

Second Workshop on  
"Variational inequalities, Nash equilibrium problems and  
applications"

Reggio Calabria, September 24-25, 2015

PROGRAM

ORGANIZERS:

Patrizia Daniele - University of Catania

Sofia Giuffrè - Mediterranea University of Reggio Calabria

Leonardo Militano - Mediterranea University of Reggio Calabria

Laura Scrimali - University of Catania

SPONSORS:

Dipartimento di Ingegneria dell'Informazione, delle Infrastrutture e dell'Energia  
Sostenibile - Mediterranea University of Reggio Calabria

Dipartimento di Matematica e Informatica - University of Catania

Istituto Nazionale di Alta Matematica "Francesco Severi"

## September 24, Thursday

10.00-10.15 Welcome ceremony

10.15- 11.00 A. MAUGERI

*On nonlinear strong duality and the infinite dimensional Lagrange Multiplier rule*

11.00-11.30 Coffee break

11.30-12.15 L. SANTOS

*Variational and quasi-variational inequalities with curl constraint*

12.15-12.40 L. SOFTOVA

*Parabolic obstacle problem in generalized in Morrey spaces*

12.40-13.05 D. PUGLISI

*Lagrange Multiplier rule in infinite dimensional*

13.05-15.00 Lunch

15.00-15.45 B. DACOROGNA

*Symplectic decomposition, Darboux theorem and ellipticity*

15.45-16.10 A. CHINNI'

*Multiple solutions for Neumann-type differential inclusion problems involving variable exponent*

16.10-16.35 G. BARLETTA

*Existence and multiplicity results for parametric variational-hemivariational inequalities*

16.35-17.00 R. LIVREA

*A critical point approach for the study of variational-hemivariational inequalities*

17.30 Visit to Reggio Calabria National Museum

20.30 Social Dinner

## September 25, Friday

9.00-9.45 S. ADLY

*The principle of least action: a wonderful story*

9.45-10.30 F. SCARCELLO

*Structural Tractability of Coalition Games, with an application to the Research Evaluation Assessment in Italy*

10.30-11.00 Coffee break

11.00-11.25 A. BARBAGALLO

*A stochastic approach for the management of an economic problem*

11.25-11.50 D. ADDONA

*On coupled systems of Kolmogorov equations with applications to stochastic differential games*

11.50-12.15 G. ZECCA

*On the Unilateral Obstacle Problem for Noncoercive Operators*

12.15-12.40 M. KRUPSKI

*Convection-Diffusion Equations with Random Initial Conditions*

12.40-13.05 C. BUCUR

*Local density of Caputo-stationary functions in the space of smooth functions*

## Second Workshop on

# “Variational inequalities, Nash equilibrium problems and applications”

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## ABSTRACTS

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**Davide Addona** (University of Milano Bicocca)

ON COUPLED SYSTEMS OF KOLMOGOROV EQUATIONS WITH APPLICATIONS TO STOCHASTIC DIFFERENTIAL GAMES

In our paper we present existence and uniqueness of classical solutions to a system of Kolmogorov equations of parabolic type. We investigate the continuity properties of the evolution operator associated, and prove an integral representation formula obtained in a different way with respect to the scalar case. Finally, we prove weighted gradient estimates and show some interesting applications. In particular, we apply our result to a system of Forward-Backward stochastic differential equations and to a stochastic differential game, proving that it is possible to write a Nash Equilibrium in terms of a mild solution of a semilinear system of PDE.

Based on a joint work with Luciana Angiuli, Luca Lorenzi e Gianmario Tessitore.

**Samir Adly** (Université de Limoges)

THE PRINCIPLE OF LEAST ACTION: A WONDERFUL STORY

In this talk accessible to a wide audience of scientists and students, we first start by reviewing some historical facts about the principle of least action from Fermat and his geometrical optics in 1657 to Feynman and his theory of quantum electrodynamics in 1942. Passing through the works of J. Bernoulli, Maupertuis, Euler, Lagrange, Hamilton, Planck and De Broglie.

Nowadays, this principle is the basis of Newtonian, relativistic, quantum mechanics and many other sciences.

Many mechanical systems are subject to conservative forces that can be derived from a differentiable or a nondifferentiable potential. The equations of motion of such systems take the form of Euler-Lagrange equations. As an application, we will study a class of nonsmooth robust controllers for lagrangian dynamical systems with non-trivial mass matrix. First the existence and uniqueness of solutions will be analyzed, then the Lyapunov stability, the Krasovskii-LaSalle invariance principle and the finite-time convergence properties will be discussed in detail.

**Annamaria Barbagallo** (University of Naples “Federico II”)

A STOCHASTIC APPROACH FOR THE MANAGEMENT OF AN ECONOMIC PROBLEM

The aim of the talk is to study a stochastic variational inequality that models the oligopolistic market equilibrium problem in conditions of uncertainty. In particular, a random Cournot-Nash equilibrium condition is given and the equivalence with the stochastic variational inequality is proved. Moreover, some existence results for the random equilibrium solutions are showed. Making use of the infinite dimensional Lagrange duality theory, the existence of Lagrange multipliers associated to the problem is obtained. In the end a numerical example is discussed.

