ESAMI di MECCANICA delle STRUTTURE - L 17

Corso P.FUSCHI - A.A. 2016-17 - prova scritta 7.06.17

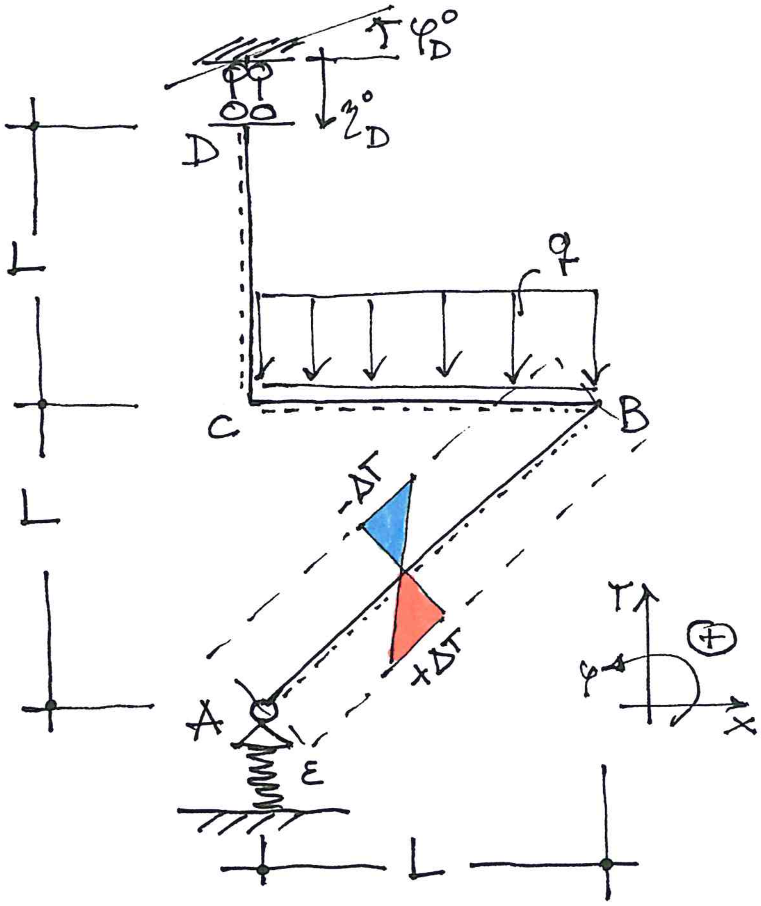
I

P.FUSCHI
A.PISANO

SOLUZIONE

Quesito n. 1

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA RIPORTATA IN FIGURA TRACCIANDO IL DIAGRAMMA DEI MOMENTI



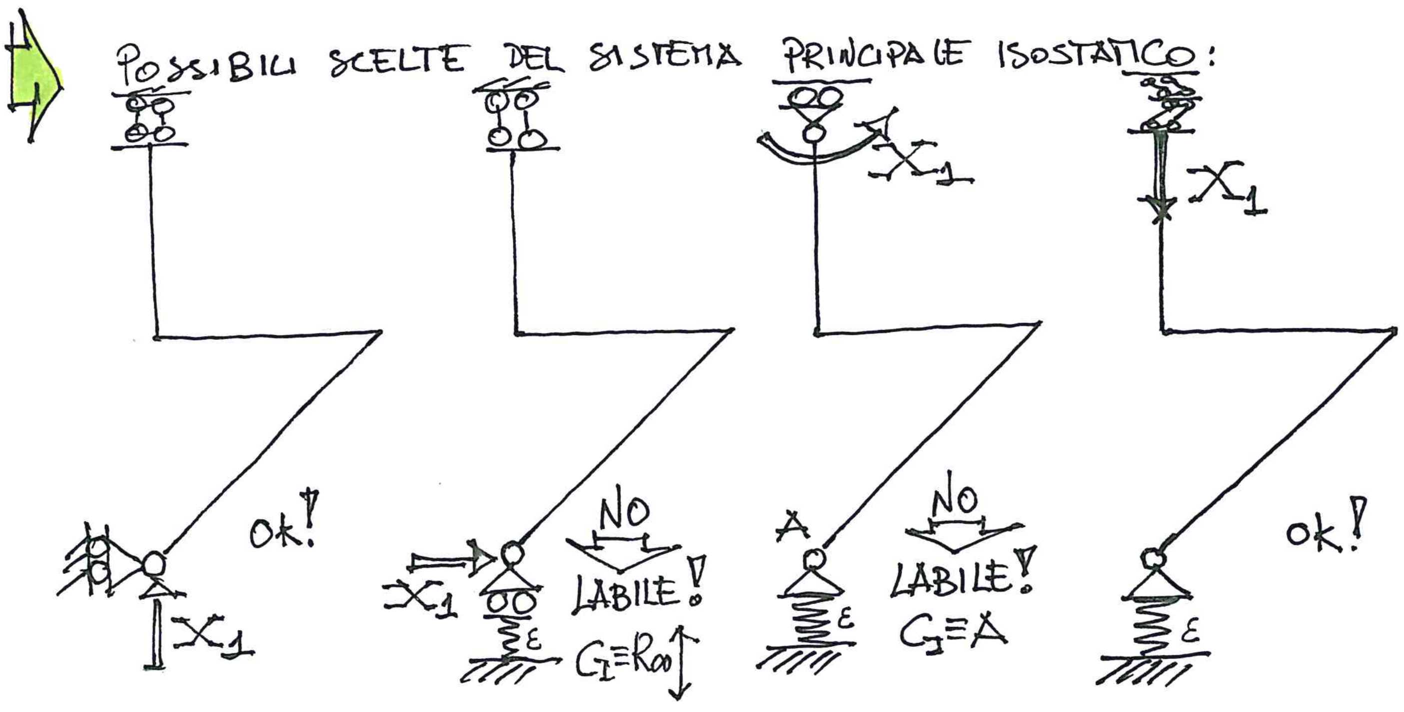
Posizioni:

$$\left|\eta_D^0\right| = \frac{qL^4}{24EI}$$

$$\left|\varepsilon\right| = \frac{L^3}{3EI}\left[\sqrt{2}+1\right]$$

$$\left|\alpha\frac{\Delta T}{h}\right| = \frac{qL^2}{3EI}\frac{\left[\sqrt{2}+1\right]}{\sqrt{2}}$$

