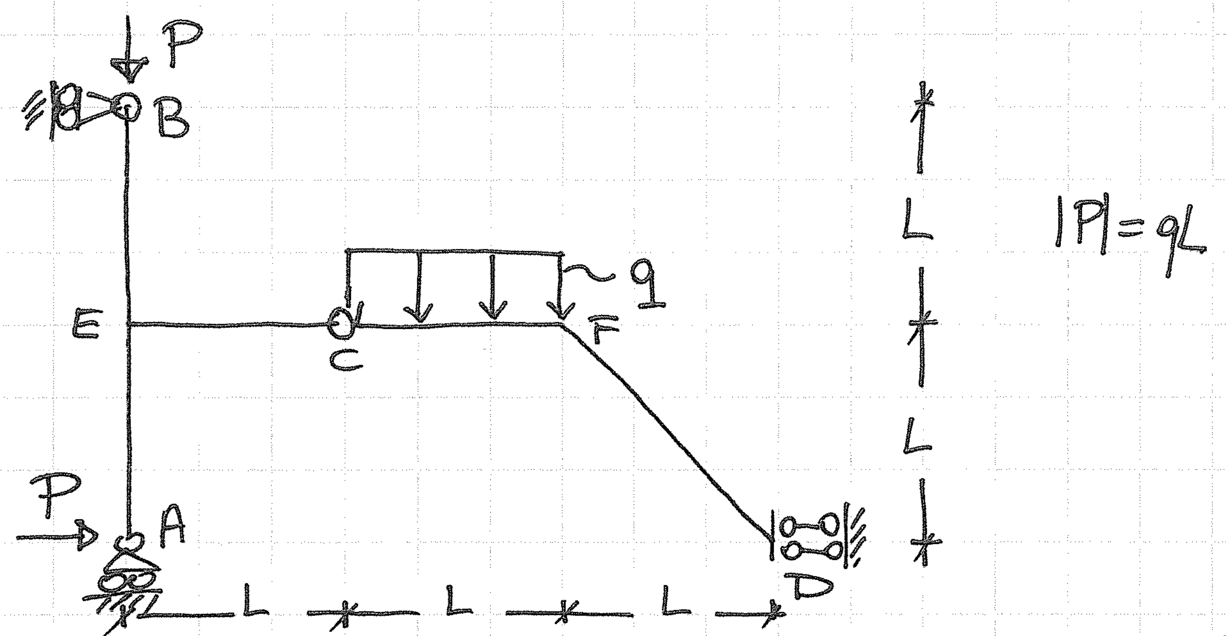
Prova scritta del 25/01/2017 Fuschi/Pisano

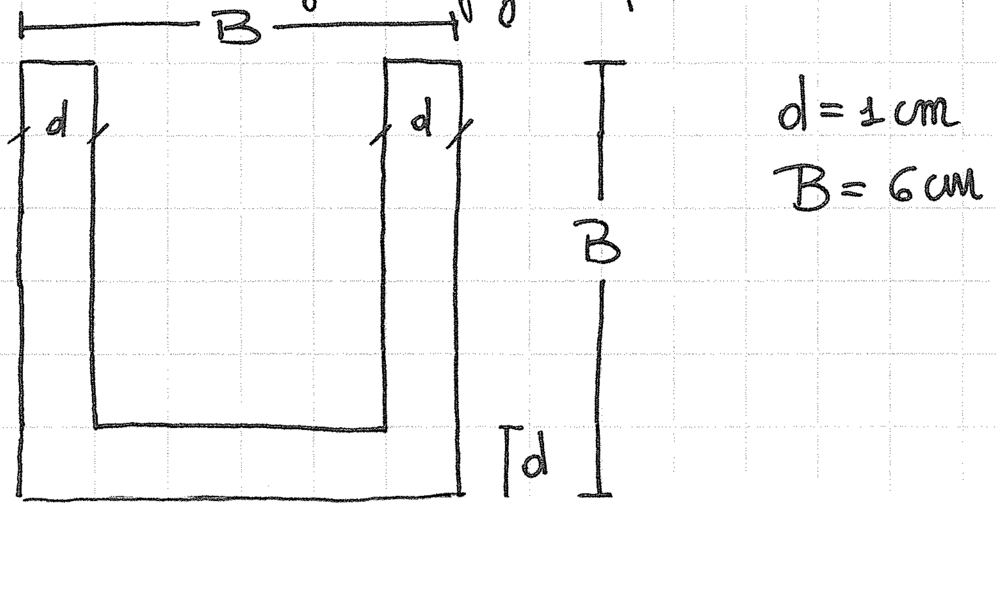
Corso di Statica AA 2016-2017

Docente del Corso A.A. Pisano $\boxed{1}$

- Es # 1 Determinare le reazioni vincolari, le funzioni caratteristiche di sollecitazione e i relativi diagrammi del sistema isostatico di travi piane di seguito riportato. Verificare l'equilibrio del nodo triplo E.



