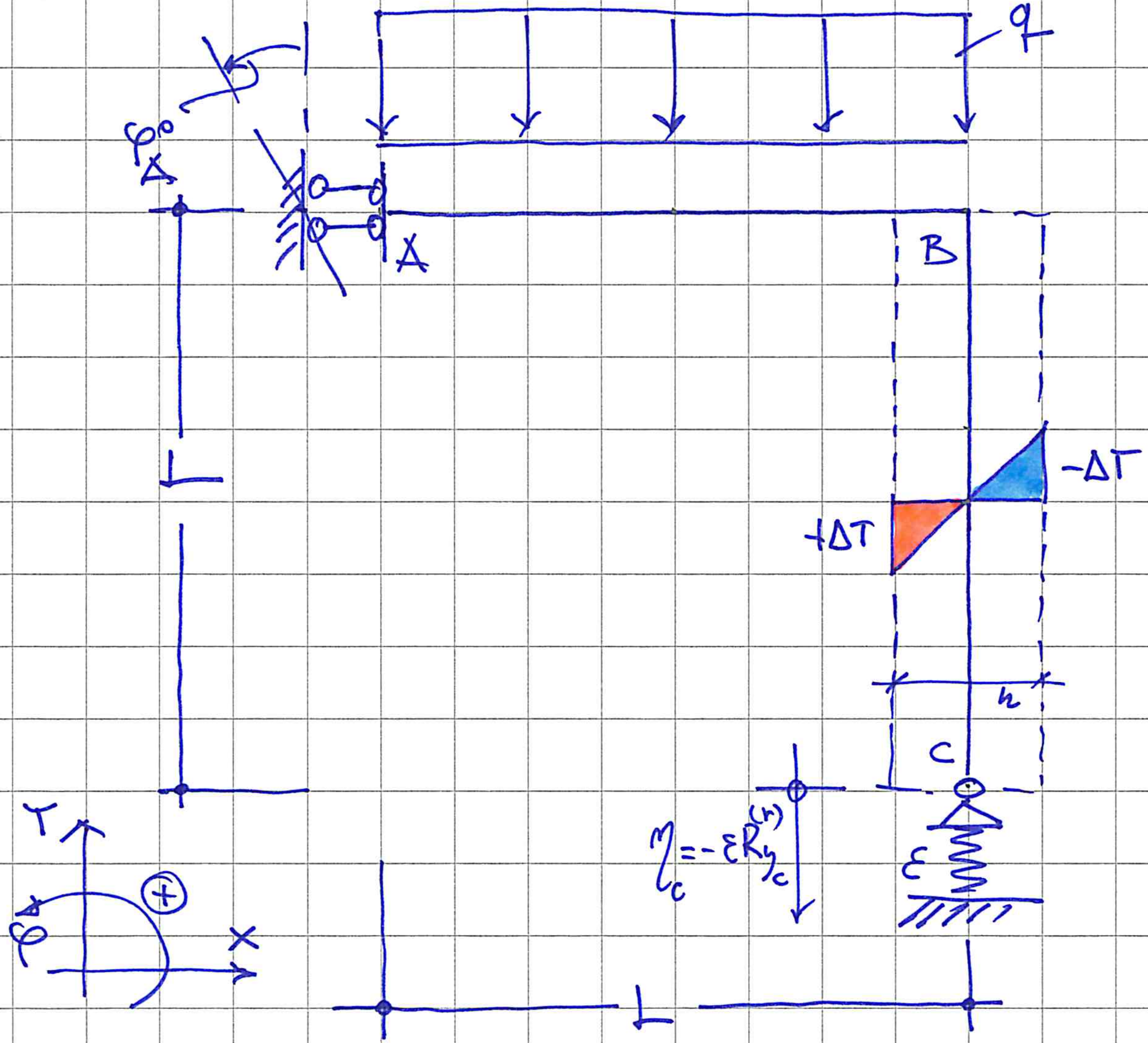
P.FUSCHI
A. PISANO

MECCANICA DELLE STRUTTURE L-17
A.A. 2016-17 CORSO: P.FUSCHI

TEST in ITINERE del 19 GENNAIO 2017

RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA SEGUENTE_
TRACCIARE IL DIAGRAMMA DEL MOMENTO TENENDO CONTO
DELLE POSIZIONI RIPORTATE IN FIGURA.



$$\left|\frac{\alpha \Delta T}{h}\right| = \frac{qL^2}{EI}; \quad \left|\varphi_A^0\right| = \frac{qL^3}{EI}; \quad |\varepsilon| = \frac{L^3}{3EI}$$

