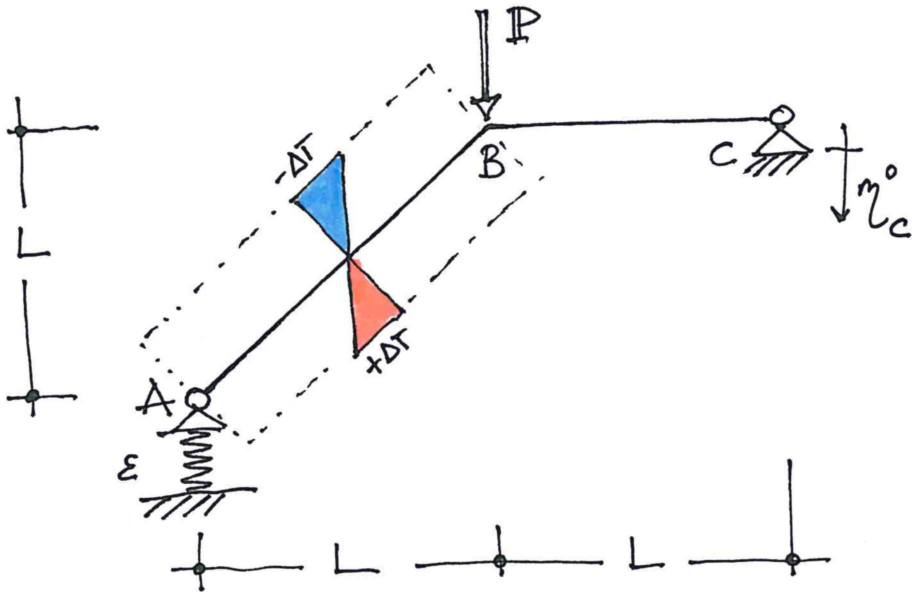
ESAMI di MECCANICA delle STRUTTURE - L 17
Corso P.FUSCHI - A.A. 2016-17 - prova scritta 8.02.17

P.FUSCHI
A.PISANO

SOLUZIONE

Quesito n.1

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA RIPORTATA IN FIGURA, TRACCIANDO IL DIAGRAMMA DEI MOMENTI



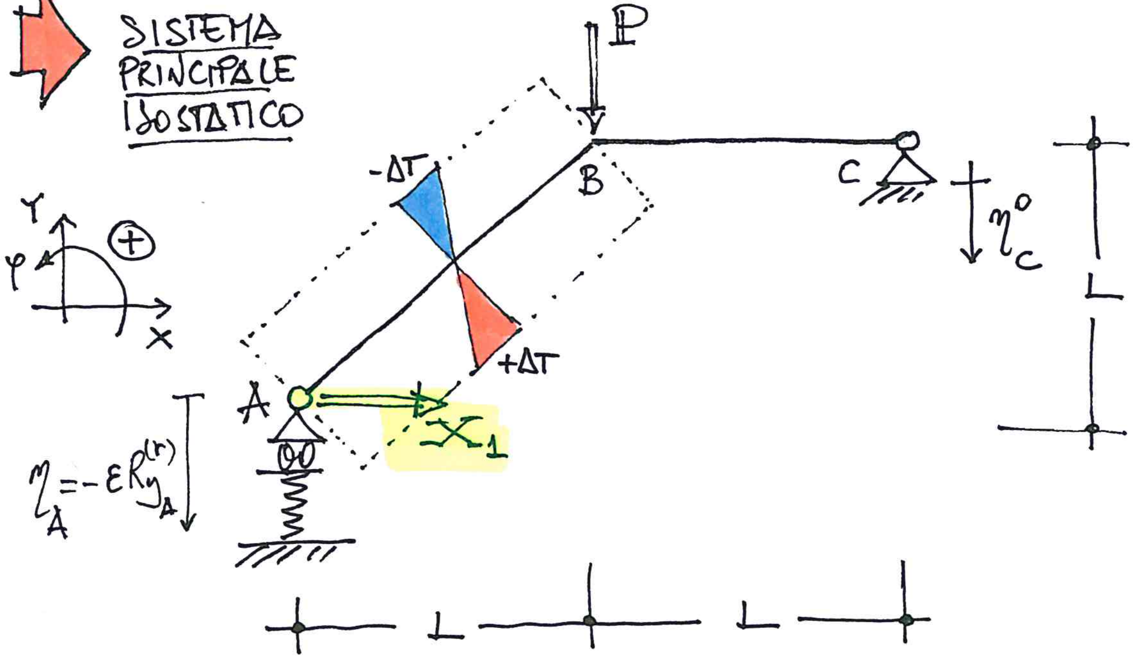
$$|\varepsilon| = \frac{L^3}{3EI}\left[\sqrt{2}+1\right]$$

$$|\eta_c^0| = \frac{PL^3}{EI}\left[\sqrt{2}+1\right]$$

$$\left|\frac{\alpha \Delta T}{h}\right| = \frac{PL}{EI}\frac{\left[\sqrt{2}+1\right]}{\sqrt{2}}$$

SOLUZIONE 1

SISTEMA PRINCIPALE ISOSTATICO



II

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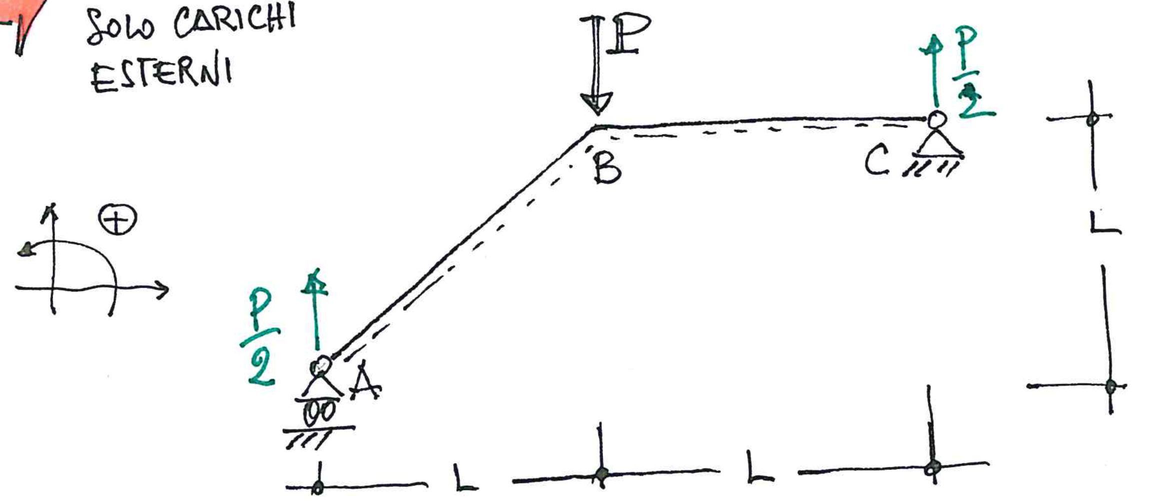
SCHEMA [0]

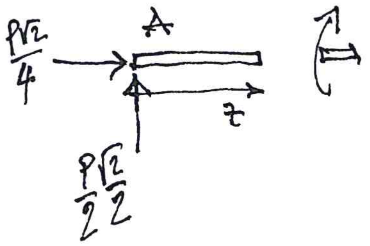
SOLO CARICHI ESTERNI



RV metodo grafico! immediata!

