

# ABSTRACT BOOKLET



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DIIES - Università Mediterranea di Reggio Calabria

Fourth Workshop on  
*Variational Inequalities, Nash Equilibrium Problems and  
Applications*

Reggio Calabria, 8-9 March 2018

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## INVITED COMMUNICATIONS

### Quasistatic evolution variational inequalities in the framework of Moreau's Sweeping process

Adly S.

**Abstract:** In this talk, we study a new variant of the Moreau's sweeping process with velocity constraint. Based on an adapted version of the Moreau's catching-up algorithm, we show the well-posedness (in the sense existence and uniqueness) of this problem in a general framework. We show the equivalence between this implicit sweeping process and a quasistatic evolution variational inequality. It is well-known that the variational formulation of many mechanical problems with unilateral contact and friction lead to an evolution variational inequality. As an application, we reformulate the quasistatic antiplane frictional contact problem for linear elastic materials with short memory as an implicit sweeping process with velocity constraint. The link between the implicit sweeping process and the quasistatic evolution variational inequality is possible thanks to some standard tools from convex analysis and is new in the literature.

Talk based on the following:

[AH] S. Adly and T. Haddad, An implicit sweeping process approach to quasistatic evolution variational inequalities. To appear in SIAM Journal on Mathematical Analysis.

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# Nash Equilibria under the lens of Computational Complexity

Greco G.

**Abstract:** The study of computational issues related to Nash equilibria has attracted much attention in the last decade within the Computer Science community. The talk will present some relevant results in this area, by focusing on strategic games and by considering mixed equilibria as well as pure and Bayesian ones. The exposition will be accessible to a broad audience and, in fact, basic notions of computational complexity will be presented, too. For the sake of concreteness, recent applications of Nash equilibria for analysing the interactions that emerge over social environments (e.g., Twitter or FaceBook) will be also discussed, again by focusing not only on modelling issues, but by also paying attention at their algorithmic underpinnings.

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# Game Theory Models of Cybercrime and Cybersecurity Investments Under Network Vulnerability

Nagurney A.

**Abstract:** The effects of cyberattacks are being felt across the globe in multiple sectors and industries. The damages incurred include direct financial damages as well as reputation issues, the loss of business, the inability to provide the expected services, opportunity costs, and the loss of trust. The world economy sustained over 460 billion in losses from cyberattacks in 2016 alone.

In this talk, I will first describe a predictive analytical multiproduct network economic model of cybercrime in financial services in which the hacked products are perishable in that their “value” deteriorates over time. I will then discuss our research on game theory models for cybersecurity investments and network vulnerability when firms compete and when they cooperate in terms of information sharing. Algorithms and computational results for both classes of models will be presented and discussed and case studies presented in retail and energy sectors with extensive sensitivity analysis results that demonstrate the benefits of cooperation. I will conclude with recent research on cybersecurity and supply chains.

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# Grouping Games: Finding Clusters in Graphs, Digraphs and Hypergraphs

Pelillo M.

**Abstract:** Clustering refers to the process of extracting maximally coherent groups from a set of objects using pairwise, or high-order, similarities. Traditional approaches to this problem are based on the idea of partitioning the input data into a predetermined number of classes, thereby obtaining the clusters as a by-product of the partitioning process. In this talk, I'll provide a brief review of recent work done in my group which offers a radically different view of the problem. In contrast to the classical approach, in fact, we attempt to provide a meaningful formalization of the notion of a cluster and we show that game theory offers an attractive and unexplored perspective that serves well our purpose. To this end, we formulate the clustering problem in terms of a non-cooperative "clustering game" and show that a natural notion of a cluster turns out to be equivalent to a classical (evolutionary) game theoretic equilibrium concept. We prove that the problem of finding the equilibria of our clustering game is equivalent to locally optimizing a polynomial function over the standard simplex, and we provide a discrete-time dynamics to perform this optimization, based on the Baum-Eagon inequality. The proposed grouping framework, which has already found applications in a variety of application fields, including computer vision, bioinformatics, security and video surveillance, etc., is general and can be applied to weighted graphs, digraphs and hypergraphs alike.

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# CONTRIBUTED COMMUNICATIONS

## A Differential Game with Exit Costs

Bagagiolo F.

**Abstract:** I will present a differential game where two players separately control their own evolution and aim to maximize and to minimize, respectively, the same functional. The game possibly stops when one of the two players leaves a given domain (one for each player). The cost functional consists of a shared integrated running cost and of three different exist costs: one for the case when the first player only leaves its own domain, one for the case when the second player only leaves its own domain, and one for the case when they simultaneously leave their domains.

The motivations for studying this kind of problem come, for example, from differential games with dynamics on networks or with hybrid dynamics.