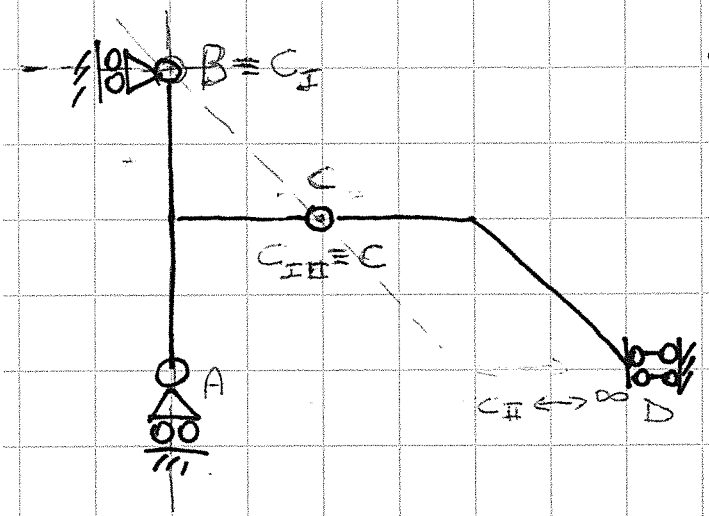
- Es#2 Con riferimento al sistema isostatico dato in #1, determinare le reazioni R_{xB} ed M_D utilizzando le equazioni di equilibrio dei cinematismi
- Es #3 Determinare i momenti centrali principali di inerzia della seguente figura piana.



Soluzione prova scritta 25/1/2017 Pisano/Fuschi

Es # 1

Analisi cinematica



CN: $l = 3N - (\mu_e + \mu_i) = 6 - (4-2) = \emptyset$

CS: $C_I \equiv B$ cerniera ideale

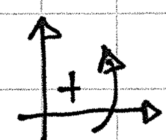
$C_{I\,II} \equiv C$

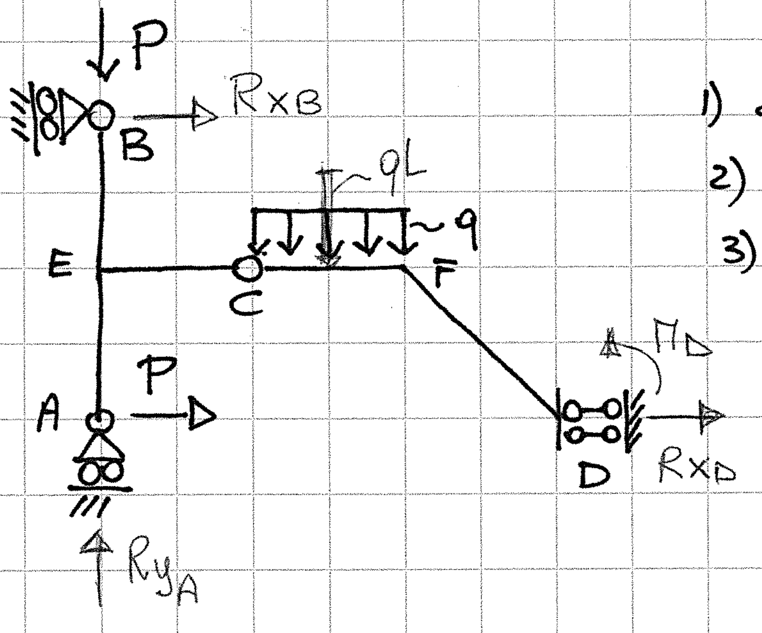
C_{II} = punto ∞ in direzione orizzontale

I tre centri non sono allineati, non è soddisfatto il II teorema delle catene cinematiche

Essendo soddisfatte sia la CN che la CS di isostaticità il sistema analizzato è isostatico!