POSSIBILI SCELTE DEL SISTEMA PRINCIPALE ISOSTATICO:



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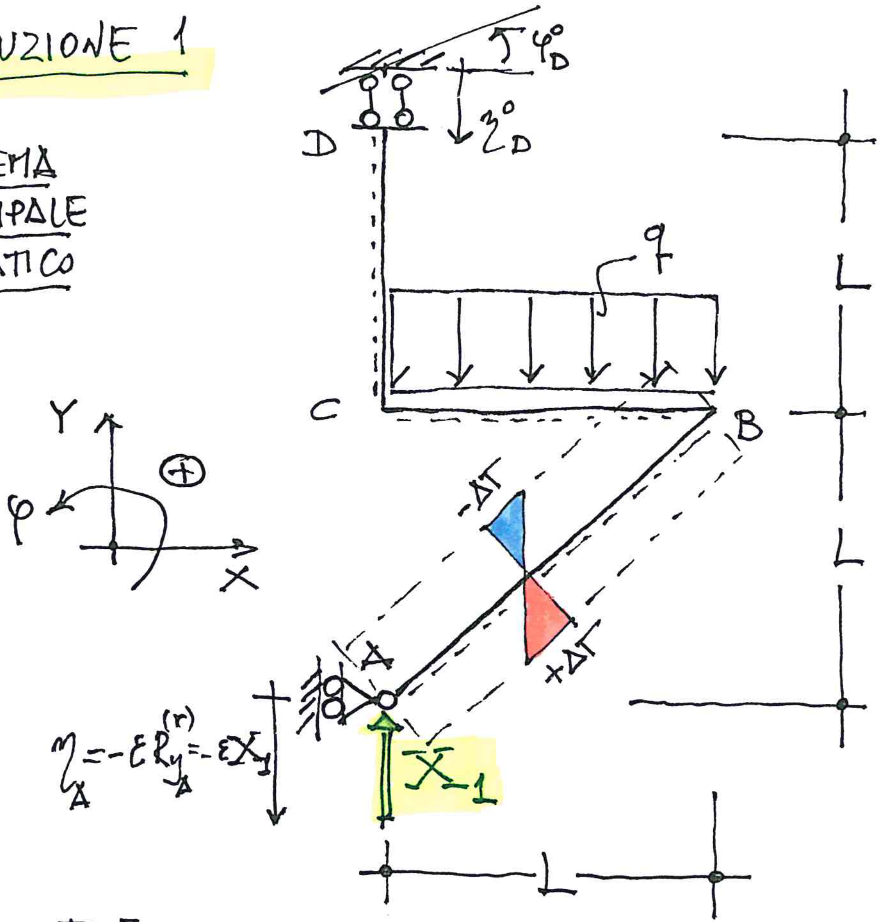
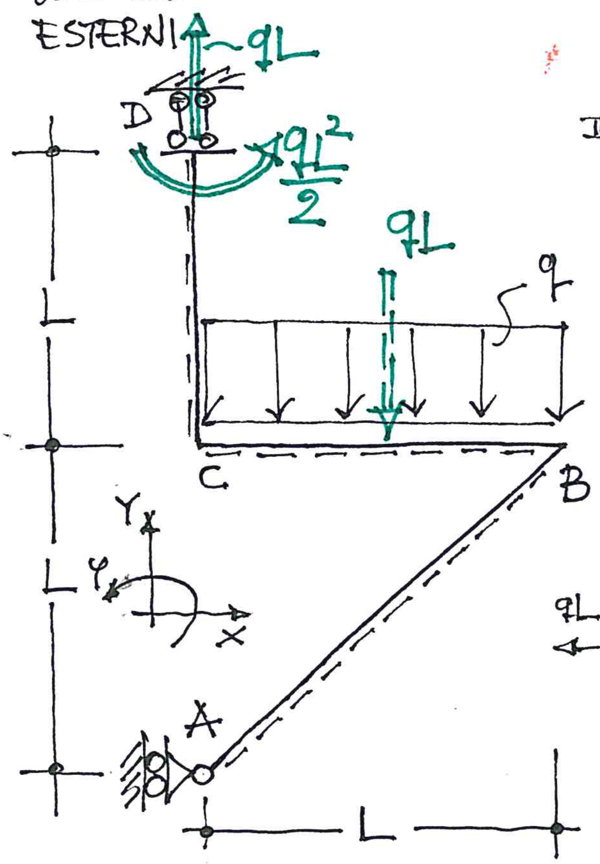
SOLUZIONE 1

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]
SOLO CARICHI ESTERNI

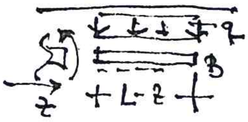


I. Si calcolano RV con metodo grafico! ... vedi figura!

II. Si calcola $M^{(0)}(z)$ sui singoli tratti, si ha:

TRATTO AB → SCARICO!