Based on a joint work with Paolo Mauro

**Giuseppina Barletta** (Mediterranea University of Reggio Calabria)

EXISTENCE AND MULTIPLICITY RESULTS FOR PARAMETRIC VARIATIONAL-HEMIVARIATIONAL INEQUALITIES

We present existence and multiplicity results for some parametric variational-hemivariational inequalities driven by a possibly nonhomogeneous operator and involving a nonlinear term that may have critical growth.

The problem is

$$(P_{\lambda,\mu}) \left\{ \begin{array}{l} \text{Find } u \in W_0^{1,p}(\Omega), u \leq \psi \text{ in } \Omega, \text{ satisfying} \\ \langle Au, v - u \rangle - \mu \int_{\Omega} |u|^{p-2} u (v - u) dx + \lambda \int_{\Omega} J^0(x, u(x); (v - u)(x)) dx \geq 0 \\ \text{for all } v \in W_0^{1,p}(\Omega), v \leq \psi \text{ in } \Omega, \end{array} \right.$$

where  $\Omega$  is a nonempty open bounded subset of  $R^N$ ,  $\lambda$  is a real parameter,  $\psi$  is a nonnegative function of  $W_0^{1,p}(\Omega)$ , the operator  $A$  is  $\langle Au, v \rangle = \int_{\Omega} (a(x, Du), Dv)_N dx$  and  $J^0$  is the generalized directional derivative with respect to the second variable of a nonsmooth function  $J(x, \xi) = -q(x)F(\xi)$ .  $q(x)$  may change sign and  $J$  may have critical growth. We avoid any assumption on the behaviour of  $J$  at  $\pm\infty$ , except the one on the growth. Our problem incorporates, as a special case the classical  $p$ -Laplacian with small perturbations.

We provide different sets of verifiable hypotheses ensuring the existence of at least three nonnegative solutions of the problem.

**Claudia Bucur** (University of Milano)

LOCAL DENSITY OF CAPUTO-STATIONARY FUNCTIONS IN THE SPACE OF SMOOTH FUNCTIONS

We consider the Caputo fractional derivative and say that a function is Caputo-stationary if its Caputo derivative is zero. We then prove that any  $C^k([0, 1])$  function can be approximated in  $[0, 1]$  by a function that is Caputo-stationary in  $[0, 1]$ , with initial point  $\alpha < 0$ . Otherwise said, Caputo-stationary functions are dense in  $C_{loc}^k(\mathbb{R})$ .

**Antonella Chinnì** (University of Messina)

MULTIPLE SOLUTIONS FOR NEUMANN-TYPE DIFFERENTIAL INCLUSION PROBLEMS INVOLVING VARIABLE EXPONENT

The existence of at least three distinct solutions for a Neumann-type differential inclusion problem involving the  $p(\cdot)$ -Laplacian is investigated. The results are obtained by using a multiple critical points theorem for locally Lipschitz continuous functionals.

**Bernard Dacorogna** (École Polytechnique Fédérale de Lausanne)  
SYMPLECTIC DECOMPOSITION, DARBOUX THEOREM AND ELLIPTICITY

Our first result concerns the classical Darboux theorem. We prove that if  $\omega_m$  is the standard symplectic form and  $f$  is a symplectic form, then we can find a diffeomorphism  $\varphi$ , with optimal regularity, satisfying

$$\varphi^*(\omega_m) = f \quad \text{and} \quad \delta[\varphi \lrcorner \omega_m] = 0$$

provided that  $f$  is a small perturbation of  $\omega_m$ . Moreover we show that the above system is elliptic and that we have uniqueness, when coupled with a Dirichlet datum.

We then apply the above result to the so-called symplectic decomposition. We show that any map  $\varphi$ , satisfying appropriate assumptions, can be written as

$$\varphi = \psi \circ \chi$$

where

$$\psi^*(\omega_m) = \omega_m \quad \text{and} \quad \delta[\chi \lrcorner \omega_m] = d\chi \lrcorner \omega_m = 0.$$

**Milosz Krupski** (University of Wrocław)  
CONVECTION-DIFFUSION EQUATIONS WITH RANDOM INITIAL CONDITIONS

The talk will be devoted to a brief overview of a Burgers-type partial differential equation

$$\partial_t u = \Delta u + \partial_x f(u) \tag{1}$$

with the initial condition  $u(0) = u_0$  given as a stationary (isotropic) random field. In general, similar equations with such random initial conditions appear in a wide range of topics, e.g in the study of the Large Scale Structure of the Universe or growing interfaces in deposition of chemical substances.

In the talk I will state the problem more rigorously and try to explain some of the difficulties that arise when dealing with nonlinear equations of this type.

The talk is meant to be straightforward and technicalities kept to the necessary minimum.

**Roberto Livrea** (Mediterranea University of Reggio Calabria)  
A CRITICAL POINT APPROACH FOR THE STUDY OF VARIATIONAL-HEMIVARIATIONAL INEQUALITIES

In the talk a brief overview of some meaningful critical point theorems for non-smooth functions will be presented. Moreover, some possible applications to suitable classes of differential inclusions, variational-hemivariational inequalities and of certain differential problems involving possible discontinuous nonlinearities will be given.