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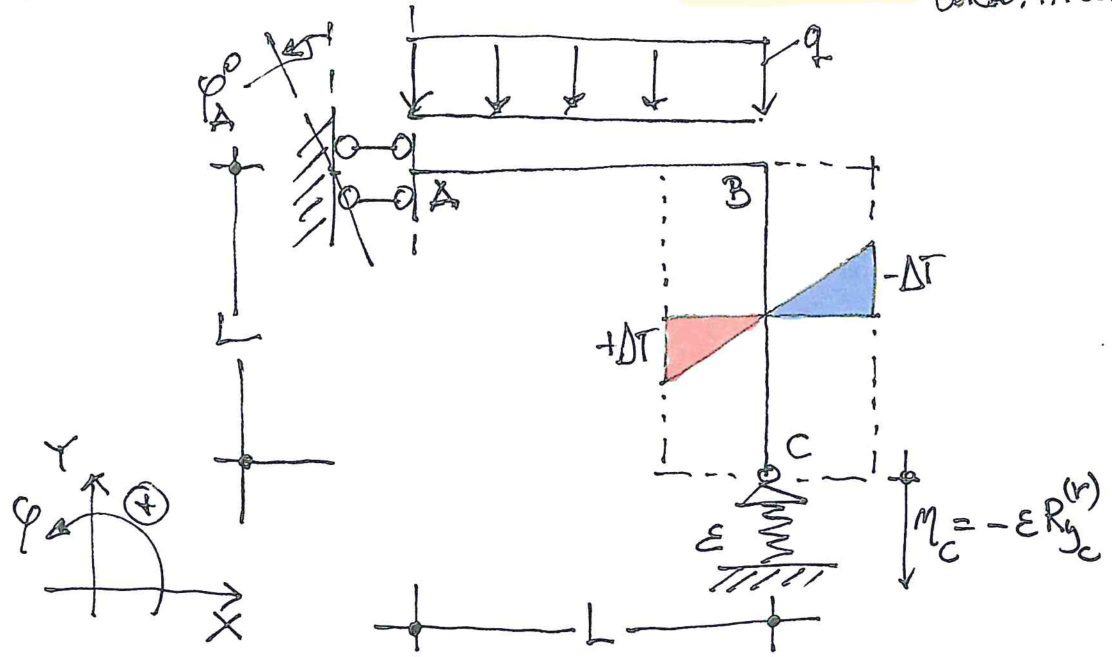


Posizioni:

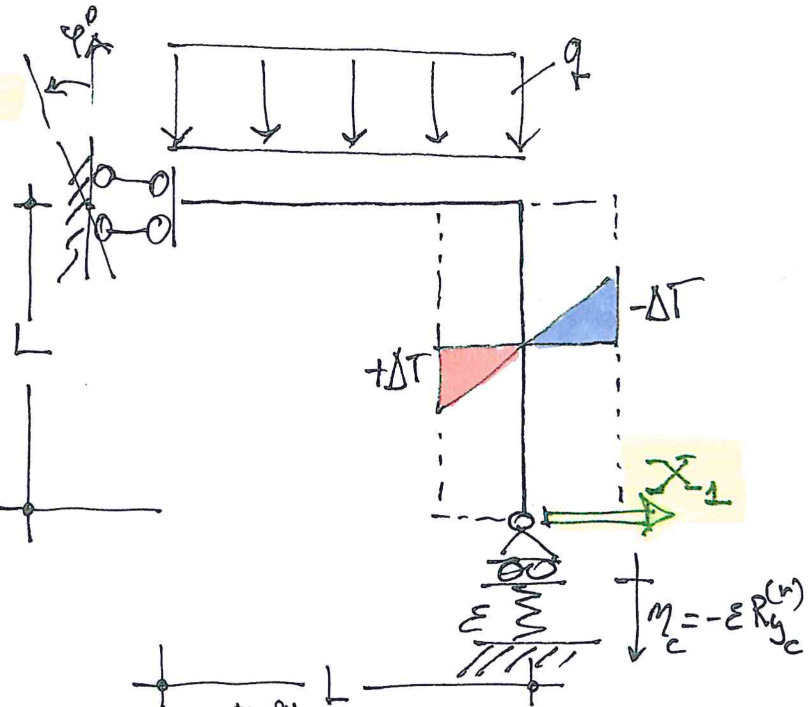
$$\left|\frac{\alpha \Delta T}{h}\right| = \frac{qL^2}{EI}$$

$$\left|\varphi_A^0\right| = \frac{qL^3}{EI}$$

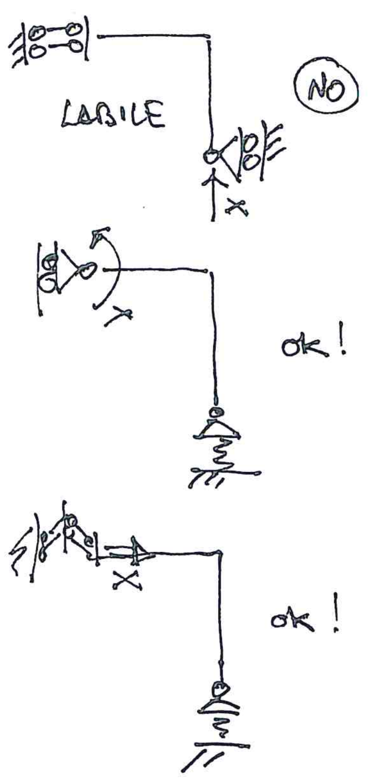
$$\left|\varepsilon\right| = \frac{L^3}{3EI}$$

