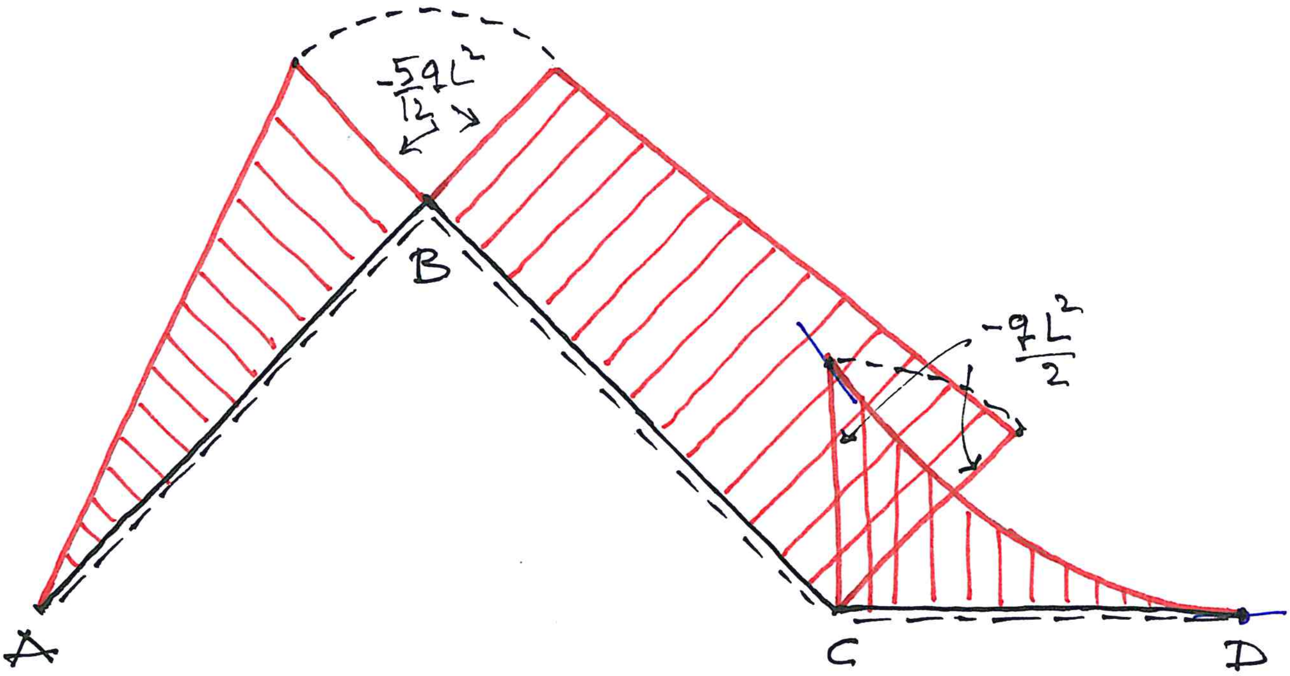
TRATTO CD $\quad 0 \le z \le L$



$$M^{(r)}(z) = -\frac{q(L-z)^2}{2}$$

$$\begin{cases} M_C = -\dfrac{qL^2}{2} \\ M_D = \phi \end{cases}$$