Calcolo delle reazioni vincolari





1) $\Sigma F_x = 0$ $\quad R_{xB} + \overset{qL}{P} + R_{xD} = 0$

2) $\Sigma F_y = 0$ $\quad R_{y_A} - \overset{qL}{P} - qL = 0$

3) $\Sigma M(B) = 0$ $\quad 2\overset{qL}{P}L - \frac{3}{2}qL^2 + M_D + 2R_{xD}L = 0$

Eq. ausiliarie

4) $\Sigma M(C)^{II} = 0$

$M_D + R_{xD}L - q\frac{L^2}{2} = 0$

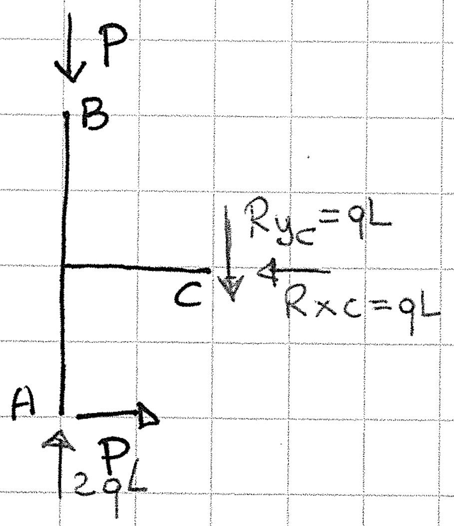
$M_D = q\frac{L^2}{2} - R_{xD}L \Rightarrow \boxed{M_D = \frac{3}{2}qL^2}$

Dalla 2) $\boxed{R_{y_A} = 2qL}$

Sostituisco 4) $\rightarrow$ 3) $\quad 2qL^2 - \frac{3}{2}qL^2 + q\frac{L^2}{2} - R_{xD}L + 2R_{xD}L = 0$ $\quad \boxed{R_{xD} = -qL}$ *