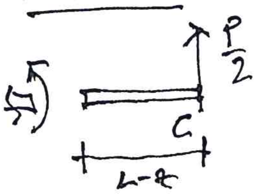
Calcoliamo $M^{(0)}(z)$:

TRATTO AB $\quad 0 \le z \le L\sqrt{2}$



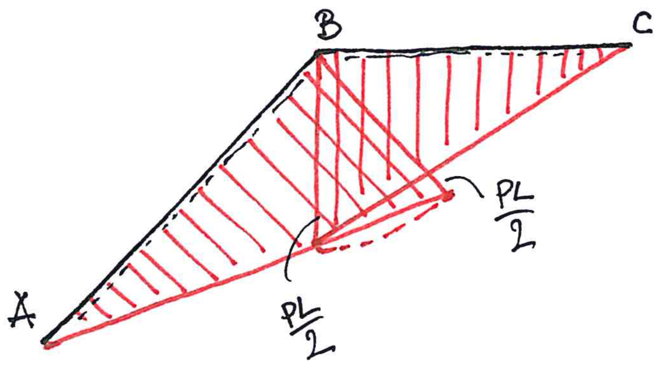
$$\boxed{M^{(0)}(z) = \frac{P\sqrt{2}}{4} \cdot z} \qquad \begin{cases} M_A = 0 \\ M_B = \frac{P\cancel{\sqrt{2}}}{\cancel{4}_{\,2}} \cdot L\cancel{\sqrt{2}} = \frac{PL}{2} \end{cases}$$

TRATTO BC $\quad 0 \le z \le L$



$$\boxed{M^{(0)}(z) = \frac{P}{2}(L-z)} \qquad \begin{cases} M_B = \frac{PL}{2} \\ M_C = \phi \end{cases}$$

Diagramma $M^{(0)}(z)$:



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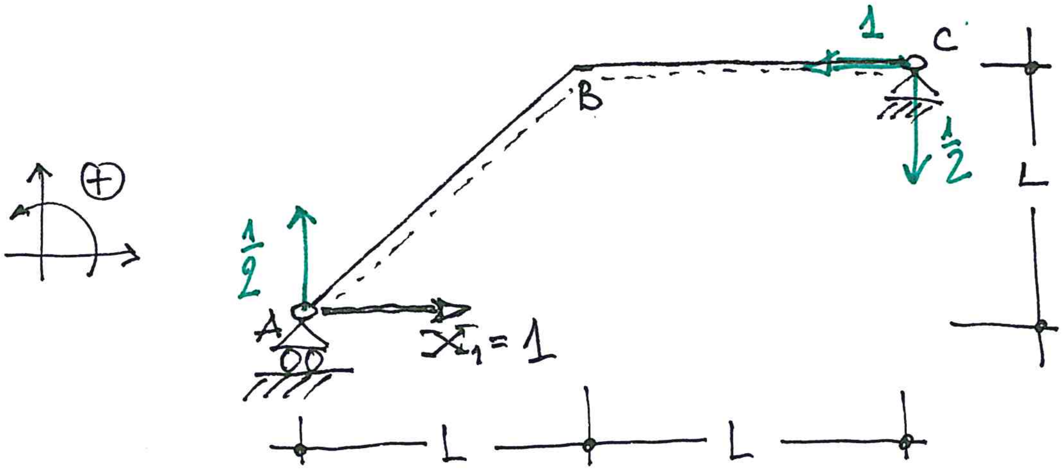
SCHEMA [1]

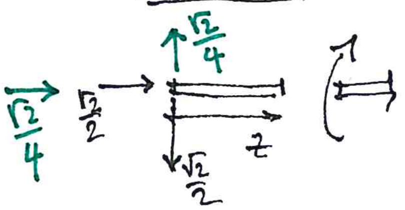
solo $X_1 = 1$



R.V. con metodo grafico!

Calcoliamo $M^{(1)}(z)$:

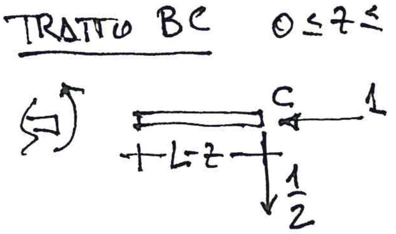
TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(1)}(z) = -\frac{\sqrt{2}}{4} z}$$

$$\begin{cases} M_A^{(1)} = 0 \\ M_B^{(1)} = -\frac{\cancel{\sqrt{2}}}{\cancel{4}_{\,2}} L \cancel{\sqrt{2}} = -\frac{L}{2} \end{cases}$$

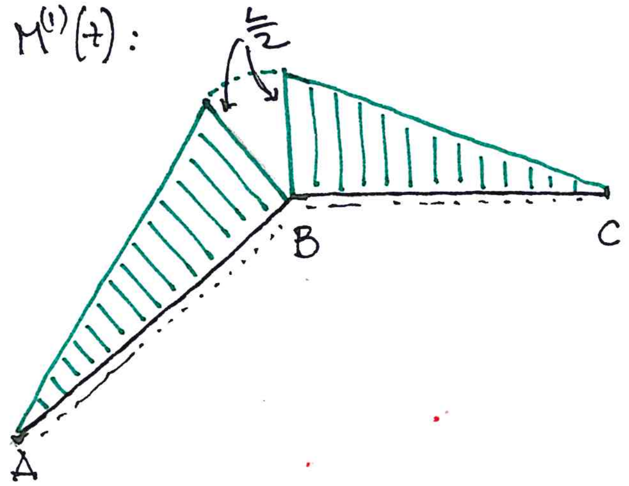
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -\frac{1}{2}(L-z)}$$

$$\begin{cases} M_B = -\frac{L}{2} \\ M_C = \phi \end{cases}$$

Diagramma $M^{(1)}(z)$:



L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve}=L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = \underbrace{X_i^{(f)}}_{=1} \cdot \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= 1 \cdot \phi + \underbrace{R_{y_A}^{(1)}}_{\frac{1}{2}} \cdot \eta_{y_A}^{(r)} + \underbrace{R_{y_C}^{(1)}}_{-\frac{1}{2}} \cdot \underbrace{\eta_C^{0}}_{<0} = -\frac{\varepsilon}{2}\left[\frac{P}{2}+\frac{X_1}{2}\right] + \frac{\eta_C^{0}}{2}$$

$$\eta_{y_A}^{(r)} \Rightarrow -\varepsilon R_{y_A}^{(r)}$$

$$R_{y_A}^{(r)} \Rightarrow R_{y_A}^{(0)} + R_{y_A}^{(1)} \cdot X_1 = \frac{P}{2} + \frac{1}{2} X_1$$

$$L_{vi} = \int_{Str} \underbrace{M^{(f)}}_{=M^{(1)}} \frac{M^{(r)}}{EI}\, dStr + \int_{Str} M^{(f)} \underbrace{\frac{\alpha \Delta T}{h}}_{>0}\, dStr =$$

con: $M^{(r)} = M^{(0)} + M^{(1)} X_1$

$$= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI}\, dStr + X_1 \int_{Str.} \frac{\left[M^{(1)}\right]^2}{EI}\, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$= \frac{1}{EI}\left\{ \int_{AB} \left[-\frac{\sqrt{2}}{4} z\right]\left[\frac{P\sqrt{2}}{4} z\right] dz + \int_{BC} \left[\frac{z-L}{2}\right]\left[\frac{P}{2}(L-z)\right] dz \right\} +$$

$$+ \frac{X_1}{EI}\left\{ \int_{AB} \left[-\frac{\sqrt{2}}{4} z\right]^2 dz + \int_{BC} \left[-\frac{1}{2}(L-z)\right]^2 dz \right\} + \frac{\alpha \Delta T}{h} \int_{AB} -\frac{\sqrt{2}}{4} z\, dz =$$

$$= \frac{1}{EI}\left[\int_0^{L\sqrt{2}} -\frac{P z^2}{8}\, dz + \int_0^{L} -\frac{P}{4} \overbrace{(L-z)^2}^{L^2+z^2-2Lz} dz \right] +$$

$$+ \frac{X_1}{EI}\left[\int_0^{L\sqrt{2}} \frac{1}{8} z^2\, dz + \int_0^{L} \frac{1}{4}\left(L^2+z^2-2Lz\right) dz \right] - \frac{\alpha \Delta T}{h} \frac{\sqrt{2}}{4} \int_0^{L\sqrt{2}} z\, dz.$$

$$= \frac{1}{EI}\left[-\frac{P}{8}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} - \frac{P}{4}\left\{ L^2[z]_0^L + \left[\frac{z^3}{3}\right]_0^L - \cancel{2}L\left[\frac{z^2}{\cancel{2}}\right]_0^L \right\}\right] +$$

$$+ \frac{X_1}{EI}\left[\frac{1}{8}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \frac{1}{4}\left\{ L^2[z]_0^L + \left[\frac{z^3}{3}\right]_0^L - \cancel{2}L\left[\frac{z^2}{\cancel{2}}\right]_0^L \right\}\right] - \frac{\alpha\Delta T}{h}\frac{\sqrt{2}}{4}\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}} =$$

$$= \frac{1}{EI}\left[-\frac{P}{\cancel{24}_{12}}\cdot L^3\cdot\cancel{2}\sqrt{2} - \frac{P}{4}\left\{ \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right\}\right] +$$

$$+ \frac{X_1}{EI}\left[\frac{1}{\cancel{24}_{12}}\cdot L^3\cdot\cancel{2}\cdot\sqrt{2} + \frac{1}{4}\left\{ \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right\}\right] - \frac{\alpha\Delta T}{h}\frac{\sqrt{2}}{\cancel{8}_4}\cdot L^2\cdot\cancel{2} =$$

$$= \frac{1}{EI}\left[-\frac{PL^3\sqrt{2}}{12} - \frac{PL^3}{12}\right] + \frac{X_1}{EI}\left[\frac{L^3\sqrt{2}}{12} + \frac{L^3}{12}\right] - \frac{\alpha\Delta T}{h}\frac{\sqrt{2}\,L^2}{4} =$$

$$= -\frac{PL^3}{12EI}\left[\sqrt{2}+1\right] + \frac{X_1L^3}{12EI}\left[\sqrt{2}+1\right] - \frac{\alpha\Delta T}{h}\frac{\sqrt{2}}{4}L^2$$