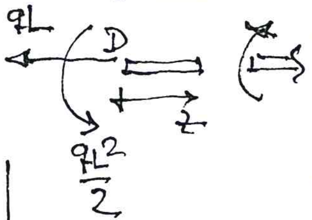
TRATTO CB $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{q\,(L-z)^2}{2}}$$

$$\begin{cases} M_C = -\dfrac{qL^2}{2} \\ M_B = \emptyset \end{cases}$$

TRATTO DC $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2}}$$

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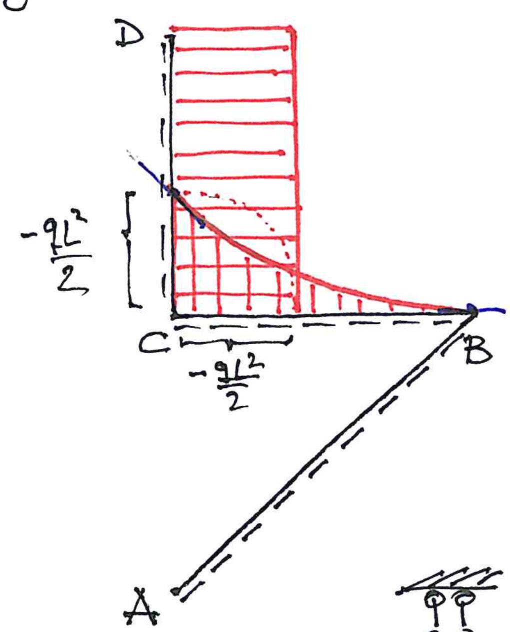
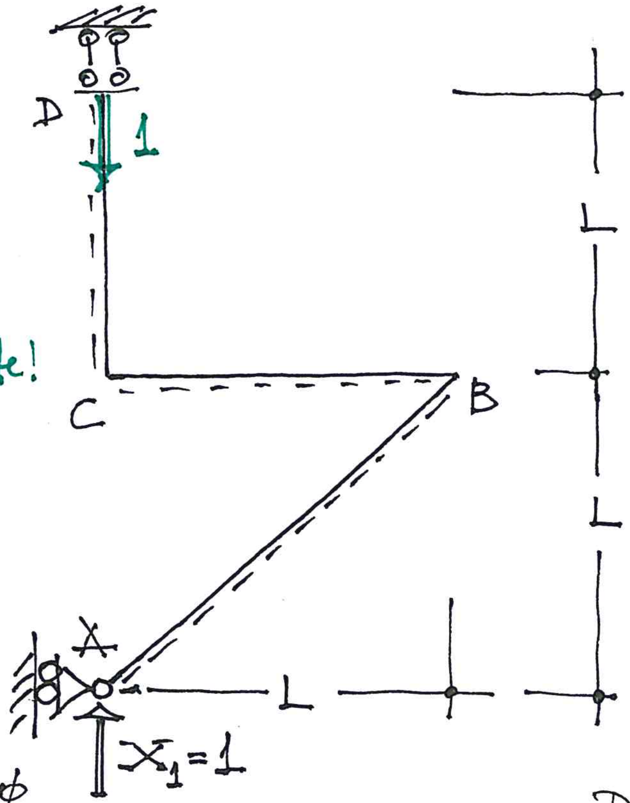
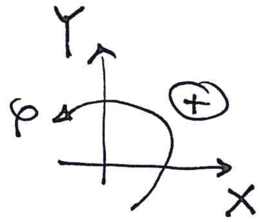
Diagramma $M^{(0)}(z)$:



SCHEMA [1]
solo $X_1 = 1$

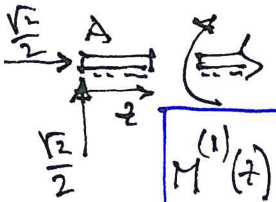




I. Si calcolano le RV con metodo grafico! ..immediate!

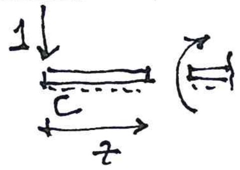
II. Si calcola $M^{(1)}(z)$ sui singoli tratti, si ha:

TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{2} \cdot z} \quad \begin{cases} M_A = \phi \\ M_B = \frac{\sqrt{2}}{2} \cdot L\sqrt{2} = L \end{cases}$$

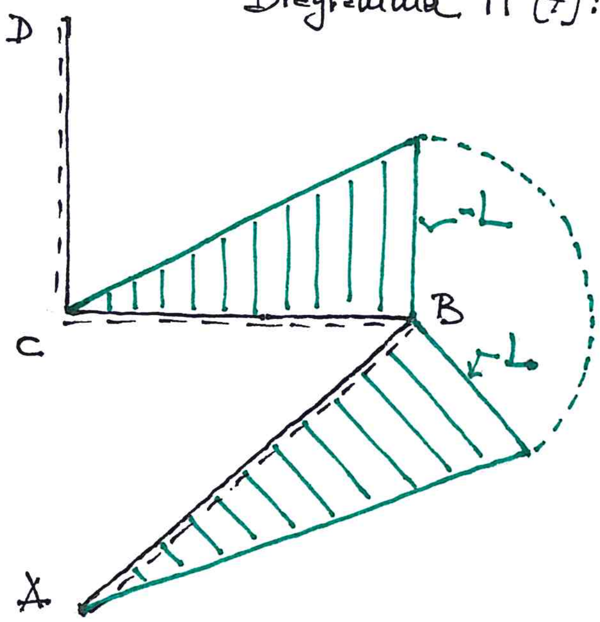
TRATTO CB $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -z} \quad \begin{cases} M_C = \phi \\ M_B = -L \end{cases}$$

TRATTO BC $0 \le z \le L$
scarico!! (solo N)

Diagramma $M^{(1)}(z)$:



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A. PISANO

L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data.