SOLUZIONE 1

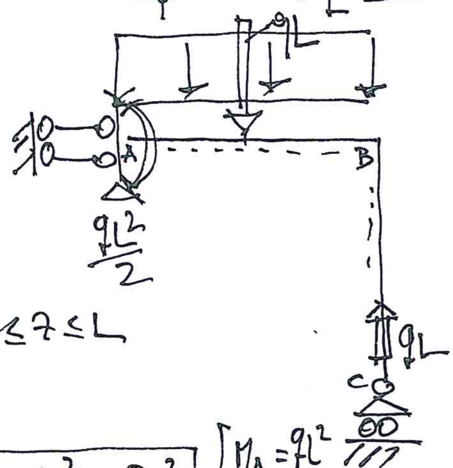
SISTEMA PRINCIPALE ISOSTATICO



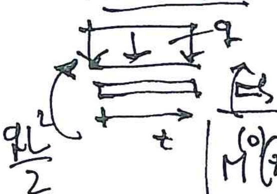
ALTRE POSSIB. SCELTE



SCHEMA [0]
SOLO CARICHI ESTERNI

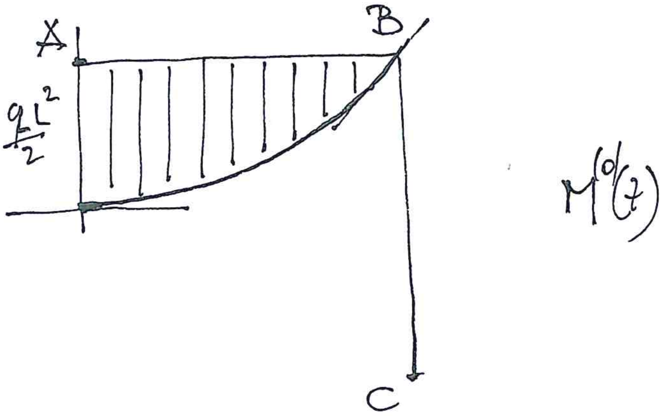


TRATTO AB $0 \le z \le L$



$$M^{(0)}(z) = \frac{qL^2}{2} - \frac{qz^2}{2}$$

$$M_A = \frac{qL^2}{2}$$
$$M_B = \phi$$

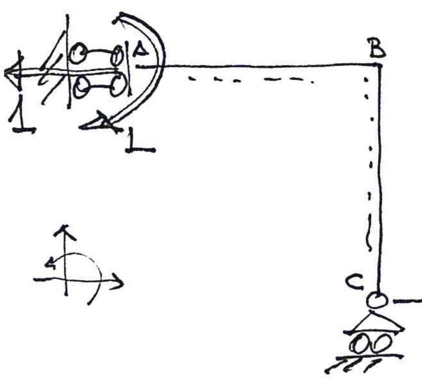


TRATTO BC $0 \le z \le L$

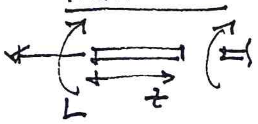
$$M^{(0)}(z) = \phi$$

SCHEMA [1]

Solo $X_1 = 1$

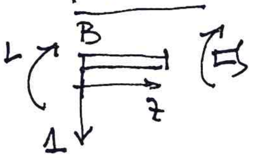


TRATTO AB $0 \le z \le L$

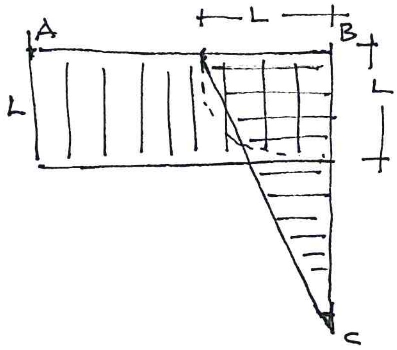


$$\boxed{M^{(1)}(z) = L}$$

TRATTO BC $0 \le z \le L$



$$\boxed{M^{(1)}(z) = L - z} \quad \begin{cases} M_B = L \\ M_C = \phi \end{cases}$$



Unica eq. di Müller-Breslau, $L_{ve} = L_{vi}$, corrispondente all'unica incognita iperstatica.

$$\underset{\approx}{L_{ve}} = X_i^{(t)} \eta_i^{(r)} + \sum_j R_j^{(t)} \eta_j^{(r)} = \underset{1}{X_1} \cdot \underset{\substack{\phi \\ \text{vinc. perf.} \\ \text{in orizz.}}}{u_{x_C}^{(r)}} + \underbrace{R_{y_C}^{(1)} \underset{\phi}{\eta_C^{(r)}}}_{\phi} + \underset{-L}{M_A^{(1)}} \cdot \underset{>0}{\varphi_A^0} = \underset{\approx}{-L\varphi_A^0} = -\frac{qL^4}{EI} \quad (\uparrow \text{ per la posiz. iniz.})$$

$$\underset{\approx}{L_{vi}} = \int_{Str} M^{(1)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} dStr = \qquad \left(M^{(r)} = M^{(0)} + M^{(1)} X_1\right)$$

$$= \frac{1}{EI} \int_{Str} M^{(1)} M^{(0)} dStr + \frac{X_1}{EI} \int_{Str} \left[M^{(1)}\right]^2 dStr + \int_{Str} M^{(1)} \frac{\alpha \overline{\Delta T}}{h} dStr =$$

$$= \frac{1}{EI} \int_{\overline{AB}} M^{(1)} M^{(0)} dStr + \frac{X_1}{EI} \left\{ \int_{\overline{AB}} \left[M^{(1)}\right]^2 dz + \int_{\overline{BC}} \left[M^{(1)}\right]^2 dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_{\overline{BC}} M^{(1)} dz =$$

$$= \frac{1}{EI} \int_0^L L \left[\frac{qL^2}{2} - \frac{qz^2}{2}\right] dz + \frac{X_1}{EI} \left\{ \int_0^L L^2 dz + \int_0^L \overbrace{(L-z)^2}^{L^2 + z^2 - 2Lz} dz \right\} + \frac{\alpha \overline{\Delta T}}{h} \int_0^L (L - z) dz =$$

$$= \frac{1}{EI} \left\{ \frac{qL^3}{2} \left[z\right]_0^L - \frac{qL}{2} \left[\frac{z^3}{3}\right]_0^L \right\} + \frac{X_1}{EI} \left\{ L^2 \left[z\right]_0^L + L^2 \left[z\right]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L \left[\frac{z^2}{2}\right]_0^L \right\} +$$