In definitiva $L_{ve} = L_{vi}$ fornisce:

$$-\frac{\varepsilon P}{4} - \frac{\varepsilon X_1}{4} + \frac{\eta_c^0}{2} = -\frac{PL^3}{12EI}\left[\sqrt{2}+1\right] + \frac{X_1L^3}{12EI}\left[\sqrt{2}+1\right] - \frac{\alpha\Delta T}{h}\frac{\sqrt{2}}{4}L^2$$

tenendo conto delle posizioni iniziali la precedente si scrive:

$$\cancel{-\frac{P}{4}\cdot\frac{L^3\left[\sqrt{2}+1\right]}{3EI}} - \frac{X_1}{4}\frac{\cancel{L^3}\cancel{\left[\sqrt{2}+1\right]}}{3\cancel{EI}} + \frac{P\cancel{L^3}}{2\cancel{EI}}\cancel{\left[\sqrt{2}+1\right]} =$$

$$= \cancel{-\frac{PL^3}{12EI}\left[\sqrt{2}+1\right]} + \frac{X_1\cancel{L^3}}{12\cancel{EI}}\cancel{\left[\sqrt{2}+1\right]} - \underbrace{\frac{\cancel{\sqrt{2}}\cancel{L^2}}{4}\,\frac{P\cancel{L}}{\cancel{EI}}\,\frac{\cancel{\left[\sqrt{2}+1\right]}}{\cancel{\sqrt{2}}}}$$

$$\frac{P}{2} + \frac{P}{4} = X_1\cdot\frac{\cancel{2}^{1}}{\cancel{12}_6} \quad\Rightarrow\quad X_1 = \cancel{6}^{3}\cdot\frac{3P}{\cancel{4}_2} = \frac{9}{2}P$$

positivo!

VERSO IPOTIZZATO CORRETTO!

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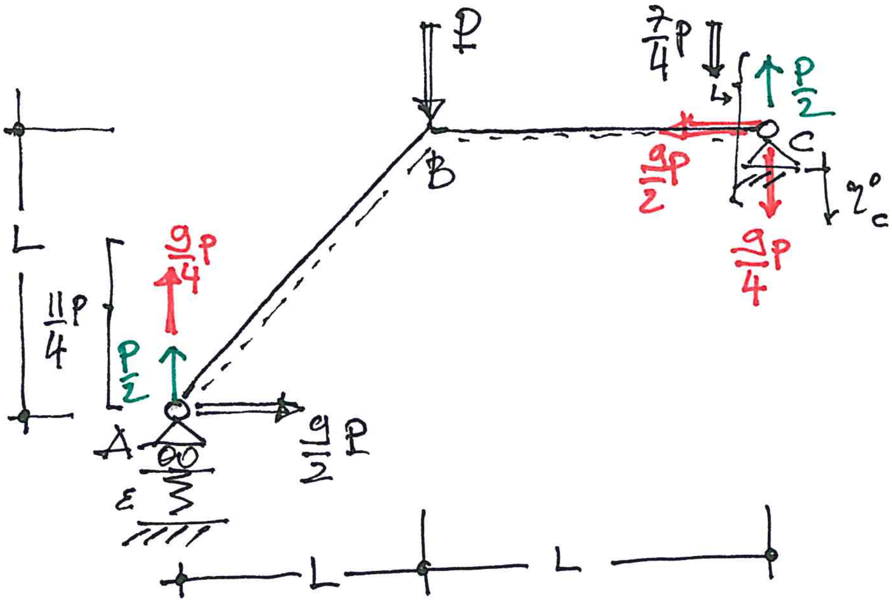
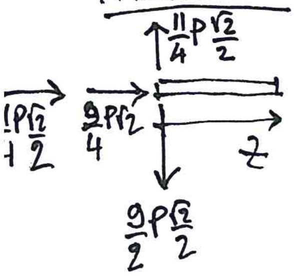
SOLUZIONE SIST. PRINCIPALE ISOSTATICO E DIAGRAMMA $M^{(r)}(z)$ FINALE



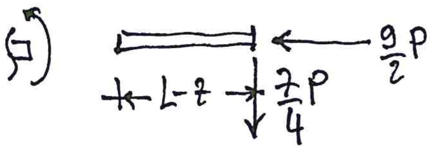
RN con metodo grafico e princ. di sovrapp. effetti

TRATTO AB $\quad 0 \le z \le L\sqrt{2}$

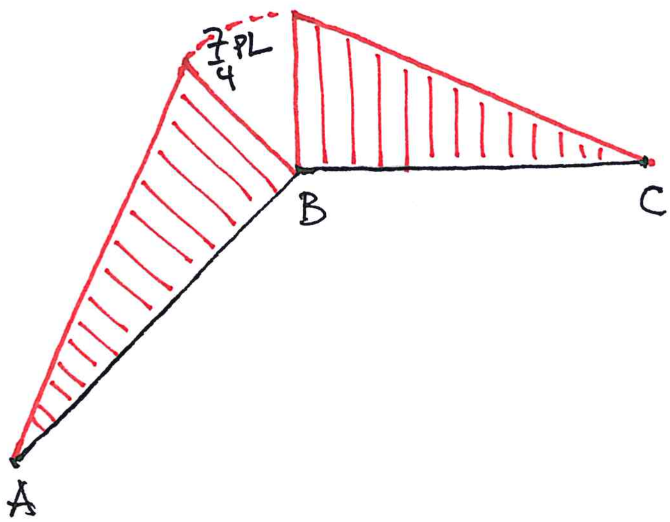


$$M^{(r)}(z) = \left[\frac{11}{8}P\sqrt{2} - \frac{9}{4}P\sqrt{2}\right] z = -\frac{7}{8}P\sqrt{2}\, z \qquad \begin{cases} M_A = 0 \\ M_B = -\frac{7}{\cancel{8}_{4}}P\cancel{\sqrt{2}} \cdot L\cancel{\sqrt{2}} = -\frac{7}{4}PL \end{cases}$$

TRATTO BC $\quad 0 \le z \le L$



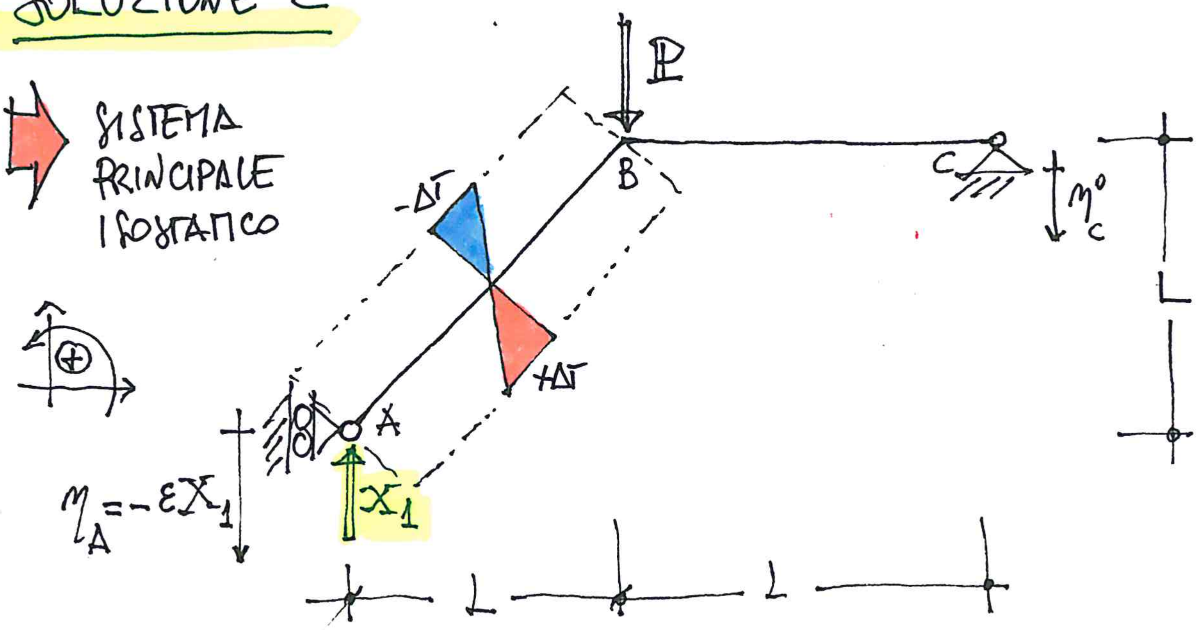
$$M^{(r)}(z) = -\frac{7}{4}P\,(L-z) \qquad \begin{cases} M_B = -\frac{7}{4}PL \\ M_C = \emptyset \end{cases}$$