**Antonino Maugeri** (University of Catania)

ON NONLINEAR STRONG DUALITY AND THE INFINITE DIMENSIONAL LAGRANGE MULTIPLIER RULE

In the present note we treat the problem regarding the strong duality in the case the equality constraint is nonlinear. Some theorems ensuring the strong duality are given and the obtained results are described by examples.

**Daniele Puglisi** (University of Catania)

LAGRANGE MULTIPLIER RULE IN INFINITE DIMENSIONAL

We treat strong duality and some equivalent condition depending (or not) on constraint functions.

**Lisa Santos** (University of Minho)

VARIATIONAL AND QUASI-VARIATIONAL INEQUALITIES WITH CURL CONSTRAINT

We prove the existence of solution of stationary and evolutionary variational and quasi-variational inequalities involving the p-curl operator and with curl constraint. In the more complicated evolutionary quasi-variational case, the constraint depends in a nonlocal way of the solution. We approximate this problem by a family of regularized systems and use a fixed point argument. After obtaining minimal a priori estimates for the solutions of the approximating problems, we pass to the weak limit, proving the existence of solutions of the original quasi-variational inequality. We exemplify the type of nonlocal constraints by coupling the quasi-variational inequality with a simple heat equation that provides a sufficient compactness.

Based on a joint work with Fernando Miranda and José Francisco Rodrigues.

**Francesco Scarcello** (University of Calabria)

STRUCTURAL TRACTABILITY OF COALITION GAMES, WITH AN APPLICATION TO THE RESEARCH EVALUATION ASSESSMENT IN ITALY

In a great number of applications, it is necessary to distribute resources or tasks to agents collaborating with each other in order to maximize the social welfare of the structure they belong to. The question in these cases is how to divide in a fair way the outcome that the structure eventually earns, say money, to the participating agents.

Coalitional game-theory provides the formal tools to study such problems, known as allocation games. Unfortunately, the computation of most solution concepts is intractable, typically  $\#P$ -hard. Motivated by this bad news, structural requirements are investigated which can be used to identify islands of tractability. In particular, we show that, over the class of allocation games, the Shapley value and the Banzhaf value can be computed in polynomial time when interactions among agents can be

formalized as graphs of bounded treewidth. This is shown by means of technical tools that are of interest in their own and that can be used for analyzing different kinds of games. We then focus on a real-world application: the distribution of funds to Italian research structures and substructures, after a research assessment program (known as VQR) that is used to evaluate the Italian research production. A fair solution based on game theory is described and analyzed. It turns out that the proposed solution enjoys all the desirable properties of fair division rules, and it could effectively be implemented in the VQR, as well as in related division-rules applications.

**Lubomira G. Softova** (Second University of Napoli)

PARABOLIC OBSTACLE PROBLEM IN GENERALIZED MORREY SPACES

We establish *Calderón-Zygmund type* estimate for the weak solutions of variational inequalities related to linear divergence form parabolic systems with discontinuous data. The *obstacle* is constrained in the frameworks of the generalized *Morrey spaces*  $M^{p,\varphi}$ ,  $p > 1$  under various conditions on the weight  $\varphi$ . The coefficients of the operator supposed to be *only measurable* in one of the space variables and to have *small mean oscillation* in the others. Regarding the non-smooth domain we suppose that its boundary is *flat in the sense of Reifenberg*.

**Gabriella Zecca** (University of Naples “Federico II”)

ON THE UNILATERAL OBSTACLE PROBLEM FOR NONCOERCIVE OPERATORS

We study the unilateral obstacle problem related to second order nonlinear elliptic equations whose model appears in the stationary diffusion-convection problem. Precisely, we are interested in operators of the type

$$-\operatorname{div}[\mathcal{A}(x, \xi) + \mathcal{B}(x, s)]$$

where  $\mathcal{A} : \Omega \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  satisfies standard growth conditions and  $\mathcal{B} : \Omega \times \mathbb{R}^n \rightarrow \mathbb{R}$  satisfies, for all  $s \in \mathbb{R}$ ,  $|\mathcal{B}(x, s)| \leq b(x)|s|$ . We obtain existence, uniqueness and regularity of solutions assuming that the growth coefficient of the convection term  $b$  lies in the Marcinkiewicz space *weak- $L^N(\Omega)$*  without assuming the smallness of the norm. Therefore, we are dealing with problems that are in general not coercive.

Based on a joint work with Luigi Greco and Gioconda Moscariello.

#### REFERENCES

- [1] L. Greco, G.Moscariello and G. Zecca: An Obstacle Problem for Noncoercive Operators, *Abstract and Applied Analysis* (2015), <http://dx.doi.org/10.1155/2015/890289>.
- [2] L. Greco, G.Moscariello and G. Zecca: Regularity for solutions to nonlinear elliptic equations, *Differential Integral Equations Vol. 26, N. 9-10* (2013), 1105- 1113.