$$+ \frac{\alpha \overline{\Delta T}}{h} \left\{ L \left[z\right]_0^L - \left[\frac{z^2}{2}\right]_0^L \right\} =$$

$$= \frac{1}{EI} \underbrace{\left\{ \frac{qL^4}{2} - \frac{qL^4}{6} \right\}}_{\frac{2}{6} qL^4 = \frac{1}{3} qL^4} + \frac{X_1}{EI} \underbrace{\left\{ L^3 + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3} \right\}}_{\frac{4L^3}{3}} + \frac{\alpha \overline{\Delta T}}{h} \underbrace{\left\{ L^2 - \frac{L^2}{2} \right\}}_{L^2/2} =$$

$$= \frac{qL^4}{3EI} + \frac{X_1}{EI} \frac{4L^3}{3} + \underbrace{\frac{\alpha \overline{\Delta T}}{h}}_{\substack{\parallel \\ \frac{qL^2}{EI} \\ \text{per la posiz. iniz.}}} \frac{L^2}{2} = \underset{\approx}{\frac{5}{6} \frac{qL^4}{EI} + \frac{X_1}{EI} \frac{4L^3}{3}} \qquad \left(\frac{qL^2}{EI} \cdot \frac{L^2}{2} = \frac{qL^4}{2EI}\right)$$

Si ha in definitiva:

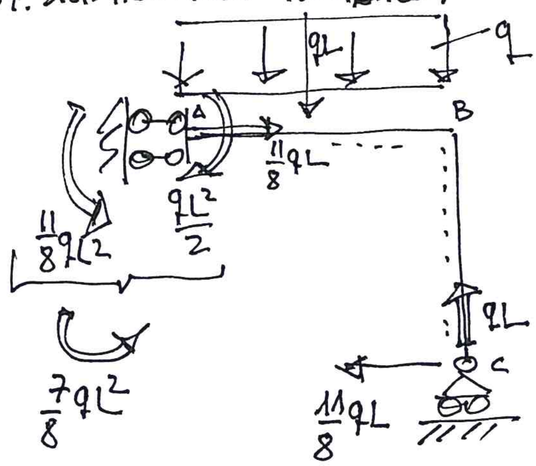
$$-\frac{qL^4}{EI} = \frac{5}{6}\frac{qL^4}{EI} + X_1 \frac{4L^3}{3EI}$$

$$\boxed{X_1} = \frac{3}{4}\left[-qL - \frac{5}{6}qL\right] = \boxed{-\frac{11}{8}qL} < 0$$

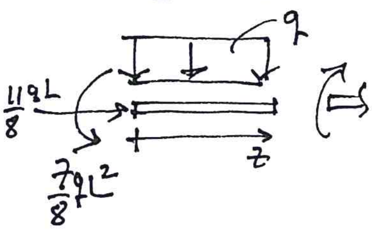
$$\left[-qL - \frac{5}{6}qL\right] = -\frac{11}{6}qL$$

VERSO OPPOSTO A QUELLO IPOTIZZATO!

SOLUZ. SIST. PRINCIPALE ISOSTATICO:

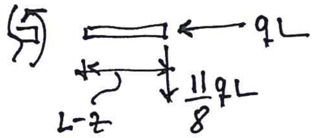


TRATTO AB $0 \le z \le L$



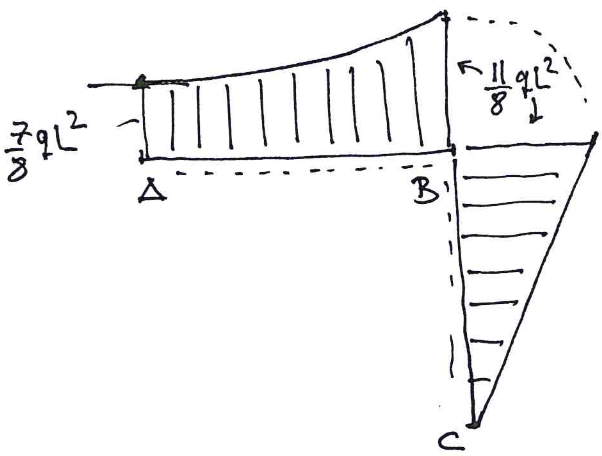
$$M^{(r)}(z) = -\frac{7}{8}qL^2 - \frac{qz^2}{2} \quad \begin{cases} M_A = -\frac{7}{8}qL^2 \\ M_B = -\frac{11}{8}qL^2 \end{cases}$$