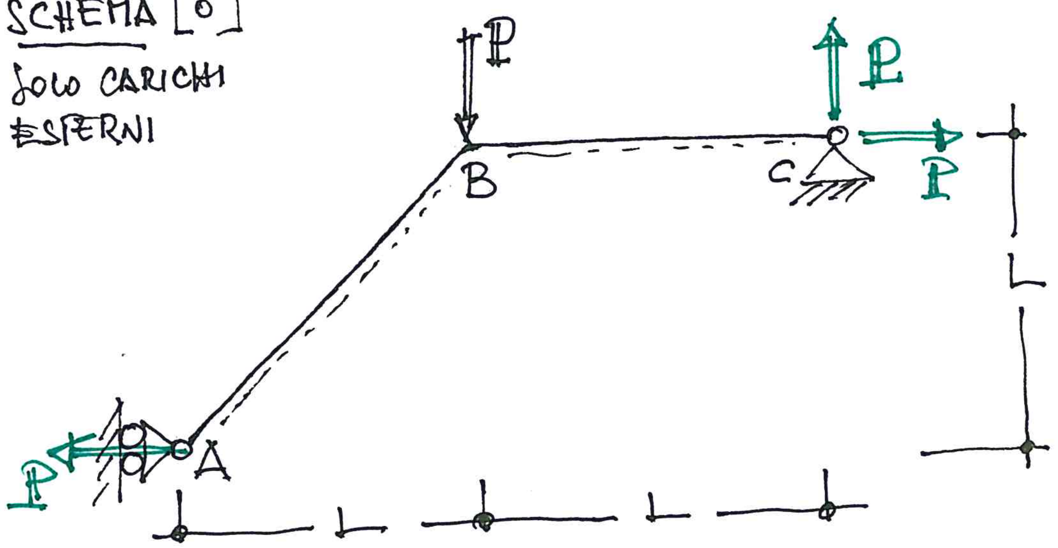
$M^{(r)}(z)$

SOLUZIONE 2

→ SISTEMA PRINCIPALE ISOSTATICO



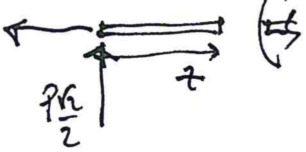
→ SCHEMA [0]
SOLO CARICHI ESTERNI



RN con metodo grafico!

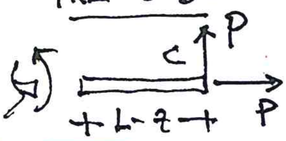
Calcoliamo $M^{(0)}(z)$:

TRATTO AB $0 \le z \le L\sqrt{2}$



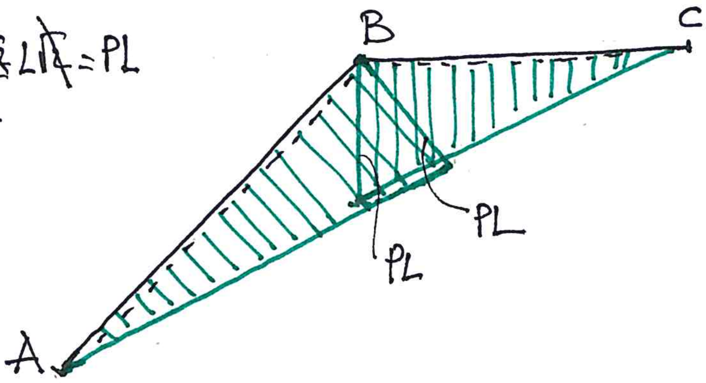
$$\boxed{M^{(0)}(z) = \frac{P\sqrt{2}}{2} z} \qquad \begin{cases} M_A = 0 \\ M_B = \frac{P\sqrt{2}}{2} L\sqrt{2} = PL \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(0)}(z) = P(L-z)} \qquad \begin{cases} M_B = PL \\ M_C = 0 \end{cases}$$

Diagramma $M^{(0)}(z)$:



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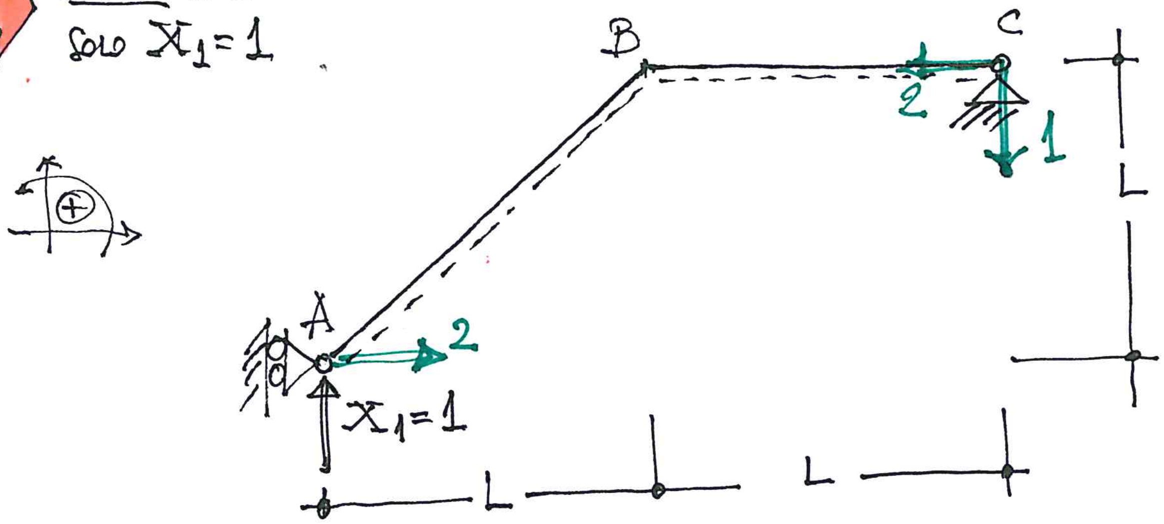
SCHEMA [1]

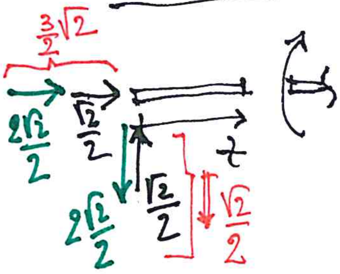
solo $X_1 = 1$



RV con metodo grafico

Calcoliamo $M^{(1)}(z)$:

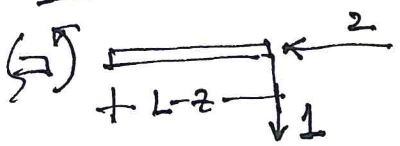
TRATTO AB $0 \le z \le L\sqrt{2}$



$$\boxed{M^{(1)}(z) = -\frac{\sqrt{2}}{2} z}$$

$$\begin{cases} M_A = \emptyset \\ M_B = -\frac{\sqrt{2}}{\cancel{2}} \cdot L\cancel{\sqrt{2}} = -L \end{cases}$$