DIAGRAMMA FINALE $M^{(r)}(z)$



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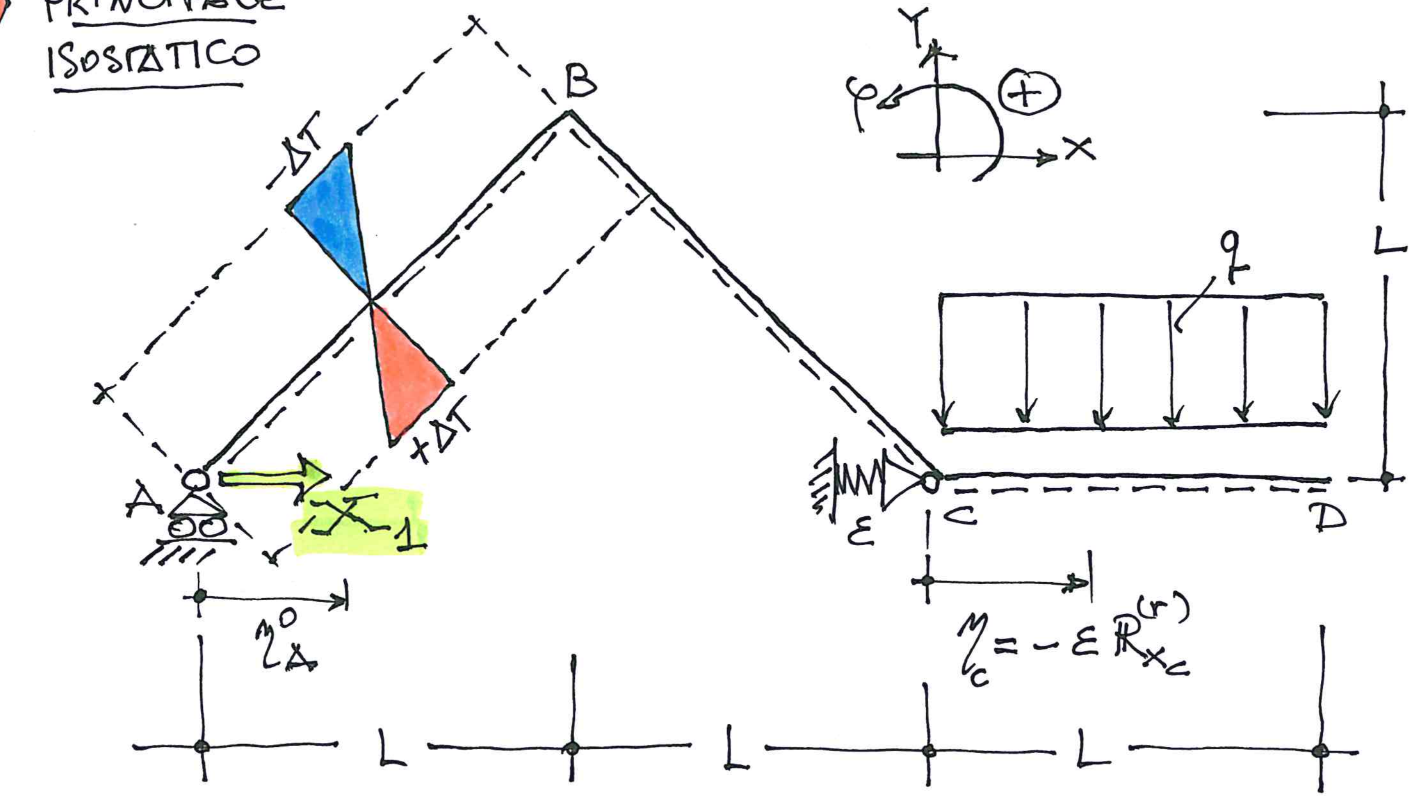
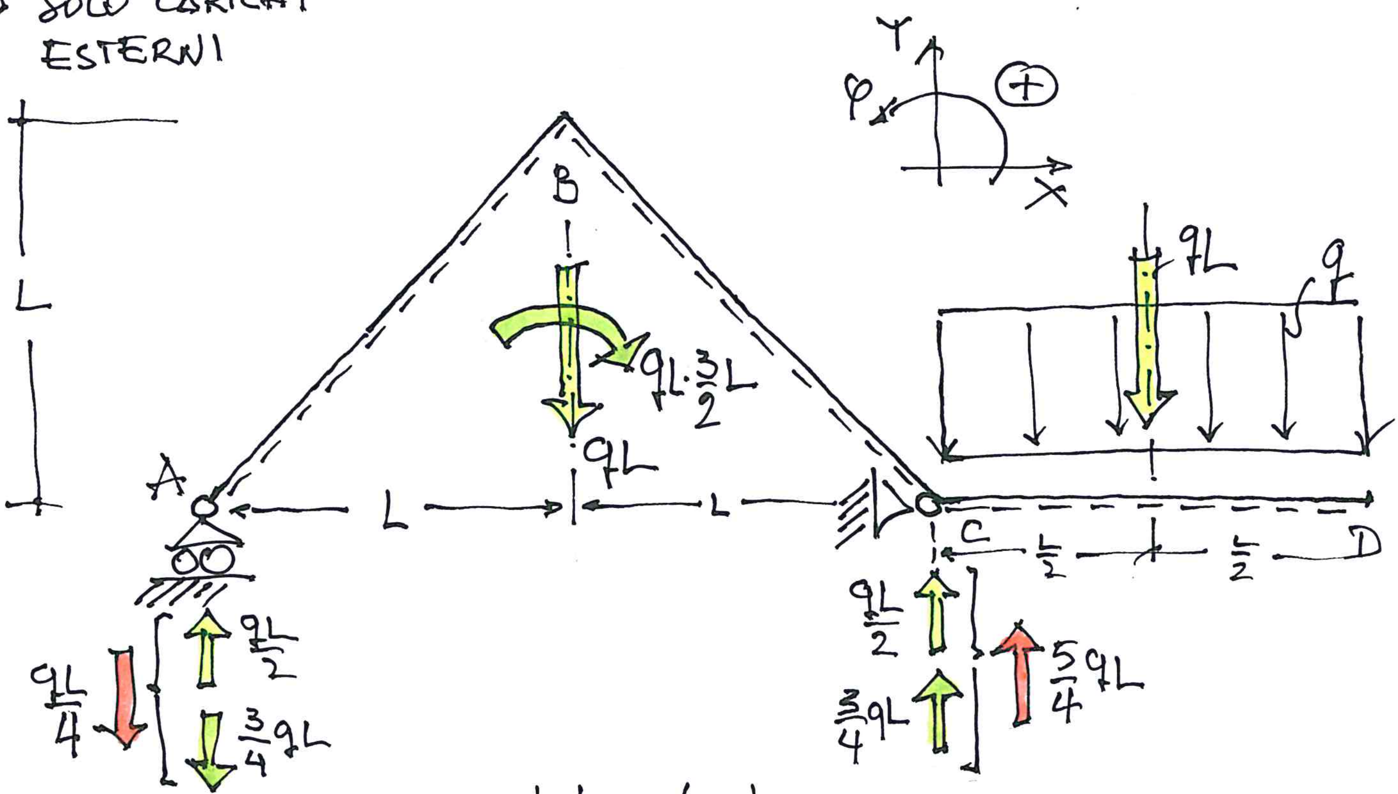
SOLUZIONE 2