TRATTO BC $0 \le z \le L$



$$M^{(r)}(z) = -\frac{11}{8}qL(L-z)$$

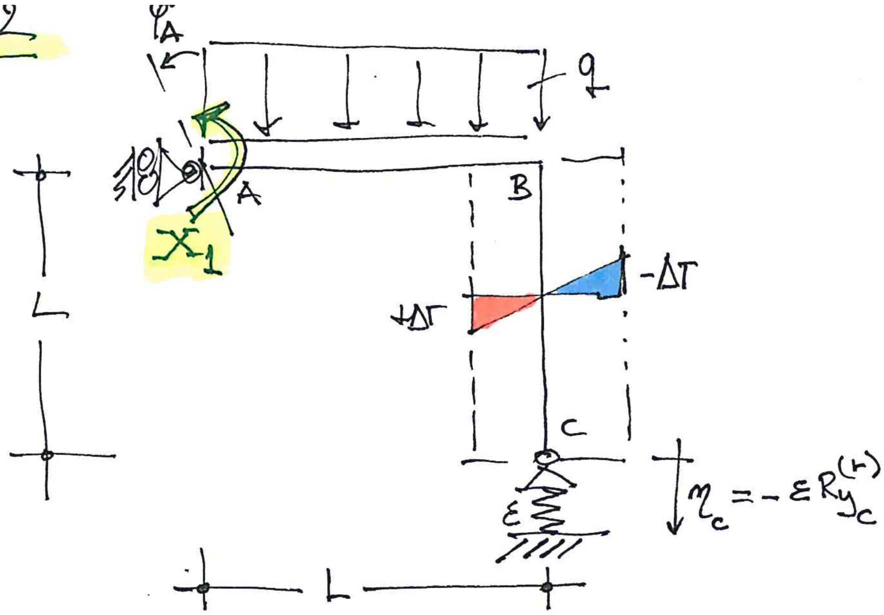
$$\begin{cases} M_B = -\frac{11}{8}qL^2 \\ M_C = \phi \end{cases}$$



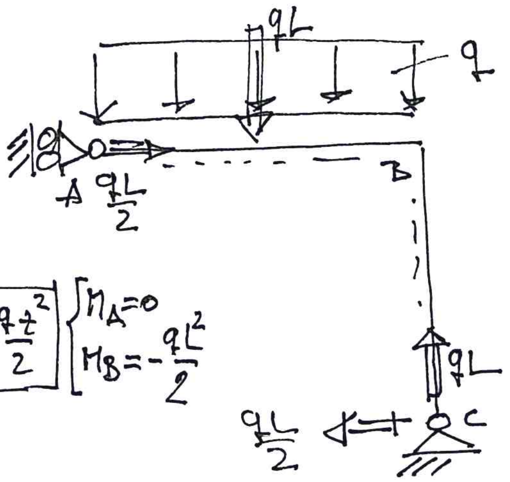
$M^{(r)}(z)$

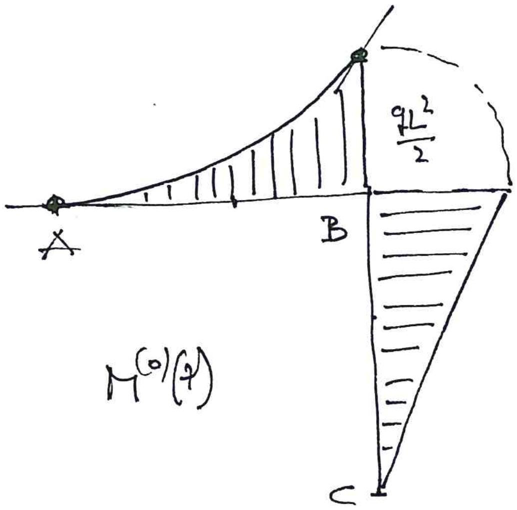
SOLUZIONE 2

SISTEMA PRINCIPALE ISOSTATICO



SCHEMA [0]
SOLO CARICHI ESTERNI





TRATTO AB $0 \le z \le L$

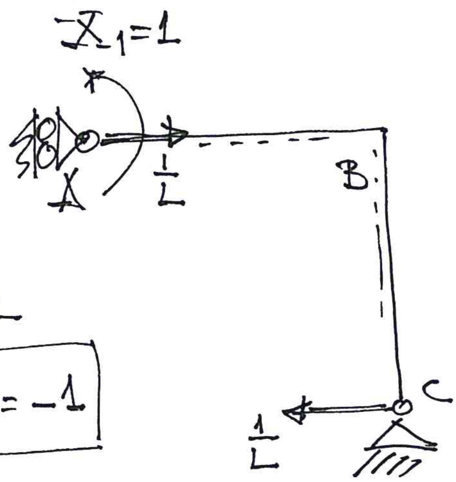
$$M^{(0)}(z) = -\frac{q z^2}{2} \qquad \begin{cases} M_A = 0 \\ M_B = -\frac{qL^2}{2} \end{cases}$$

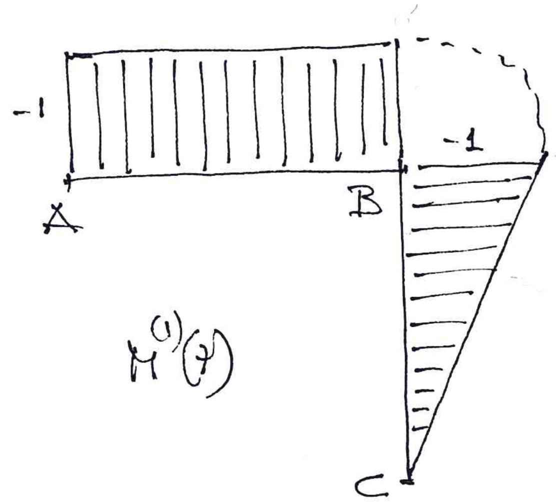
TRATTO BC $0 \le z \le L$

$$M^{(0)}(z) = -\frac{qL}{2}(L-z) \qquad \begin{cases} M_B = -\frac{qL^2}{2} \\ M_C = \varnothing \end{cases}$$

SCHEMA [1]
SOLO $X_1 = 1$

$X_1 = 1$





TRATTO AB $0 \le z \le L$

$$M^{(1)}(z) = -1$$

TRATTO BC $0 \le z \le L$

$$M^{(1)}(z) = -\frac{1}{L}(L-z) \qquad \begin{cases} M_B = -1 \\ M_C = \varnothing \end{cases}$$