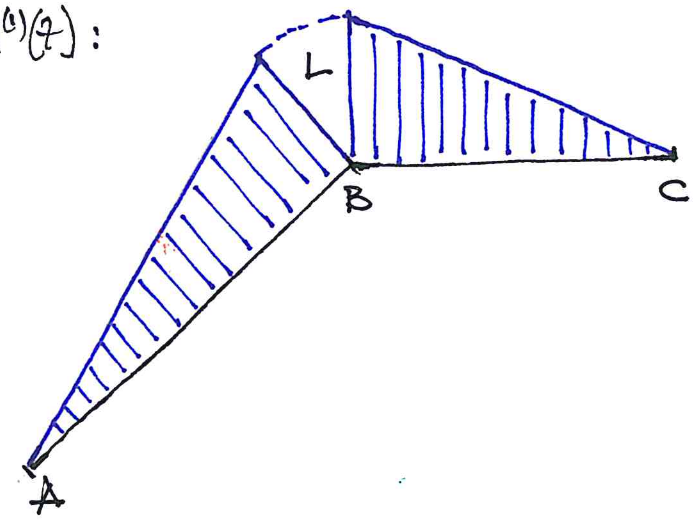
TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = -(L-z)}$$

$$\begin{cases} M_B = -L \\ M_C = \emptyset \end{cases}$$

Diagramma $M^{(1)}(z)$:



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L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve}=L_{vi}$ assumendo come sistema lavorante e fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)}\eta_i^{(r)} + \sum_j R_j^{(f)}\eta_j^{(r)} =$$

$$\underbrace{X_1}_{1}\cdot\underbrace{\eta_A^{(r)}}_{-\varepsilon \underbrace{R_{y_A}^{(r)}}_{X_1}} + \underbrace{R_{y_C}^{(f)}}_{R_{y_C}^{(1)} = -1}\;\underbrace{\eta_C^{o}}_{<0 \text{ verso il basso}} = -\varepsilon X_1 + \eta_C^{o}$$

$$L_{vi} = \int_{str} M^{(f)}\frac{M^{(r)}}{EI}\,ds_{tr} + \int_{str} M^{(f)}\alpha\frac{\Delta T}{h}\,ds_{tr} =$$

$$\text{con}\quad \begin{cases} M^{(r)} = M^{(0)} + M^{(1)}X_1 \\ M^{(f)} = M^{(1)} \end{cases}$$

$$= \int_{str} M^{(1)}\frac{M^{(0)}}{EI}\,ds_{tr} + X_1\int_{str}\frac{\left[M^{(1)}\right]^2}{EI}\,ds_{tr} + \int_{str} M^{(1)}\frac{\alpha\Delta T}{h}\,ds_{tr} =$$

$$= \frac{1}{EI}\left\{\int_{\overline{AB}}\left[-\frac{\sqrt{2}}{2}z\right]\left[\frac{P\sqrt{2}}{2}z\right]dz + \int_{\overline{BC}}\left[-(L-z)\right]\left[P(L-z)\right]dz\right\} +$$

$$+\frac{X_1}{EI}\left\{\int_{\overline{AB}}\left[\frac{\sqrt{2}}{2}z\right]^2 dz + \int_{\overline{BC}}\overbrace{(L-z)^2}^{L^2+z^2-2Lz}dz\right\} + \frac{\alpha\Delta T}{h}\int_{\overline{AB}} -\frac{\sqrt{2}}{2}z\,dz =$$

$$= \frac{1}{EI}\left\{-\frac{P}{2}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} - P\left\{L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - \cancel{2}L\left[\frac{z^2}{\cancel{2}}\right]_0^L\right\}\right\} +$$

$$+\frac{X_1}{EI}\left\{\frac{1}{2}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - \cancel{2}L\left[\frac{z^2}{\cancel{2}}\right]_0^L\right\} - \frac{\alpha\Delta T}{h}\frac{\sqrt{2}}{2}\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}} =$$

$$= -\frac{P}{EI}\left\{\frac{L^3\sqrt{2}}{3} + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right\} + \frac{X_1}{EI}\left\{\frac{L^3\sqrt{2}}{3} + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right\} - \frac{\alpha\Delta T}{h}\frac{\sqrt{2}}{2}L^2 =$$

$$= -\frac{PL^3}{3EI}\left[\sqrt{2}+1\right] + \frac{X_1}{EI}\frac{L^3}{3}\left[\sqrt{2}+1\right] - \alpha\frac{\Delta t}{h}\frac{\sqrt{2}}{2}L^2$$

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In definitiva, tenendo conto delle posizioni iniziali, l'equazione $L_{ve} = L_{vi}$ si scrive:

$$-\frac{\cancel{L^3}}{\cancel{EI}}\frac{\cancel{[\sqrt{2}+1]}}{3}X_1 + \frac{P\cancel{L^3}}{\cancel{EI}}\cancel{[\sqrt{2}+1]} =$$

$$= -\frac{P\cancel{L^3}}{3\cancel{EI}}\cancel{[\sqrt{2}+1]} + \frac{\cancel{L^3}}{3\cancel{EI}}\cancel{[\sqrt{2}+1]}X_1 - \frac{P\cancel{L}}{\cancel{EI}\cancel{\sqrt{2}}}\cdot\cancel{[\sqrt{2}+1]}\cdot\frac{\cancel{\sqrt{2}}}{2}\cancel{L^2}$$

da cui:

$$P + \frac{P}{3} + \frac{P}{2} = \left[\frac{1}{3} + \frac{1}{3}\right]X_1 \Rightarrow \frac{6P+2P+3P}{6} = \frac{2}{3}X_1$$

$$X_1 = \frac{\cancel{3}}{2}\,\frac{11P}{\cancel{6}_{2}} = \frac{11P}{4}$$

POSITIVA!
VERSO IPOTIZZATO CORRETTO!

cfr. anche RV calcolata a p. VI per la soluz. 1 !!

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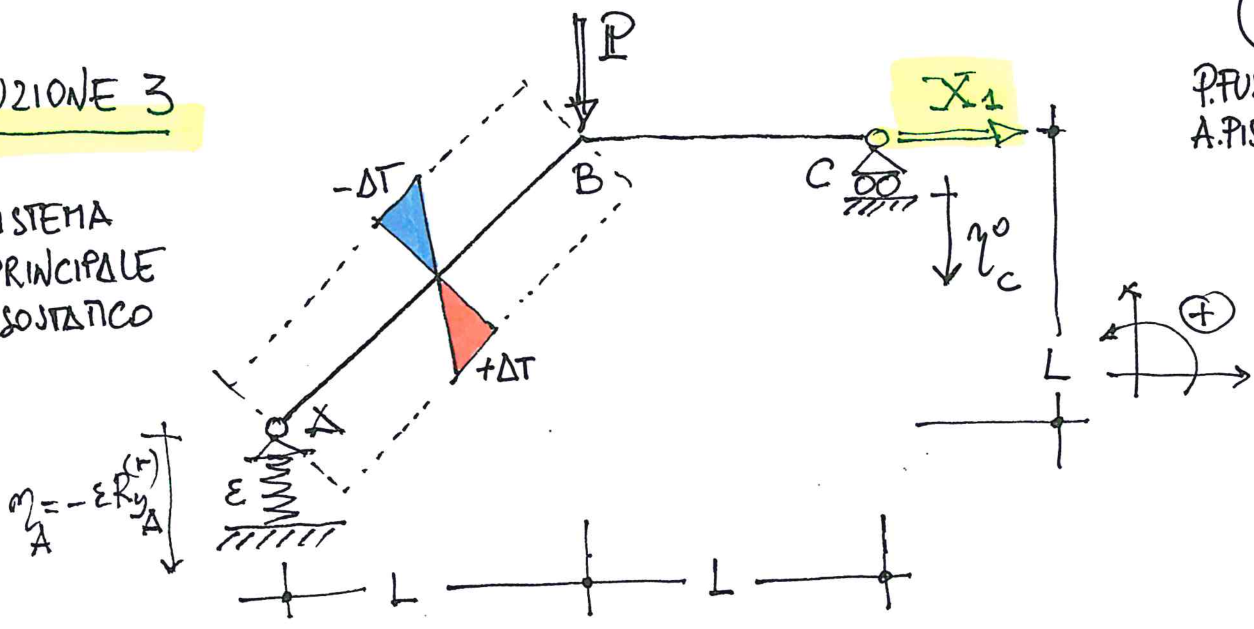
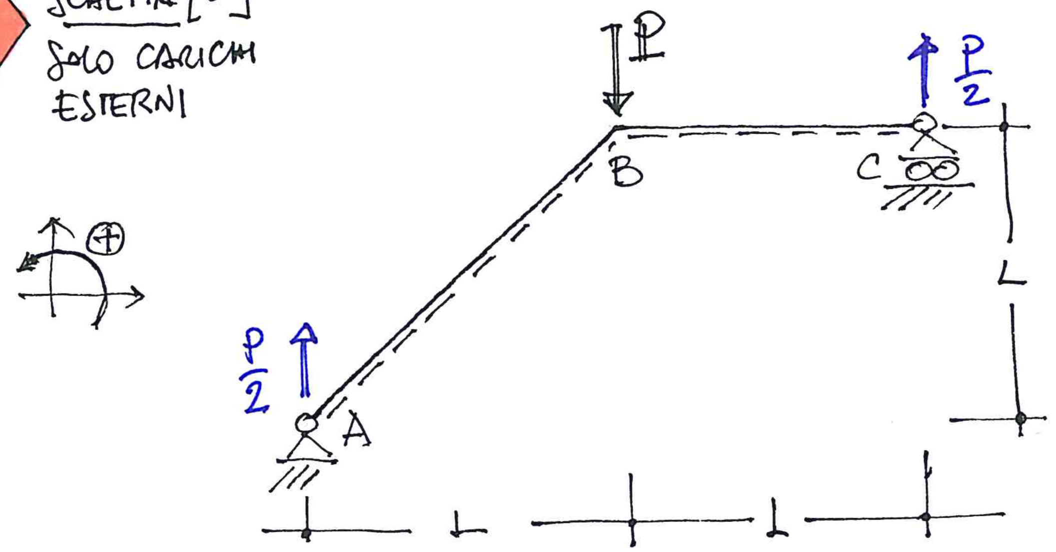
SOLUZIONE 3