Using suitable non-anticipative strategies, I will prove that the lower and upper value of the game are continuous functions and are the unique viscosity solutions of two suitable Dirichlet problems for a Hamilton-Jacobi-Isaacs equation, respectively. In particular, the boundary conditions are formulated in a variational inequality way, as it is standard for optimal control problems with exit cost, but instead rather new for differential games with exit costs.

From such uniqueness results, we can also get, under some hypotheses, the existence of an equilibrium for the game.

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# A Financial Model for a Multi-Period Portfolio Optimization Problem with a Variational Formulation

Colajanni G.

**Abstract:** We aim at presenting a mathematical model, based on networks, which allows us to formulate a new multi-period portfolio selection problem as a Markowitz mean-variance optimization problem with intermediaries and the addition of transaction costs and taxes (on the capital gain). Moreover, by means of the proposed Integer Nonlinear Programming (INLP) Problem, it is possible to establish when it is suitable to buy and to sell financial securities, not only while maximizing the profits but also while minimizing the risk which is weighted by an aversion degree or risk inclination value. We find the related optimality conditions, which provide us with a variational inequality formulation. Some existence and uniqueness results as well as the Lagrange formulation are stated and some numerical examples are studied.

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# Precise Morrey Regularity of the Weak Solutions to a Kind of Quasilinear Systems with Discontinuous Data

Fattorusso L.

**Abstract:** We consider the Dirichlet problem for a class of quasilinear elliptic systems in domain with irregular boundary. The principal part satisfies componentwise coercivity condition while the lower order terms are Caratheodory maps having Morrey regularity in  $x$  and verifying controlled growth conditions with respect to the other variables. We have obtained boundedness of the weak solutions to the problem that permits to apply an iteration procedure in order to an optimal Morrey regularity of the gradient of the solutions.

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# Quantitative estimates for very weak solutions to some Partial Differential Equations arising in Geometric Function Theory

Farroni F.

**Abstract:** In this talk, we provide existence, uniqueness and the precise behavior for solutions to elliptic and parabolic PDEs which are typically related to some Variational Problems in the Geometric Function Theory

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# Cournot-Nash-Walras Games

Flåm S.D.

**Abstract:** Considering use of renewable resources as games, this paper suggests that property rights be replaced by user rights. By assumption, effective use is paid for - or valued - via direct deals or double auctions. These may serve to restore or secure substantial parts of the resource rent. Residual parts will remain though, with users who supply oligopolistic product markets. The model marries a perfect market, in user rights, to a strategic game, in outputs. Broadly, Walrasian exchange connects to Nash equilibrium and Cournot oligopoly.

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# Consistent Equilibrium in Oligopoly coincides with Nash Equilibrium in the Upper Level Game

Flores-Muñiz J.G.

**Abstract:** Recently, a new concept of conjectural variations equilibrium (CVE) was investigated, in which the conjectural variations (represented via the influence coefficients of each agent) affected the structure of the Nash equilibrium. According to this concept, the agents behave as follows: each agent chooses his/her most favorable action taking into account that every rival's strategy is a conjectured function of his own strategy.

The main obstacle in the way of admitting this concept is its consistency. The equilibrium is consistent if the conjectural best response of each agent coincides with the conjectured reaction function of the same. However, such a definition is too restrictive since if the number of players  $n$  is higher than two then one cannot compare  $n$  reaction functions with  $n(n-1)$  conjectured best responses.

In order to cope with such a conceptual difficulty arising in many-player games, a completely new approach was proposed in our previous papers. Namely, we supposed that each player makes conjectures not about the optimal response functions of the other players but only about first-order variations of the market price depending upon his/her infinitesimal output variations. In the equilibrium, each agent applies a verification procedure and check if his influence coefficient is consistent with those of the rest of the agents. In a case when the CVE is consistent for each agent, we call it interior (in contrast to the exterior equilibrium, in which not all conjectures must be consistent with others).

As in general, the consistent conjectures need not coincide with those of Nash, an interesting question arises: Is there any relationship between the classical Nash equilibrium and the CVE? The question is answered positively in this talk.

Based on a joint work with V. Kalashnikov, N. Kalashnykova

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# Solving equilibrium problems using extended mathematical programming and SELKIE

Kim Y.

**Abstract:** We introduce an extended mathematical programming (EMP) framework along with its solver SELKIE for specifying equilibrium problems and their variational forms and computing solutions of them from modeling languages. The problem classes that we focus on are generalized Nash equilibrium, multiple optimization problems with equilibrium constraints, and (quasi-) variational inequalities. Our framework defines a new set of constructs that allow a natural translation of the model from one formulation to another computationally more tractable form without requiring the modeler to supply derivatives. In the context of many independent agents in the equilibrium, SELKIE transforms a given model into a set of submodels based on a grouping of agents in a flexible and adaptable way to achieve a more robust and faster solution path. Diagonalization is then applied to those submodels possibly with parallel computations for making full use of computational resources. This decomposition-based approach makes SELKIE effective for large-scale equilibrium problems. We can choose a subsolver to use for each submodel so that a highly efficient process can be employed tailored to a certain problem type. For stronger convergence results and numerical stability, primal and dual proximal perturbations are implemented. Examples illustrating the flexibility and effectiveness of SELKIE are given. Our framework and SELKIE have been implemented and are available within GAMS/EMP.