Si ha:

$$L_{ve} = \underset{\overset{\shortparallel}{1}}{X_i^{(f)}} \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \underbrace{\eta_A^{(r)}}_{\substack{-\varepsilon \underbrace{R_{y_A}^{(r)}}_{X_1}}} + \underbrace{M_D^{(1)}}_{\substack{\shortparallel \\ \emptyset}} \varphi_D^{(o)} + \underbrace{R_{y_D}^{(1)}}_{-1} \cdot \underbrace{\eta_D^{o}}_{\substack{<0 \\ \text{verso il basso}}} = -\varepsilon X_1 + \eta_D^{o}$$

$$L_{vi} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(f)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$\to M^{(f)} = M^{(1)} \qquad \to M^{(r)} = M^{(o)} + M^{(1)} X_1$$

$$= \int_{Str} M^{(1)} \frac{M^{(o)}}{EI} dStr + X_1 \int_{Str} \frac{[M^{(1)}]^2}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$L^2 + z^2 - 2Lz$$

$$= \frac{1}{EI} \left[\int_{CB} [-z] \left[-\frac{q}{2} (L-z)^2 \right] dz \right] +$$

$$+ \frac{X_1}{EI} \left[\int_{AB} \left[\frac{\sqrt{2}}{2} z \right]^2 dz + \int_{CB} [-z]^2 dz \right] + \frac{\alpha \overline{\Delta T}}{h} \int_{AB} \frac{\sqrt{2}}{2} z \, dz =$$

$$= \frac{1}{EI} \left[\int_0^L \left(\frac{q}{2} L^2 z + \frac{q}{2} z^3 - qL z^2 \right) dz \right] +$$

$$+ \frac{X_1}{EI} \left[\int_0^{L\sqrt{2}} \frac{1}{2} z^2 dz + \int_0^L z^2 dz \right] + \frac{\alpha \overline{\Delta T}}{h} \frac{\sqrt{2}}{2} \int_0^{L\sqrt{2}} z \, dz =$$

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$$= \frac{1}{EI}\left\{ \frac{qL^2}{2}\left[\frac{z^2}{2}\right]_0^L + \frac{q}{2}\left[\frac{z^4}{4}\right]_0^L - qL\left[\frac{z^3}{3}\right]_0^L \right\} +$$

$$+ \frac{X_1}{EI}\left\{ \frac{1}{2}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \left[\frac{z^3}{3}\right]_0^L \right\} + \frac{\alpha\overline{\Delta T}}{h}\frac{\sqrt{2}}{2}\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}} =$$

$$= \frac{1}{EI}\left[\frac{qL^4}{4} + \frac{qL^4}{8} - \frac{qL^4}{3} \right] + \frac{X_1}{EI}\left[\frac{L^3\sqrt{2}}{3} + \frac{L^3}{3} \right] + \frac{\alpha\overline{\Delta T}}{h}\cdot\frac{\sqrt{2}}{2}L^2 =$$

$$= \frac{qL^4}{EI}\cdot\frac{1}{24} + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] + \frac{\alpha\overline{\Delta T}}{h}\cdot\frac{\sqrt{2}}{2}L^2$$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\varepsilon X_1 + \eta_D^0 = \frac{qL^4}{24EI} + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] + \frac{\alpha\overline{\Delta T}}{h}\cdot\frac{\sqrt{2}}{2}L^2$$

quest'ultima, tenendo conto delle posizioni iniziali, si scrive:

$$-X_1\cdot\frac{2\cancel{L^3}}{\cancel{3EI}}\cancel{\left[\sqrt{2}+1\right]} + \cancel{\frac{qL^4}{24EI}} = \cancel{\frac{qL^4}{24EI}} + \frac{q\cancel{L^3}}{\cancel{3EI}}\frac{\cancel{\left[\sqrt{2}+1\right]}}{\cancel{\sqrt{2}}}\cdot\frac{\sqrt{2}}{2}\cancel{L^2}$$

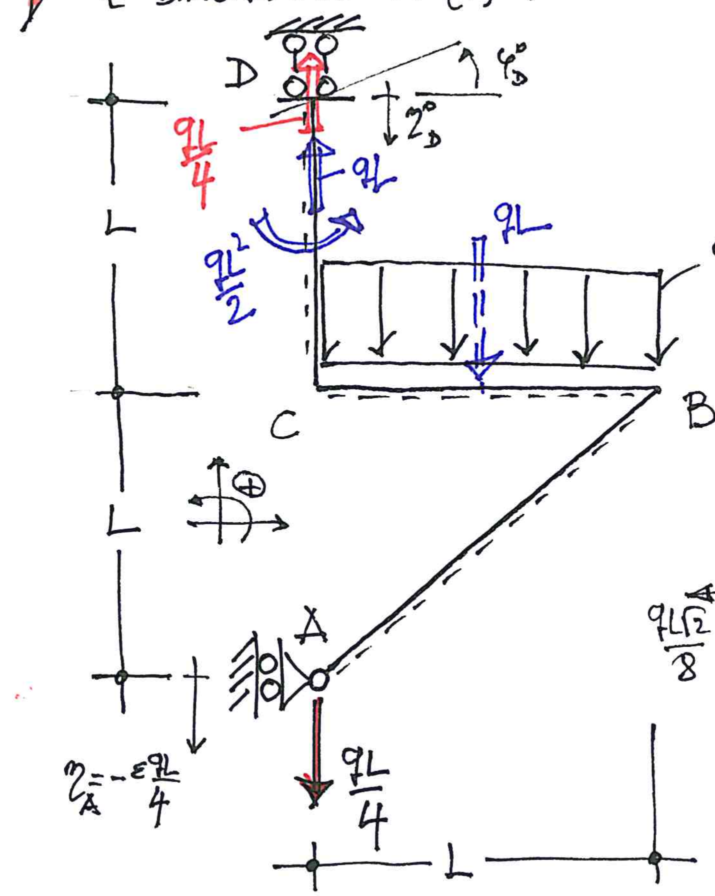
da cui: $X_1 = -\frac{qL}{4}$ ⇒ NEGATIVA!