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]
SOLO CARICHI ESTERNI



RV con metodo grafico!

La situazione di carico è identica a quella già esaminata a p. II per la SOLUZIONE 1, cui si rimanda. Si riportano qui di seguito, per comodità operativa, le leggi di $M^{(0)}(z)$ nei due tratti!

TRATTO AB $\quad 0 \le z \le L\sqrt{2}$

$$\boxed{M^{(0)}(z) = \frac{P\sqrt{2}}{4} \cdot z}$$

TRATTO BC $\quad 0 \le z \le L$

$$\boxed{M^{(0)}(z) = \frac{P}{2}(L - z)}$$

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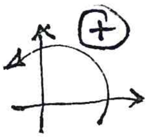
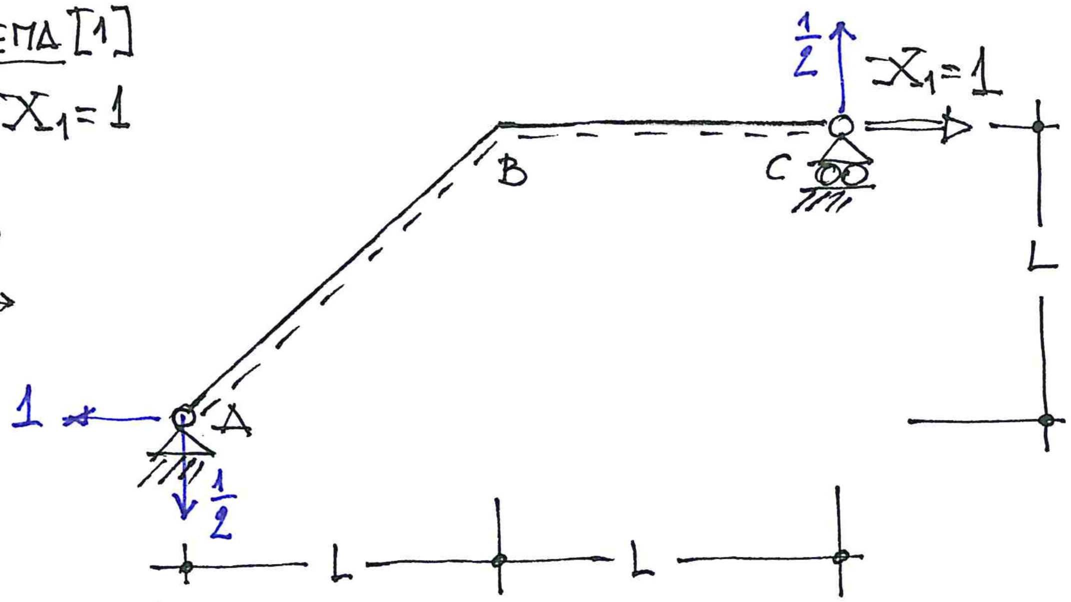
A. PISANO

SCHEMA [1]

Solo $X_1 = 1$

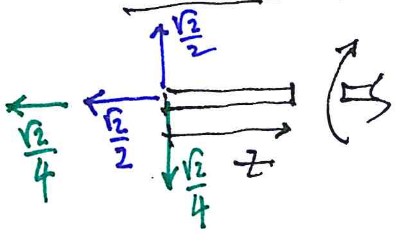




RV con metodo grafico!

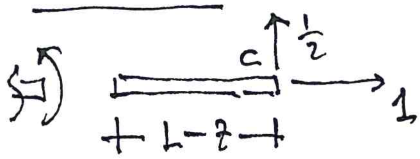
Calcoliamo $M^{(1)}(z)$:

TRATTO AB $\quad 0 \leq z \leq L\sqrt{2}$



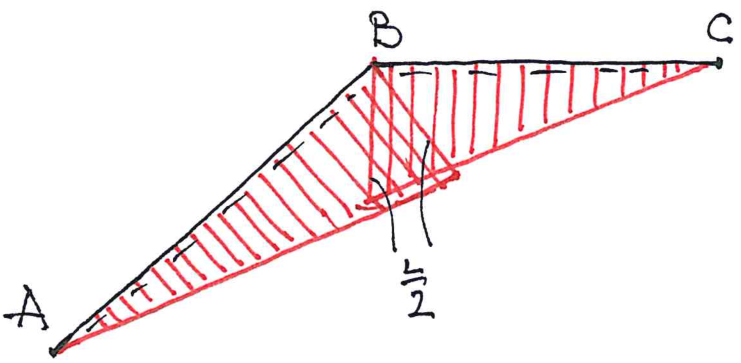
$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{4} \cdot z} \qquad \begin{cases} M_A = \emptyset \\ M_B = \frac{\cancel{\sqrt{2}}}{\cancel{4}_2} \cdot L\cancel{\sqrt{2}} = \frac{L}{2} \end{cases}$$

TRATTO BC $\quad 0 \leq z \leq L$



$$\boxed{M^{(1)}(z) = \frac{1}{2}(L - z)} \qquad \begin{cases} M_B = \frac{L}{2} \\ M_C = \emptyset \end{cases}$$

Diagramma $M^{(1)}(z)$:



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L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= \underbrace{1 \cdot \phi}_{\phi} + \underbrace{R_{y_A}^{(1)}}_{-\frac{1}{2}} \cdot \underbrace{\eta_A}_{-\varepsilon R_{y_A}^{(r)}} + \underbrace{R_{y_C}^{(1)}}_{\frac{1}{2}} \cdot \underset{<0}{\eta_C^{0}} = \frac{\varepsilon}{2}\left[\frac{P}{2} - \frac{X_1}{2}\right] - \frac{\eta_C^{0}}{2}$$

$$R_{y_A}^{(r)} = R_{y_A}^{(0)} + R_{y_A}^{(1)} X_1 = \frac{P}{2} - \frac{1}{2} X_1$$

$$L_{vi} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(f)} \underbrace{\alpha \frac{\Delta T}{h}}_{>0} dStr =$$

con $M^{(r)} = M^{(0)} + M^{(1)} X_1$
$M^{(f)} = M^{(1)}$

$$= \int_{Str} \frac{M^{(1)} M^{(0)}}{EI} dStr + X_1 \int_{Str} \frac{[M^{(1)}]^2}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \frac{1}{EI}\left\{ \int_{\overline{AB}} \left[\frac{P\sqrt{2}}{4} \cdot z\right]\left[\frac{\sqrt{2}}{4} z\right] dz + \int_{\overline{BC}} \left[\frac{L-z}{2}\right]\left[\frac{P}{2}(L-z)\right] dz \right\} +$$

$$\frac{X_1}{EI}\left\{ \int_{\overline{AB}} \left[\frac{\sqrt{2}}{4} z\right]^2 dz + \int_{\overline{BC}} \left[\frac{1}{2}(L-z)\right]^2 dz \right\} +$$

$$+ \int_{\overline{AB}} \left[\frac{\sqrt{2}}{4} z\right] \frac{\alpha \Delta T}{h} dz =$$