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]
SOLO CARICHI ESTERNI



I. Si calcolano RV con metodo grafico!

II. Le leggi di $M^{(0)}(t)$ sono IDENTICHE a quelle della Soluz. n. 1 ∇ cfr. p. III

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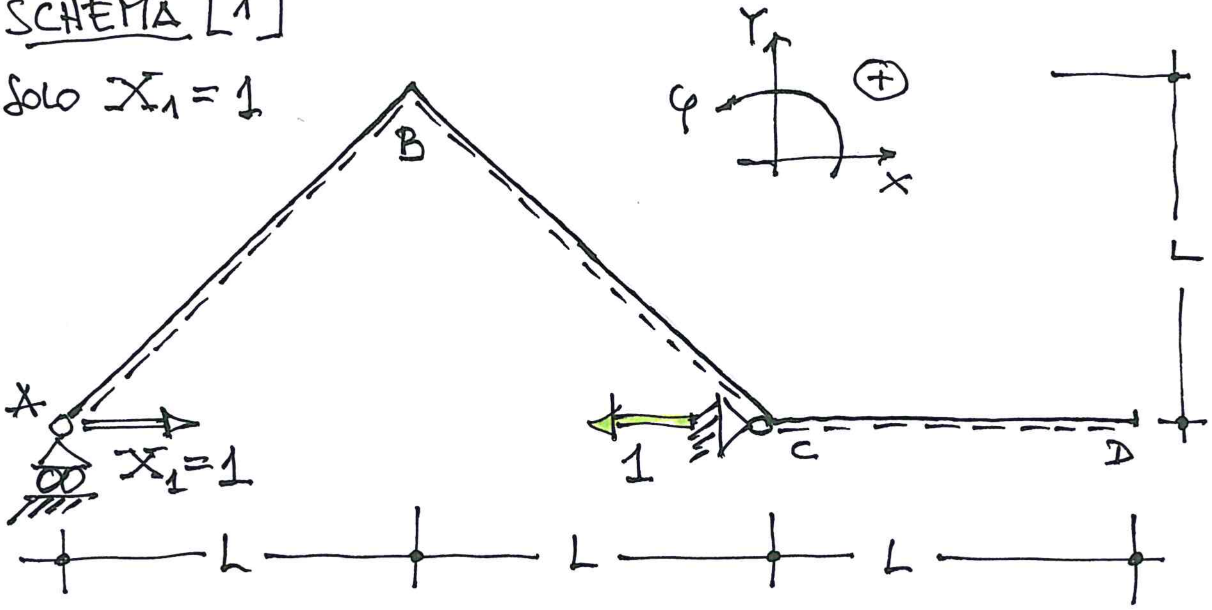
SCHEMA [1]