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# Applications of a General Market Model to Network Equilibrium Problems

Konnov I.

**Abstract:** We present a general market model with divisible commodities that admits different implementation mechanisms associated with proper information exchange schemes. Using its equivalent variational inequality reformulation, we obtain an existence result on unbounded sets. We describe an extension of the network flow equilibrium problem with elastic demands and show it is a particular case of the general market model. This enables us to obtain new existence results for this model and to find its solution with iterative methods. We describe implementation of linearization type methods to this problem.

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# Dual Decomposition Approach for Dynamic Spatial Equilibrium Models: an Analysis of the Spot Gas Markets

Oggioni G.

**Abstract:** We consider the general problem of managing a dynamic system of spatially distributed auction markets of a homogeneous commodity that are joined by transmission lines in a network. At each market, traders and buyers are represented by their offer/bid price functions and volume bounds. We propose a set of equilibrium type conditions for this system of markets and show that it is equivalent to a single-level variational inequality problem. In order to find its solution, which yields an equilibrium trajectory, we propose to apply a combined proximal point and dual type methods. Numerical experiments are conducted on the spot gas markets in the Netherlands and in the United Kingdom.

Based on a joint work with E. Allevi, A. Gnudi, I.V. Konnov

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# Variational Properties of a Functional for Motion Compensated Video Inpainting

Riey G.

**Abstract:** In the last years variational methods have been widely used in computer vision in order to solve several kinds of problems (for instance: edge detection, denoising, inpainting, motion estimation). In this framework the common strategy consists in minimizing a suitable functional. Typically, the considered functionals involve the sum of terms which enforce fidelity to the data and terms with regularizing effects on the solutions. Recently, Lauze and Nielsen proposed a variational model for simultaneous video inpainting and motion estimation. Their approach requires the minimization of an integral functional, whose integrand has linear growth with respect to the partial derivatives of first and second order of the unknown functions. We modify their functional in order to achieve better variational properties and we study the corresponding relaxed functional, which gives information about numerical algorithms designed for the original functional. Minimizers of the relaxed functional are functions of Bounded Variation (BV). In two dimensions, differently from Sobolev functions, BV functions can be discontinuous along curves and that allows the reconstruction of the video content, which is discontinuous along the boundaries of moving objects.

Based on a joint work R. March

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# Generalized Morrey regularity of the solutions to a kind of nonlinear equations of p-Laplacian type

Softova L.

**Abstract:** We consider the Dirichlet problem for a kind of nonlinear elliptic equations in divergence form in domains of Reifenberg type. We study the regularity of the solutions in the generalized Morrey spaces.

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# An optimal control problem for nonlinear elliptic equations with unbounded coefficients

Zecca G.

**Abstract:** We consider the Dirichlet problem for a kind of nonlinear elliptic equations in divergence form in domains of Reifenberg type. We study the regularity of the solutions in the generalized Morrey spaces. We study an optimal control problem associated to a Dirichlet boundary value problem of the type

$$(BVP) \quad \operatorname{div} \left[ \beta(x) \nabla u(x) + \left( A \frac{x}{|x|^2} + g(x) \right) u(x) \right] = \operatorname{div} \mathcal{F}, \quad u \in W_0^{1,p}(\Omega),$$

$1 < p \leq 2$ , where  $\Omega$  is a bounded regular domain of  $\mathbb{R}^n$ ,  $0 \in \Omega$ ,  $\beta : \Omega \rightarrow \mathbb{R}$  is an unbounded function satisfying  $\beta(x) \geq \lambda_0 > 0$  a.e.,  $A$  is a *suitably small* constant, and  $g \in L^\infty(\Omega; \mathbb{R}^n)$ . We consider the vector field  $\mathcal{F}$  as the control and the corresponding *weak solution*  $u$  to (BVP) as the state. Our aim is to find the optimal vector field  $\mathcal{F} \in L^p(\Omega)$  so that the corresponding state  $u \in W_0^{1,p}(\Omega)$  is close to the desired profile in  $L^p(\Omega)$  while the norm of  $u$  in  $W^{1,p}(\Omega)$  is not too large. We prove that, for every  $p$  less than 2 and suitably close to 2, (BVP) admits an unique *weak solution* and for such values of  $p$ , we prove the existence of optimal pairs.

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