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$$= \frac{1}{EI}\left[\int_0^{L\sqrt{2}} \frac{P}{8} z^2 dz + \int_0^L \frac{P}{4} \overbrace{(L-z)^2}^{L^2+z^2-2Lz} dz\right] +$$

$$+ \frac{X_1}{EI}\left[\int_0^{L\sqrt{2}} \frac{z^2}{8} dz + \int_0^L \frac{1}{4}(L-z)^2 dz\right] + \frac{\alpha\bar{\Delta T}}{h}\frac{\sqrt{2}}{4}\int_0^{L\sqrt{2}} z\, dz =$$

$$= \frac{1}{EI}\left\{\frac{P}{8}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \frac{P}{4}\left[L^2[z]_0^L + \left[\frac{z^3}{3}\right]_0^L - \cancel{2}L\left[\frac{z^2}{\cancel{2}}\right]_0^L\right]\right\} +$$

$$+ \frac{X_1}{EI}\left\{\frac{1}{8}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + \frac{1}{4}\left[L^2[z]_0^L + \left[\frac{z^3}{3}\right]_0^L - \cancel{2}L\left[\frac{z^2}{\cancel{2}}\right]_0^L\right]\right\} + \frac{\alpha\bar{\Delta T}}{h}\frac{\sqrt{2}}{4}\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}} =$$

$$= \frac{P}{4EI}\left[\frac{L^3\sqrt{2}}{3} + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right] + \frac{X_1}{EI}\left[\frac{L^3\sqrt{2}}{12} + \cancel{\frac{L^3}{4}} + \frac{L^3}{12} - \cancel{\frac{L^3}{4}}\right] + \frac{\alpha\bar{\Delta T}}{h}\frac{\sqrt{2}}{4}L^2 =$$

$$= \frac{PL^3}{12EI}[\sqrt{2}+1] + \frac{X_1 L^3}{12EI}[\sqrt{2}+1] + \frac{\alpha\bar{\Delta T}}{h}\frac{\sqrt{2}}{4}L^2.$$

In definitiva, tenendo conto delle posizioni iniziali, l'equazione $L_{ve} = L_{vi}$ fornisce:

$$\frac{\cancel{L^3}}{6\cancel{EI}}\cancel{[\sqrt{2}+1]}\left\{\frac{P}{2} - \frac{X_1}{2}\right\} - \frac{P\cancel{L^3}}{2\cancel{EI}}\cancel{[\sqrt{2}+1]} =$$

$$= \frac{P\cancel{L^3}}{12\cancel{EI}}\cancel{[\sqrt{2}+1]} + \frac{X_1\cancel{L^3}}{12\cancel{EI}}\cancel{[\sqrt{2}+1]} + \frac{P\cancel{L}}{\cancel{EI}}\frac{\cancel{[\sqrt{2}+1]}}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{4}\cancel{L^2}$$

da cui:
$$\cancel{\frac{P}{12}} - \frac{X_1}{12} - \frac{P}{2} = \cancel{\frac{P}{12}} + \frac{X_1}{12} + \frac{P}{4}$$

$$\frac{X_1}{\cancel{6}_3} = -\frac{3P}{\cancel{4}_2} \Rightarrow X_1 = -\frac{9}{2}P$$

NEGATIVA! VERSO OPPOSTO A QUELLO IPOTIZZATO!

$\Downarrow$

cfr. anche RV calcolata a p. VI per la sol. 1. ok!

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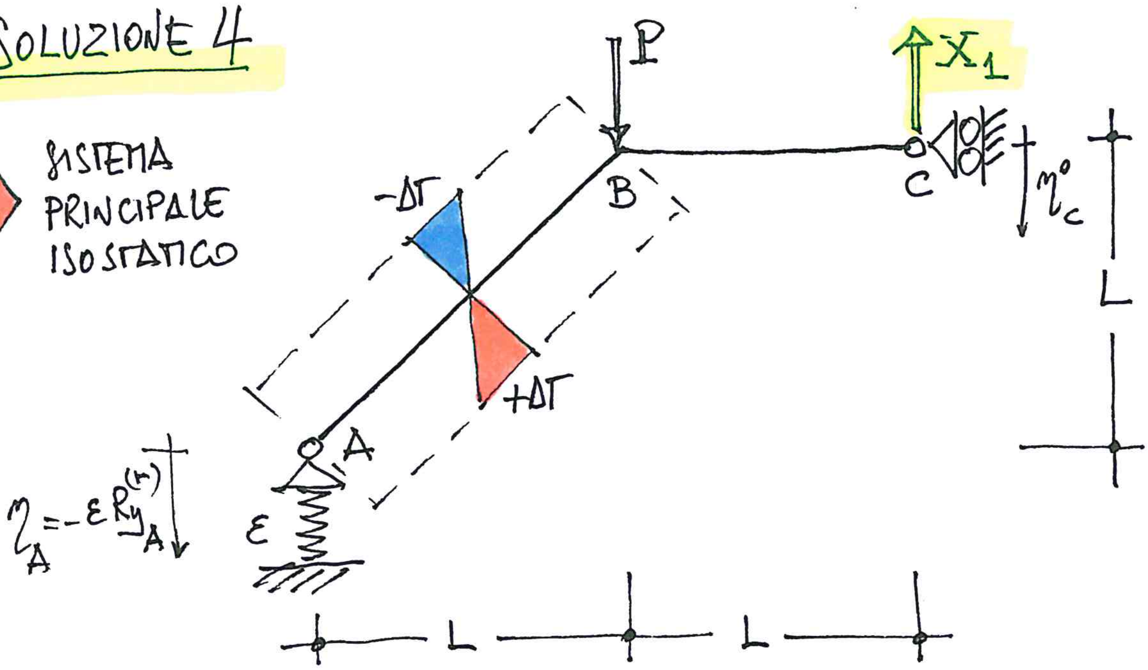
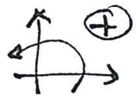
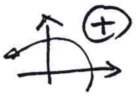
SOLUZIONE 4

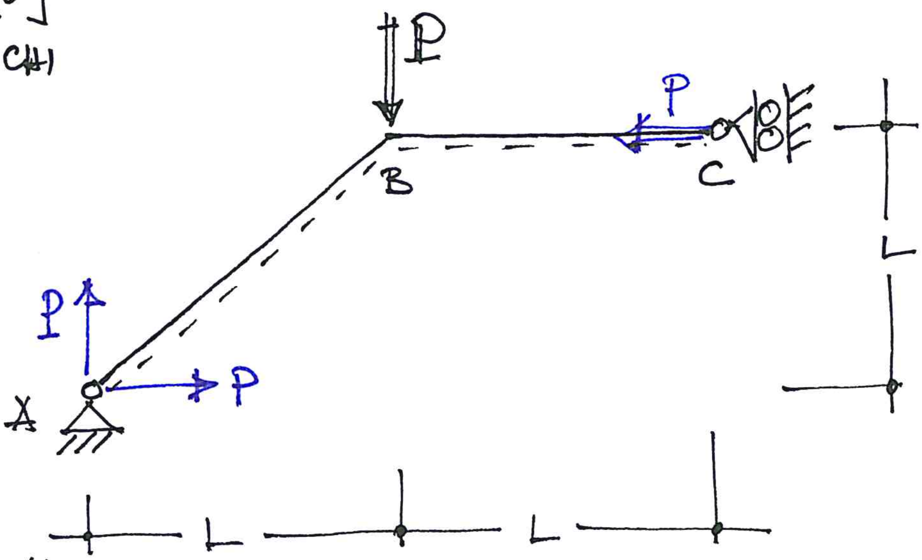
SISTEMA PRINCIPALE ISOSTATICO





SCHEMA [0]
SOLO CARICHI ESTERNI

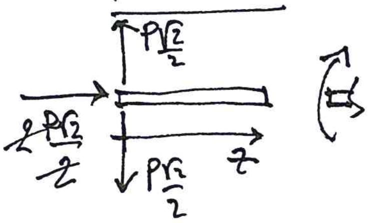




RV con metodo grafico!