⇓

Verso effettivo CONTRARIO A QUELLO IPOTIZZATO! CHE ERA VERSO L'ALTO! quindi ESSA VALE $\frac{qL}{4}$ VERSO IL BASSO!

P. FUSCHI
A. PISANO

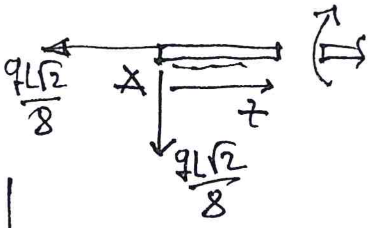
SOLUZIONE SISTEMA PRINCIPALE ISOSTATICO E DIAGRAMMA $M^{(r)}(z)$ FINALE



1. RV con metodo grafico e principio di sovrapp. degli effetti!

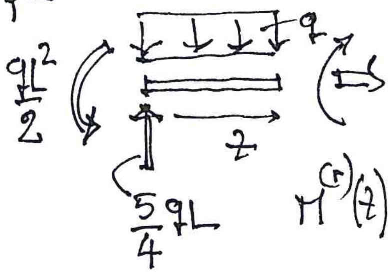
2. Calcolo di $M^{(r)}(z)$ sui singoli tratti:

TRATTO AB $0 \le z \le L\sqrt{2}$



$$M^{(r)}(z) = -\frac{qL\sqrt{2}}{8}\cdot z \quad \begin{cases} M_A = \phi \\ M_B = -\frac{qL^2}{4} \end{cases}$$

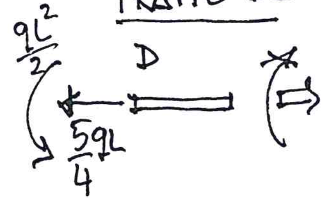
TRATTO CB $0 \le z \le L$



$$M^{(r)}(z) = \frac{5}{4} qL\cdot z - \frac{qz^2}{2} - \frac{qL^2}{2}$$

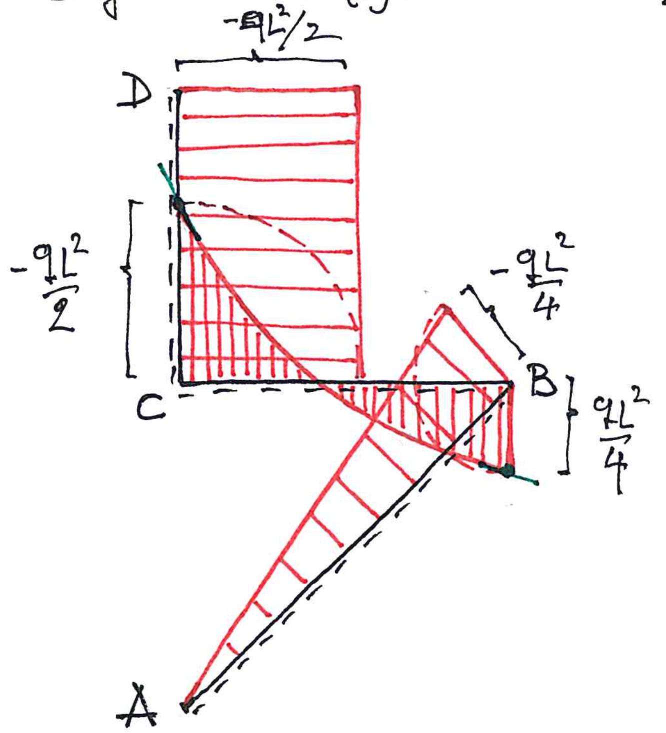
$$\begin{cases} M_C = -\frac{qL^2}{2} \\ M_B = \frac{qL^2}{4} \end{cases}$$

TRATTO DC $0 \le z \le L$



$$M^{(r)}(z) = -\frac{qL^2}{2}$$

Diagramma $M^{(r)}(z)$:



Soluzione n. 2

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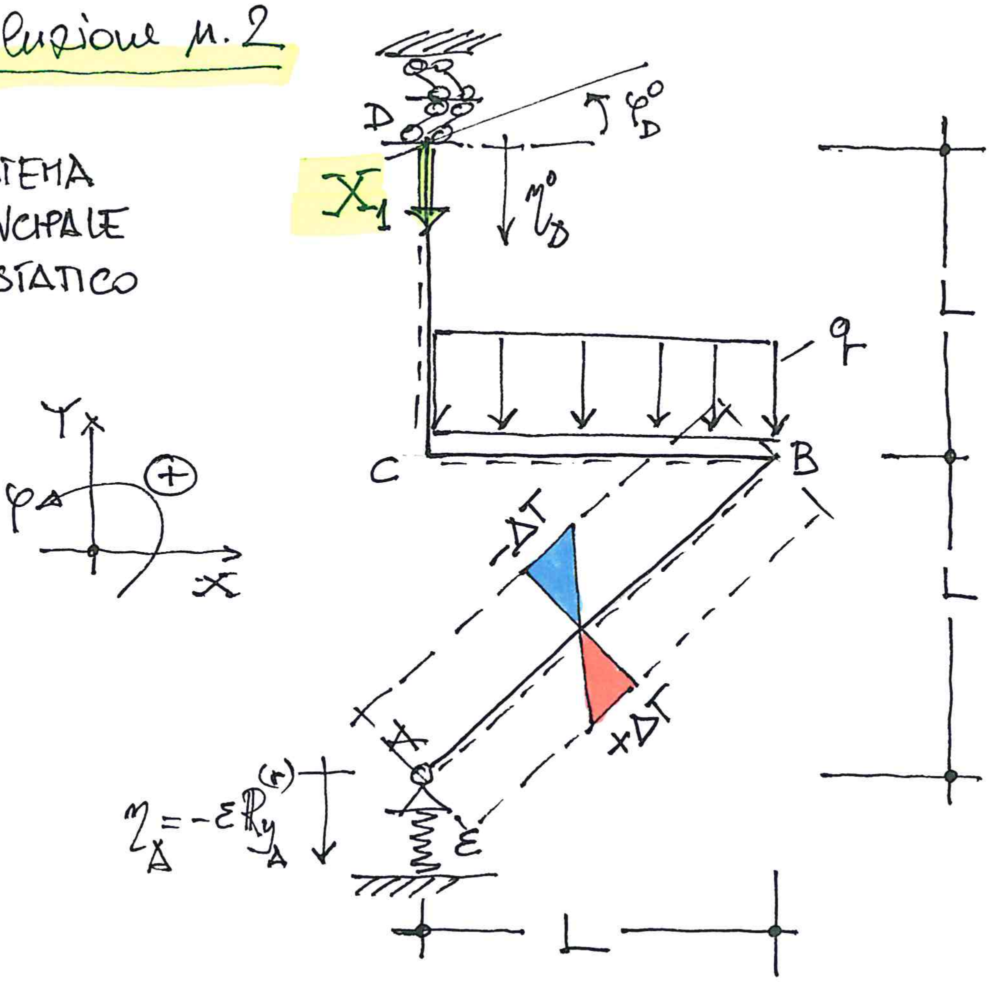
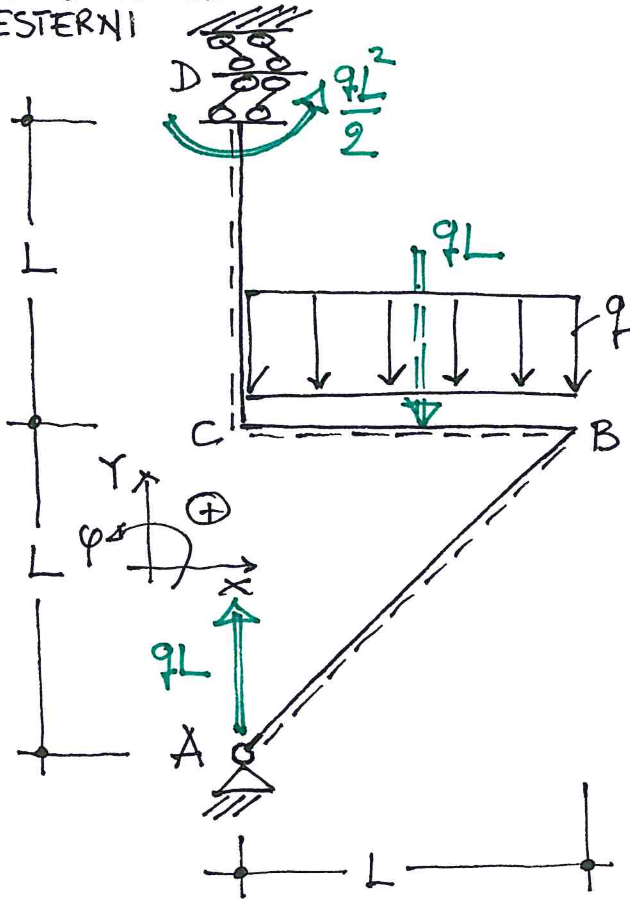
A. PISANO

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]

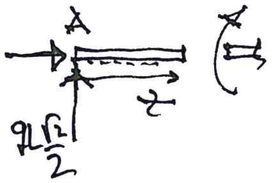
SOLO CARICHI ESTERNI



I. Si calcolano le RV con metodo grafico... vedi figura!

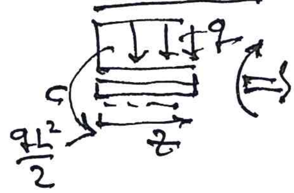
II. Si calcola $M^{(0)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $\quad 0 \le z \le L\sqrt{2}$



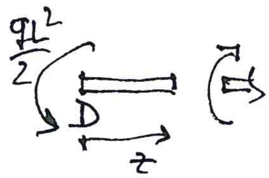
$$\boxed{M^{(0)}(z) = \frac{qL\sqrt{2}}{2} \cdot z} \qquad \begin{cases} M_A = 0 \\ M_B = qL^2 \end{cases}$$

TRATTO CB $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2} - \frac{qz^2}{2}} \qquad \begin{cases} M_C = -\frac{qL^2}{2} \\ M_B = -q \end{cases}$$

TRATTO DC $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = -\frac{qL^2}{2}}$$