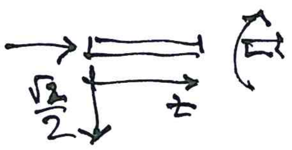
solo $X_1 = 1$



I. Si calcolano le RV con metodo grafico!... immediate!

II. Si calcola $M^{(1)}(z)$ sui singoli tratti, si ha:

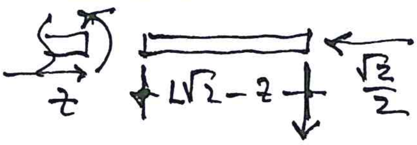
TRATTO AB $0 \leq z \leq L\sqrt{2}$



$$\boxed{M^{(1)}(z) = -\frac{\sqrt{2}}{2} z}$$

$$\begin{cases} M_A = \phi \\ M_B = -\frac{\sqrt{2}}{2} \cdot L\sqrt{2} = -L \end{cases}$$

TRATTO BC $0 \leq z \leq L\sqrt{2}$

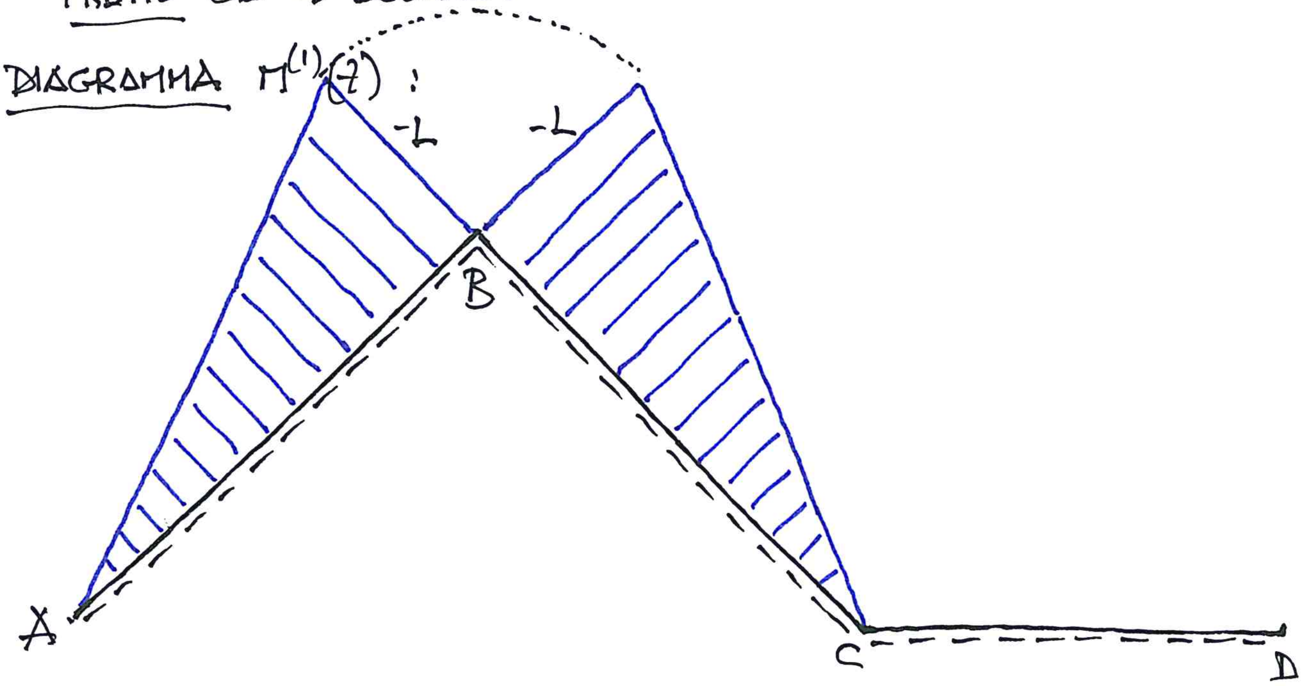


$$\boxed{M^{(1)}(z) = -\frac{\sqrt{2}}{2}\left[L\sqrt{2} - z\right]}$$

$$\begin{cases} M_B = -L \\ M_C = \phi \end{cases}$$

TRATTO CD → SCARICO!