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Unica eq. di Müller-Breslau, $L_{ve} = L_{vi}$, corrispondente all'unica incognita iperstatica –

$$L_{ve} = X_i^{(f)} \eta_i^{(r)} + \sum_j R_j^{(f)} \eta_j^{(r)} =$$

$$= \underbrace{X_1 \cdot \varphi_A^0}_{\text{CONCORDI}} + \underbrace{R_{y_C}^{(1)} \cdot \eta_C^{(r)}}_{\phi} = \varphi_A^0 = \frac{qL^3}{EI}$$

$$L_{vi} = \int_{Strut} M^{(1)} \frac{M^{(f)}}{EI} dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr =$$

$$(M^{(f)} = M^{(0)} + X_1 M^{(1)})$$

$$= \frac{1}{EI} \int_{Strut.} M^{(1)} M^{(0)} dStr + \frac{X_1}{EI} \int_{Str} [M^{(1)}]^2 dStr + \frac{\alpha \Delta T}{h} \int_{Str} M^{(1)} dStr =$$

$$= \frac{1}{EI}\left[\int_{\overline{AB}} M^{(1)} M^{(0)} dz + \int_{\overline{BC}} M^{(1)} M^{(0)} dz\right] + \frac{X_1}{EI}\left[\int_{\overline{AB}} [M^{(1)}]^2 dz + \int_{\overline{BC}} [M^{(1)}]^2 dz\right] +$$

$$+ \frac{\alpha \Delta T}{h} \int_{\overline{BC}} M^{(1)} dz =$$

$$= \frac{1}{EI}\left\{\int_0^L -1 \cdot \left[-\frac{qz^2}{2}\right] dz + \int_0^L -\frac{1}{L}(L-z) \cdot \left[-\frac{qL}{2}(L-z)\right] dz\right\} + \quad (L^2 + z^2 - 2Lz)$$

$$+ \frac{X_1}{EI}\left\{\int_0^L 1 \cdot dz + \int_0^L \left[-\frac{1}{L}(L-z)\right]^2 dz\right\} + \frac{\alpha \Delta T}{h} \int_0^L -\frac{1}{L}(L-z) dz = \quad \left(-1 + \frac{z}{L}\right)$$

$$= \frac{1}{EI}\left\{\frac{q}{2}\left[\frac{z^3}{3}\right]_0^L + \frac{q}{2}\left\{L^2[z]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L\right\}\right\} +$$

$$+ \frac{X_1}{EI}\left\{[z]_0^L + \frac{1}{L^2}\left\{L^2[z]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L\right\}\right\} +$$

$$+ \frac{\alpha \Delta T}{h}\left\{-[z]_0^L + \frac{1}{L}\left[\frac{z^2}{2}\right]_0^L\right\} =$$

$$= \frac{1}{EI}\left[\frac{qL^3}{6} + \frac{qL^3}{2} + \frac{qL^3}{6} - \frac{qL^3}{2}\right] + \frac{X_1}{EI}\left\{L + L + \frac{L}{3} - L\right\} + \frac{\alpha \Delta T}{h}\left\{-L + \frac{L}{2}\right\} =$$

$$= \frac{qL^3}{3EI} + \frac{4 X_1 L}{3EI} - \underbrace{\frac{\alpha \Delta T}{h}}_{\frac{qL^2}{EI}} \frac{L}{2} = \frac{4}{3}\frac{X_1 L}{EI} - \frac{qL^3}{6EI}$$

$$\left(\frac{\alpha \Delta T}{h}\frac{L}{2} = \frac{qL^3}{2EI}\right)$$

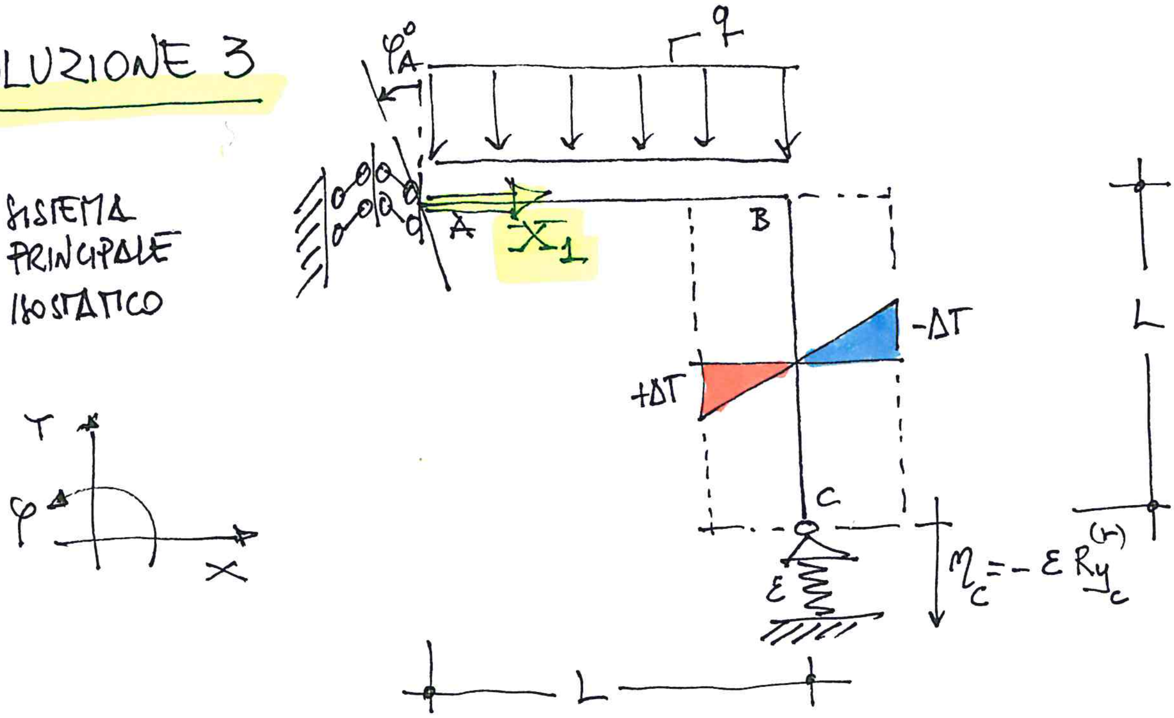
Si ha in definitiva:

$$\frac{qL^{3}}{EI} = \frac{4}{3}\frac{X_1 L}{EI} - \frac{qL^{3}}{6EI} \Rightarrow X_1 = \frac{3}{4}\left[qL^2 + \frac{qL^2}{6}\right] = \frac{7}{8}qL^2 > 0 \text{ POSITIVO}$$

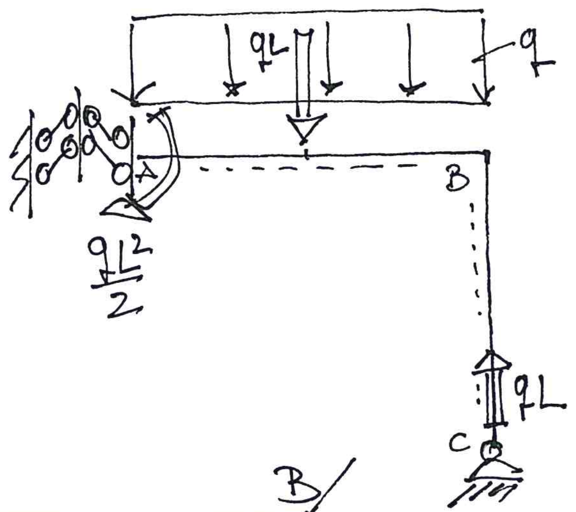
- VERSO IPOTIZZATO CORRETTO!
- cfr. ANCHE CON R_V di PAG. 3! tutto ok!!

SOLUZIONE 3

→ SISTEMA PRINCIPALE ISOSTATICO



→ SCHEMA [0] SOLO CARICHI ESTERNI



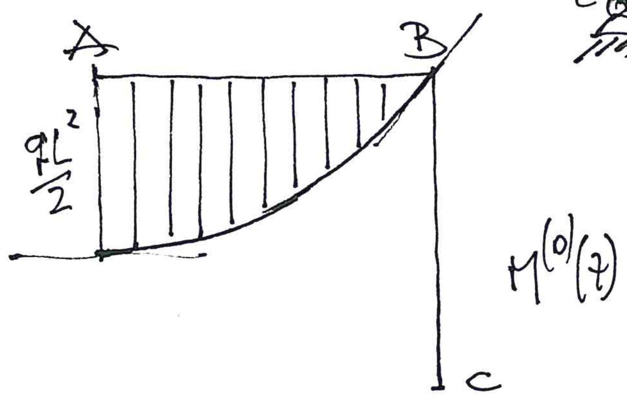
TRATTO AB $0 \le z \le L$

$$M^{(0)}(z) = \frac{qL^2}{2} - \frac{qz^2}{2}$$

$$\begin{cases} M_A = \frac{qL^2}{2} \\ M_B = \emptyset \end{cases}$$

TRATTO BC $0 \le z \le L$

$$M^{(0)}(z) = \emptyset$$



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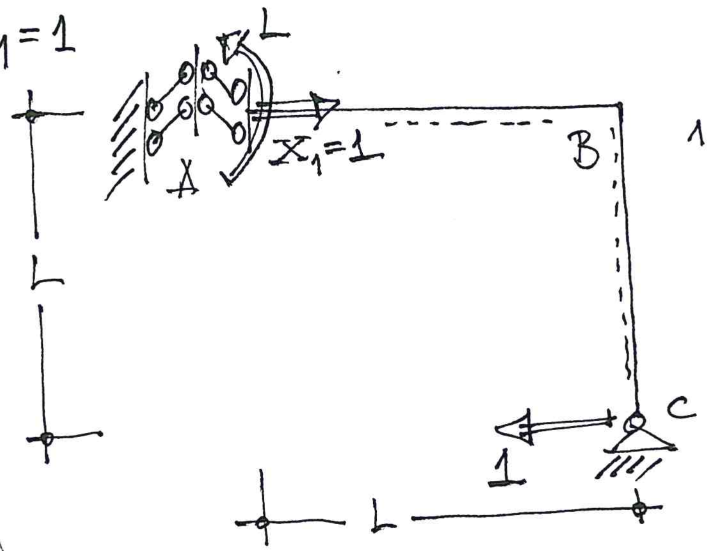
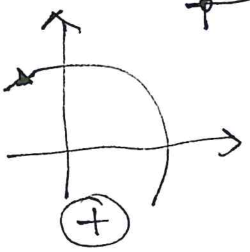
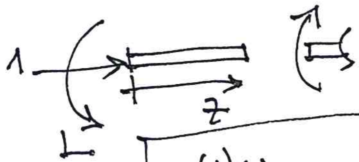
SCHEMA [1]