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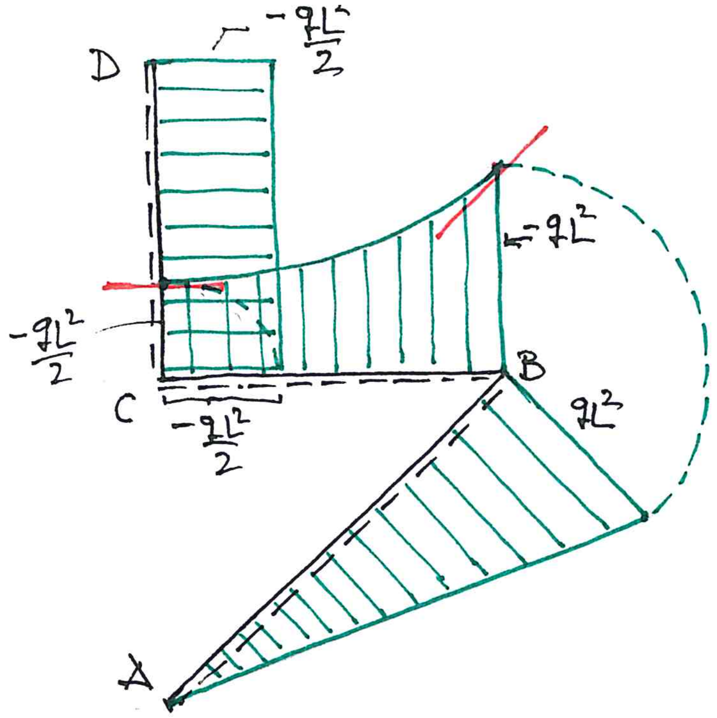
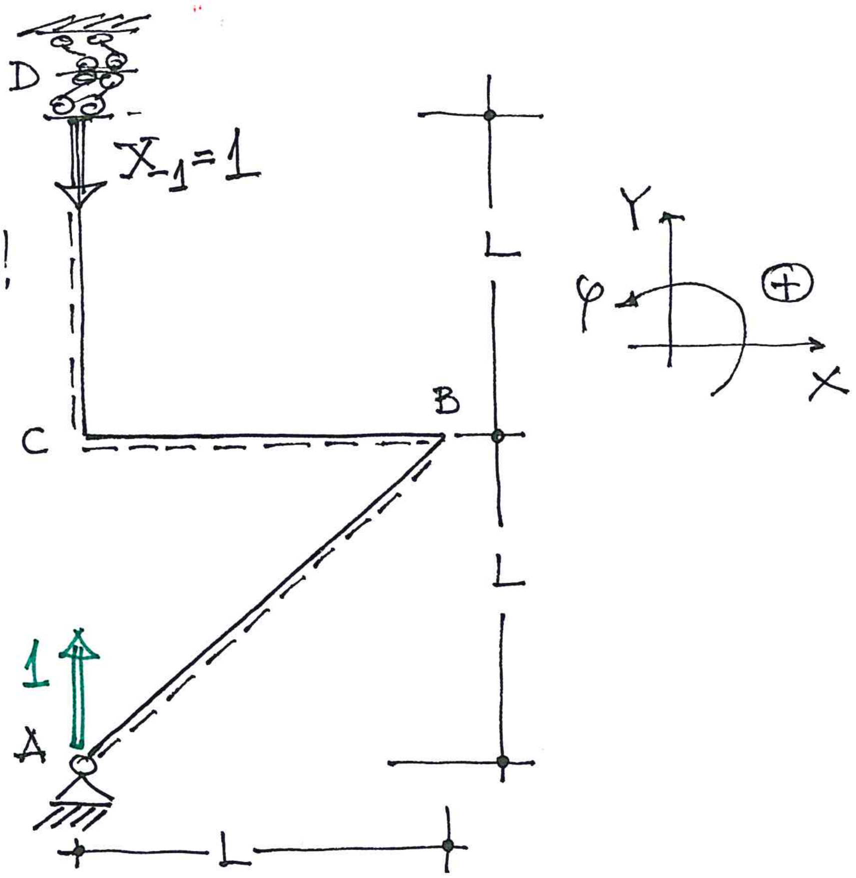
Diagramma $M^{(0)}(z)$



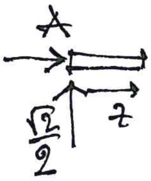
SCHEMA [1]
Solo $X_1 = 1$



I. Si calcolano RV con metodo grafico!.. immediate!

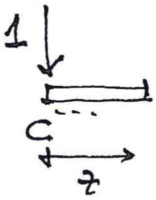
II. Si calcola $M^{(1)}(z)$ sui singoli tratti. Si ha:

TRATTO AB $0 \leq z \leq L\sqrt{2}$



$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{2} \cdot z} \quad \begin{cases} M_A = \phi \\ M_B = L \end{cases}$$

TRATTO CB $0 \leq z \leq L$



$$\boxed{M^{(1)}(z) = -z} \quad \begin{cases} M_C = \phi \\ M_B = -L \end{cases}$$

TRATTO DC $0 \leq z \leq L$

$$M^{(1)}(z) = \phi$$

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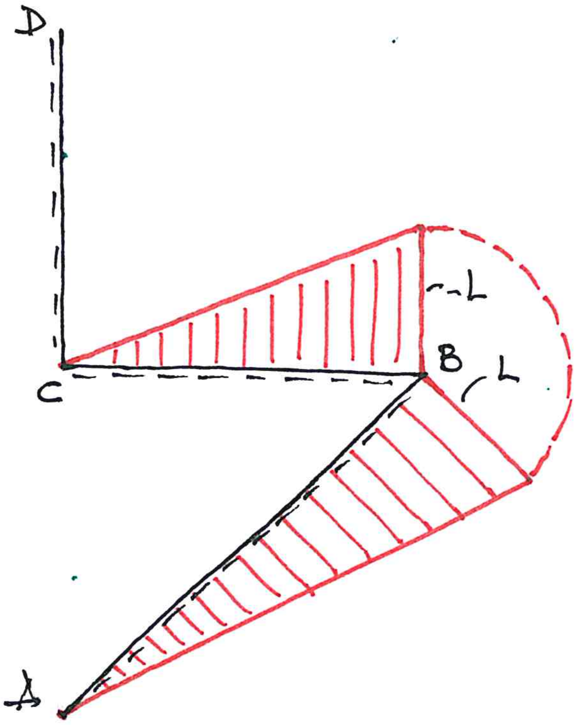
Diagramma $M^{(1)}(z)$:



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve}=L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \cdot \eta_i^{(r)} + \sum_j R_j^{(f)}\, \eta_j^{(r)} =$$

$$= (-1)(-\eta_D^{0}) + \underbrace{R_{y_A}^{(1)}}_{1} \cdot \underbrace{\eta_A^{(r)}}_{-\varepsilon R_{y_A}^{(r)}} = \eta_D^{0} - \varepsilon\left(qL + X_1\right)$$

$$R_{y_A}^{(r)} = \underbrace{R_{y_A}^{(0)}}_{qL} + \underbrace{R_{y_A}^{(1)}}_{1} \cdot X_1$$

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$$L_{vi} = \int_{Str} \overset{\to M^{(1)}}{M^{(f)}} \, \frac{\overset{\to M^{(0)}+M^{(1)}X_1}{M^{(r)}}}{EI} \, dStr + \int_{Str} M^{(f)} \, \frac{\alpha \overline{\Delta T}}{h} \, dStr =$$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} \, dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI} \, dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} \, dStr =$$

$$= \frac{1}{EI} \left\{ \int_{\overline{AB}} \left[\frac{\sqrt{2}}{2} z\right] \cdot \left[\frac{qL\sqrt{2}}{2} z\right] dz + \int_{\overline{CB}} \left[-z\right] \left[-\frac{qL^2}{2} - \frac{qz^2}{2}\right] dz \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \int_{\overline{AB}} \left(\frac{\sqrt{2}}{2} z\right)^2 dz + \int_{\overline{CB}} \left[-z\right]^2 dz \right\} + \int_{\overline{AB}} \frac{\sqrt{2}}{2} z \, \frac{\alpha \overline{\Delta T}}{h} \, dz =$$

$$= \frac{1}{EI} \left\{ \int_0^{L\sqrt{2}} \frac{qL}{2} z^2 \, dz + \int_0^L \left(\frac{qL^2}{2} z + \frac{q}{2} z^3\right) dz \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \int_0^{L\sqrt{2}} \frac{1}{2} z^2 \, dz + \int_0^L z^2 \, dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \cdot \frac{\sqrt{2}}{2} \int_0^{L\sqrt{2}} z \, dz =$$

$$= \frac{1}{EI} \left\{ \frac{qL}{2} \left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \frac{qL^2}{2} \left[\frac{z^2}{2}\right]_0^L + \frac{q}{2} \left[\frac{z^4}{4}\right]_0^L \right\} +$$

$$+ \frac{X_1}{EI} \left\{ \frac{1}{2} \left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \left[\frac{z^3}{3}\right]_0^L \right\} + \frac{\alpha \overline{\Delta T}}{h} \frac{\sqrt{2}}{2} \left[\frac{z^2}{2}\right]_0^{L\sqrt{2}} =$$

$$= \frac{1}{EI} \left[\frac{qL^4\sqrt{2}}{3} + \frac{qL^4}{4} + \frac{qL^4}{8} \right] + \frac{X_1}{EI} \left[\frac{\sqrt{2}}{3} L^3 + \frac{L^3}{3} \right] + \frac{\alpha \overline{\Delta T}}{h} \frac{\sqrt{2}}{2} L^2 =$$

$$= \frac{qL^4\sqrt{2}}{3EI} + \frac{3}{8} \frac{qL^4}{EI} + \frac{X_1 L^3}{3EI} \left[\sqrt{2} + 1\right] + \frac{\alpha \overline{\Delta T}}{h} \frac{\sqrt{2}}{2} L^2$$

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In definitiva $L_{ve} = L_{vi}$ fornisce:

$$\eta_D^0 - \varepsilon\left[qL + X_1\right] = \frac{qL^4}{3EI}\sqrt{2} + \frac{3}{8}\frac{qL^4}{EI} + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] + \alpha\frac{\Delta T}{h}\frac{\sqrt{2}}{2}L^2$$

quest'ultima, tenendo conto delle posizioni iniziali, si scrive:

$$\frac{qL^4}{24EI} - \frac{L^3}{3EI}\left[\sqrt{2}+1\right]\left[qL+X_1\right] = \frac{qL^4}{3EI}\sqrt{2} + \frac{3}{8}\frac{qL^4}{EI} + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] + \frac{qL^2}{3EI}\frac{\left[\sqrt{2}+1\right]}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{2}L^2$$

e ancora:

$$\frac{qL^4}{24EI} - \frac{L^3}{3EI}\left[\sqrt{2}+1\right]qL - \frac{L^3}{3EI}\left[\sqrt{2}+1\right]X_1 = \frac{qL^4}{3EI}\sqrt{2} + \frac{3}{8}\frac{qL^4}{EI} + \frac{X_1 L^3}{3EI}\left[\sqrt{2}+1\right] + \frac{qL^4\left[\sqrt{2}+1\right]}{6EI}$$

quest'ultima, semplificando ulteriormente, fornisce:

$$-\frac{5}{6}qL^4\left[\sqrt{2}+1\right] = \frac{2}{3}L^3\left[\sqrt{2}+1\right]X_1$$

e, in definitiva, si ha:

$$X_1 = -\frac{5}{4}qL$$

NEGATIVA! VERSO OPPOSTO A QUELLO IPOTIZZATO! ESSA È CIOÈ VERSO L'ALTO!

OK! cfr. con la R.V. di pag VI ottenuta in precedenza!