DIAGRAMMA $M^{(1)}(z)$:



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L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \eta_A^0 + \underbrace{R_{x_C}^{(1)}}_{=-1} \cdot \underbrace{\eta_C^{(r)}}_{=-\varepsilon \underbrace{R_{x_C}^{(r)}}_{R_{x_C}^{(0)} + R_{x_C}^{(1)} X_1}} = \eta_A^0 - \varepsilon X_1$$

$$R_{x_C}^{(0)} = \emptyset \qquad R_{x_C}^{(1)} = -1$$

$$L_{vi} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} dStr =$$

$$M^{(f)} \equiv M^{(1)} \qquad M^{(r)} = M^{(0)} + M^{(1)} X_1$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \frac{1}{EI} \left[\int_{\overline{AB}} \left[-\frac{\sqrt{2}}{2} z \right] \left[-\frac{qL\sqrt{2}}{8} z \right] dz + \int_{\overline{BC}} -\frac{\sqrt{2}}{2} \left[L\sqrt{2} - z \right] \left[-\frac{qL^2}{4} - \frac{qL\sqrt{2}}{8} z \right] dz \right] +$$

$$+ \frac{X_1}{EI} \left[\int_{\overline{AB}} \left[-\frac{\sqrt{2}}{2} z \right]^2 dz + \int_{\overline{BC}} \left[-\frac{\sqrt{2}}{2} \left(L\sqrt{2} - z \right) \right]^2 dz \right] +$$

$$+ \frac{\alpha \Delta T}{h} \int_{\overline{AB}} -\frac{\sqrt{2}}{2} z \, dz$$

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N.B. Confrontando l'ultima espressione con quella analoga della soluzione n. 1 (a pag. V) si osserva che il primo addendo (con termine $\frac{1}{EI}$ in evidenza) è uguale a quello già sviluppato per la soluz. 1 cambiato di segno; il secondo addendo (con $\frac{X_1}{EI}$) è identico a quello visto in precedenza in quanto le leggi di $M^{(1)}$, pur essendo negative in questo caso sono da considerarsi al quadrato; il terzo addendo infine è di nuovo pari a quello di pag. V cambiato di segno!

Risulta quindi (cfr. p. VI):

$$L_{vi} = \frac{qL^4}{4EI}\sqrt{2} + \frac{X_1}{EI}\frac{2}{3}L^3\sqrt{2} - \frac{\alpha\Delta T}{h}\frac{\sqrt{2}}{2}L^2$$

In definitiva $L_{ve} = L_{vi}$ fornisce in questo caso:

$$\eta_A^0 - \varepsilon X_1 = \frac{qL^4}{4EI}\sqrt{2} + \frac{X_1}{EI}\frac{2L^3\sqrt{2}}{3} - \frac{\alpha\Delta T}{h}\frac{\sqrt{2}}{2}L^2$$

Tenendo conto delle posizioni iniziali e semplificando si ha:

$$\cancel{\frac{qL^4}{4EI}\sqrt{2}} - \frac{L^3\cancel{\sqrt{2}}}{3EI}X_1 = \cancel{\frac{qL^4}{4EI}\sqrt{2}} + \frac{X_1}{EI}\frac{2L^3}{3}\cancel{\sqrt{2}} - \frac{\alpha\Delta T}{h}\frac{\cancel{\sqrt{2}}}{2}L^2$$

$$\frac{L^3}{\cancel{EI}}X_1 = \frac{qL^2}{3\cancel{EI}}\frac{L^2}{2} \quad \Rightarrow \quad X_1 = \frac{qL}{6} \quad \text{positiva!}$$

Ok! cfr. p. VIII con la RV ottenuta a partire dalla soluz. 1!