solo $X_1 = 1$



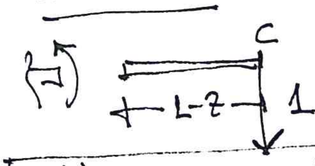


TRATTO AB $0 \le z \le L$

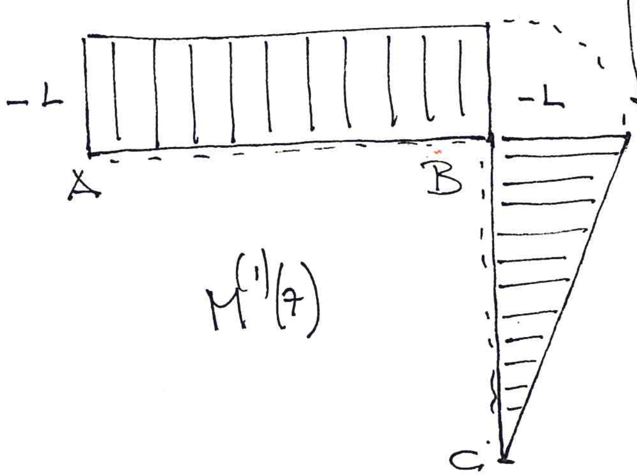


$$M^{(1)}(z) = -L$$

TRATTO BC $0 \le z \le L$



$$M^{(1)}(z) = -1\cdot(L-z) \qquad \begin{cases} M_B = -L \\ M_C = \phi \end{cases}$$



$M^{(1)}(z)$

Unica equazione di Müller-Breslau, $L_{ve} = L_{vi}$, corrispondente all'unica incognita iper.

$$\underset{\sim}{L_{ve}} = X_i^{(f)}\,\eta_i^{(r)} + \sum_j R_j^{(f)}\,\eta_j^{(r)} =$$

$$= \underbrace{1\cdot\phi}_{\phi} + \underbrace{M_A^{(1)}}_{L}\cdot\varphi_A^o + \underbrace{R_{y_C}^{(1)}}_{\phi}\cdot\eta_C^{(r)} = L\cdot\underbrace{\varphi_A^o}_{\frac{qL^3}{EI}} = \underset{\sim}{\frac{qL^4}{EI}}$$

$$L_{vi} = \int_{str}\frac{M^{(1)}M^{(f)}}{EI}\,ds_{tr} + \int_{str} M^{(1)}\frac{\alpha\Delta T}{h}\,ds_{tr} =$$

$$M^{(f)} = M^{(0)} + M^{(1)}X_1$$

$$= \frac{1}{EI}\int_{str} M^{(1)}M^{(0)}\,ds_{tr} + \frac{X_1}{EI}\int_{str}\left[M^{(1)}\right]^2 ds_{tr} + \frac{\alpha\Delta T}{h}\int_{str} M^{(1)}\,ds_{tr} =$$

$$= \frac{1}{EI}\int_{\overline{AB}} M^{(1)}M^{(0)}\,dz + \frac{X_1}{EI}\left[\int_{\overline{AB}}\left[M^{(1)}\right]^2 dz + \int_{\overline{BC}}\left[M^{(1)}\right]^2 dz\right] + \frac{\alpha\Delta T}{h}\int_{\overline{BC}} M^{(1)}\,dz =$$

P. FUSCHI
A. PISANO

$$= \frac{1}{EI}\left[\int_0^L -L\cdot\left[\frac{qL^2}{2}-\frac{qz^2}{2}\right]dz\right] + \frac{X_1}{EI}\left[\int_0^L L^2\,dz + \int_0^L (L-z)^2\,dz\right] +$$

$$+ \frac{\alpha\Delta T}{h}\int_0^L [z-L]\,dz =$$

$$= \frac{1}{EI}\left[-\frac{qL^3}{2}[z]_0^L + \frac{qL}{2}\left[\frac{z^3}{3}\right]_0^L\right] + \frac{X_1}{EI}\left[L^2[z]_0^L + L^2[z]_0^L + \left[\frac{z^3}{3}\right]_0^L - 2L\left[\frac{z^2}{2}\right]_0^L\right] +$$

$$+ \frac{\alpha\Delta T}{h}\left\{\left[\frac{z^2}{2}\right]_0^L - L[z]_0^L\right\} =$$

$$= \frac{1}{EI}\left[-\frac{qL^4}{2} + \frac{qL^4}{6}\right] + \frac{X_1}{EI}\left[L^3 + \cancel{L^3} + \frac{L^3}{3} - \cancel{L^3}\right] + \frac{\alpha\Delta T}{h}\left[\frac{L^2}{2} - L^2\right] =$$

$$= -\frac{qL^4}{3EI} + \frac{4X_1L^3}{3EI} - \underbrace{\frac{\alpha\Delta T\,L^2}{h\,2}}_{\underbrace{\frac{qL^2}{EI}\cdot\frac{L^2}{2}}_{\frac{qL^4}{2EI}}} = -\frac{5}{6}\frac{qL^4}{EI} + \frac{4}{3}\frac{X_1L^3}{EI}$$

In definitiva si ha:

$$\frac{qL^4}{EI} = -\frac{5}{6}\frac{qL^4}{EI} + \frac{4}{3}\frac{X_1L^3}{EI}$$

$$X_1 = \frac{3}{4}\left[qL + \frac{5}{6}qL\right] = \frac{11}{8}qL > 0 \quad \Rightarrow \text{VERSO IPOTIZZATO CORRETTO!}$$

$$\frac{11}{6}qL$$

cfr. anche con R_{X_A} di pag. 3! tutto ok!