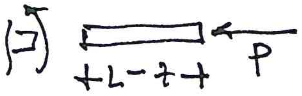
Calcoliamo $M^{(0)}(z)$:

TRATTO AB $0 \le z \le L\sqrt{2}$



$$M^{(0)}(z) = \phi$$

TRATTO BC $0 \le z \le L$



$$M^{(0)}(z) = \phi$$

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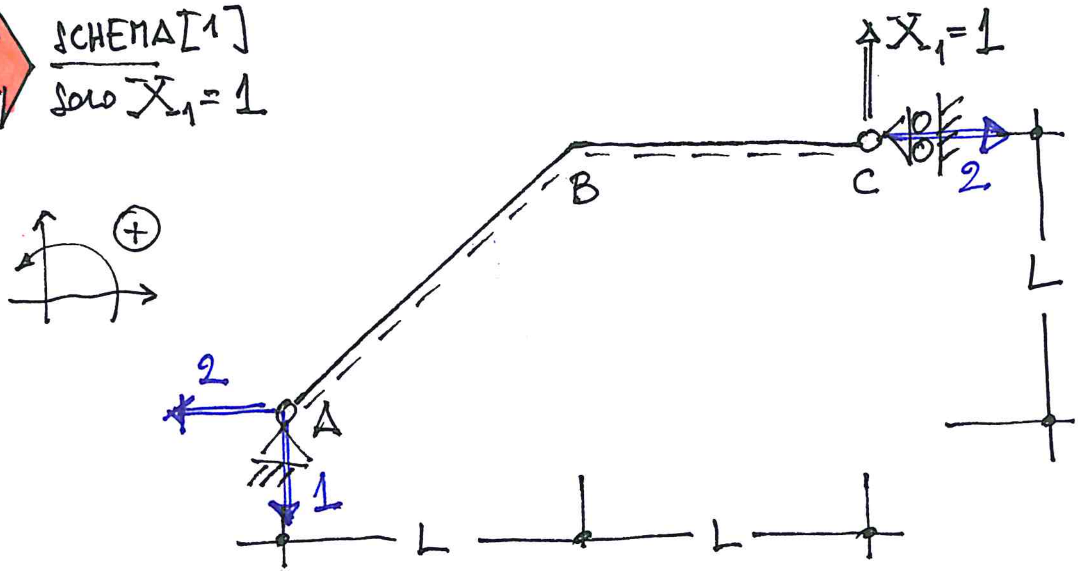
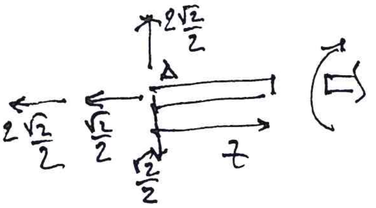
SCHEMA [1]
solo $X_1 = 1$



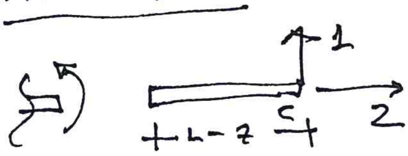
RV con metodo grafico!

Calcoliamo $M^{(1)}(z)$: TRATTO AB $0 \le z \le L\sqrt{2}$



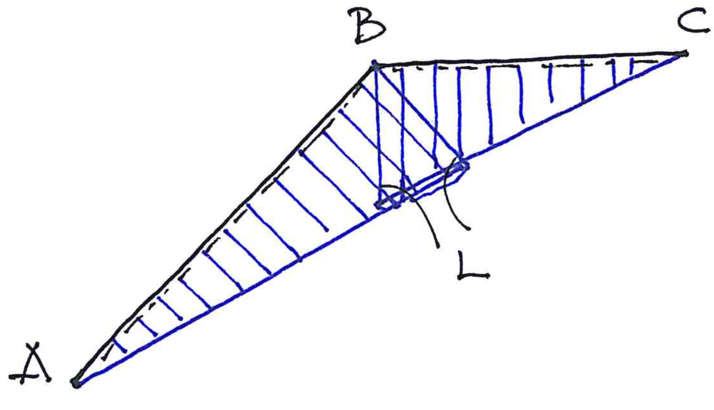
$$\boxed{M^{(1)}(z) = \frac{\sqrt{2}}{2} z} \qquad \begin{cases} M_A = 0 \\ M_B = \frac{\sqrt{2}}{2} L\sqrt{2} = L \end{cases}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = L - z} \qquad \begin{cases} M_B = L \\ M_C = \phi \end{cases}$$

Diagramma $M^{(1)}(z)$:



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L'unica equazione di Müller-Breslau, corrispondente all'unica incognita iperstatica X_1, si scrive nella forma $L_{ve} = L_{vi}$ assumendo come sistema lavorante o fittizio lo schema [1] e come sistema reale la struttura iperstatica data. Si ha:

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= \underbrace{X^{(1)}}_{=1} \underbrace{\eta_c^{0}}_{<0} + \underbrace{R_{y_A}^{(1)}}_{=-1} \cdot \underbrace{\eta_A^{(r)}}_{-\varepsilon R_{y_A}^{(r)}} = -\eta_c^{0} + \varepsilon\left[P - X_1\right]$$

$$R_{y_A}^{(r)} = \underbrace{R_{y_A}^{(0)}}_{=P} + X_1 \underbrace{R_{y_A}^{(1)}}_{=-1}$$

$$L_{vi} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI}\, dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$\begin{cases} M^{(r)} = M^{(0)} + M^{(1)} X_1 \\ M^{(f)} = M^{(1)} \end{cases}$$

$$= \int_{Str} \frac{M^{(1)} M^{(0)}}{EI}\, dStr + X_1 \int_{Str} \frac{\left[M^{(1)}\right]^2}{EI}\, dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h}\, dStr =$$

$$= \frac{1}{EI} \underbrace{\left\{ \int_{\overline{AB}} \left[\frac{\sqrt{2}}{2} z\right] \cdot \emptyset\, dz + \int_{\overline{BC}} \left[L - z\right] \cdot \emptyset\, dz \right\}}_{\text{tutto nullo!!}} +$$

$$+ \frac{X_1}{EI} \left[\int_{\overline{AB}} \frac{z^2}{2}\, dz + \int_{\overline{BC}} \underbrace{(L-z)^2}_{L^2 + z^2 - 2Lz}\, dz \right] + \int_{\overline{AB}} \left(\frac{\sqrt{2}}{2} z\right) \alpha \frac{\Delta T}{h}\, dz =$$

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$$= \frac{X_1}{EI}\left\{\frac{1}{2}\left[\frac{z^3}{3}\right]_0^{L\sqrt{2}} + L^2\left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - \cancel{2}L\left[\frac{z^2}{\cancel{2}}\right]_0^L\right\} +$$

$$+ \frac{\alpha\overline{\Delta T}}{h}\left\{\frac{\sqrt{2}}{2}\left[\frac{z^2}{2}\right]_0^{L\sqrt{2}}\right\} =$$

$$= \frac{X_1}{EI}\left\{\frac{L^3\sqrt{2}}{3} + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right\} + \frac{\alpha\overline{\Delta T}}{h}\frac{\sqrt{2}}{\cancel{4}_2}L^2\cancel{2} =$$

$$= \frac{X_1}{EI}\frac{L^3}{3}\left[\sqrt{2}+1\right] + \frac{\alpha\overline{\Delta T}}{h}\frac{\sqrt{2}}{2}L^2$$

In definitiva, tenendo conto delle posizioni iniziali, l'equazione $L_{ve} = L_{vi}$ fornisce:

$$-\frac{P\cancel{L^3}}{\cancel{EI}}\cancel{\left[\sqrt{2}+1\right]} + \frac{\cancel{L^3}}{3\cancel{EI}}\cancel{\left[\sqrt{2}+1\right]}\left\{P - X_1\right\} =$$

$$= \frac{X_1}{\cancel{EI}}\frac{\cancel{L^3}}{3}\cancel{\left[\sqrt{2}+1\right]} + \frac{P\cancel{L}}{\cancel{EI}}\frac{\cancel{\left[\sqrt{2}+1\right]}}{\cancel{\sqrt{2}}}\frac{\cancel{\sqrt{2}}}{2}\cancel{L^2}$$

da cui:

$$-P + \frac{P}{3} - \frac{X_1}{3} = \frac{X_1}{3} + \frac{P}{2}$$

$$\frac{2}{3}X_1 = -P + \frac{P}{3} - \frac{P}{2} = -\frac{7}{6}P$$

$$\frac{2}{\cancel{3}}X_1 = -\frac{7}{\cancel{6}_2}P \quad \Rightarrow \quad X_1 = -\frac{7}{4}P$$

NEGATIVA! VERSO OPPOSTO!

cfr. anche RV di p. VI! OK!

END