Sostituisco R_{xD} in 1) $\boxed{R_{xB} = 0}$

RV interne

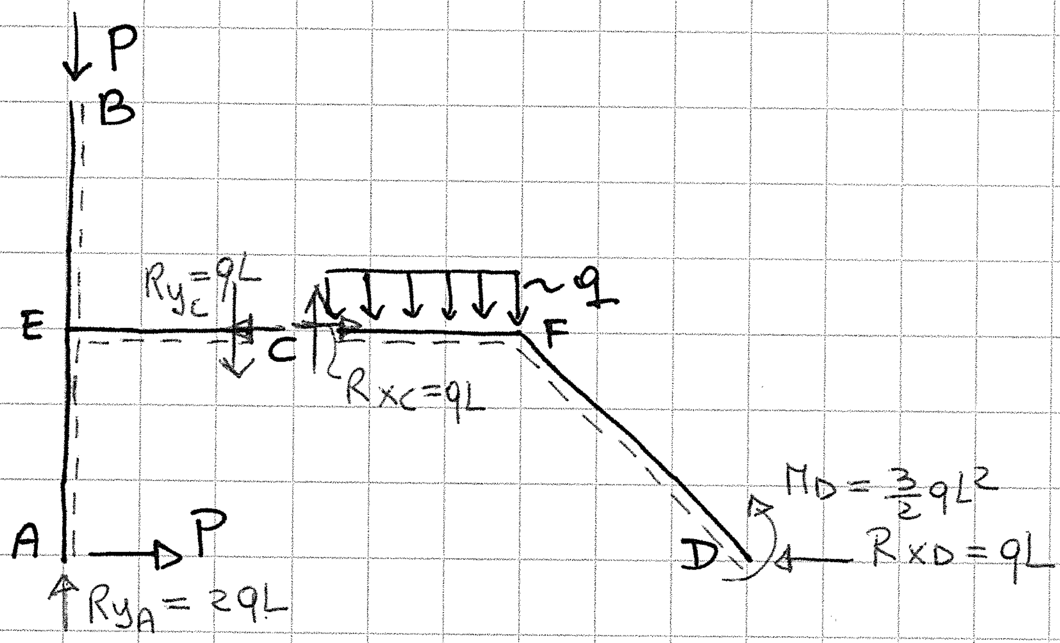


$$Rx_C = qL$$

$$Ry_C = 2qL - qL = qL$$

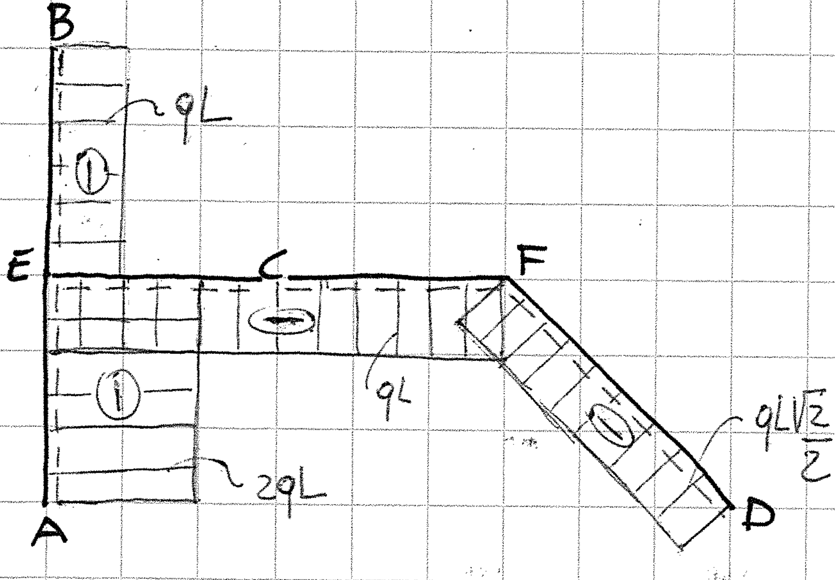
$$\Sigma M(A) = qL^2 - qL^2 = 0 \text{ ok}$$

In definitiva si ha:

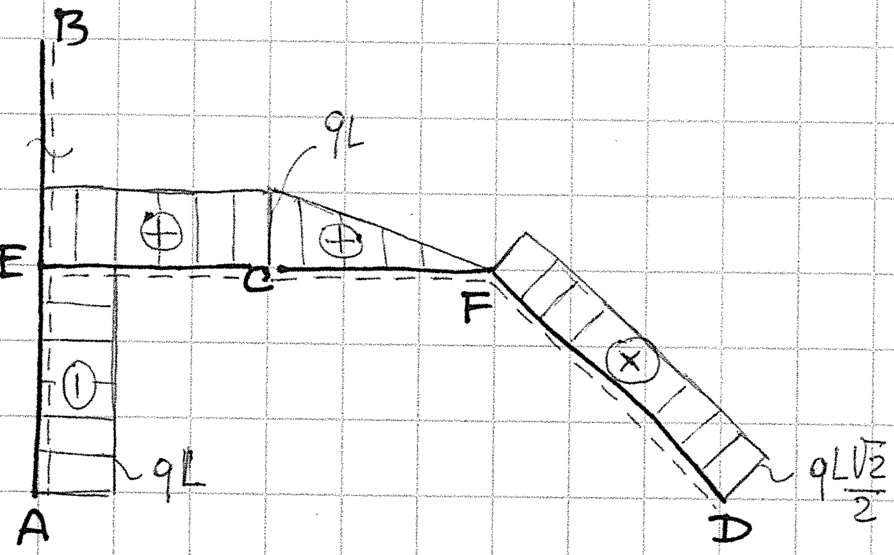


$$\boxed{\text{Diagrammi delle caratteristiche di sollecitazione}}$$

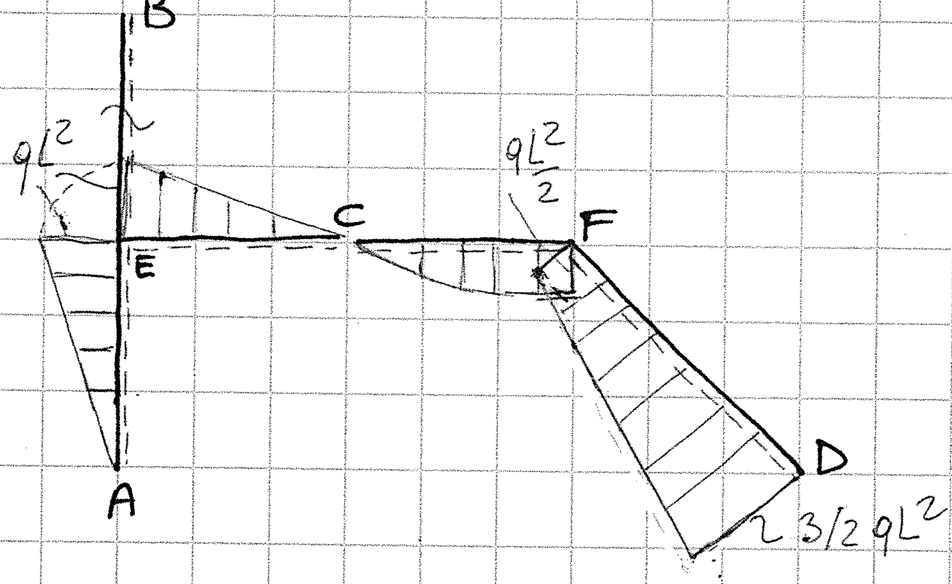
Sforzo normale



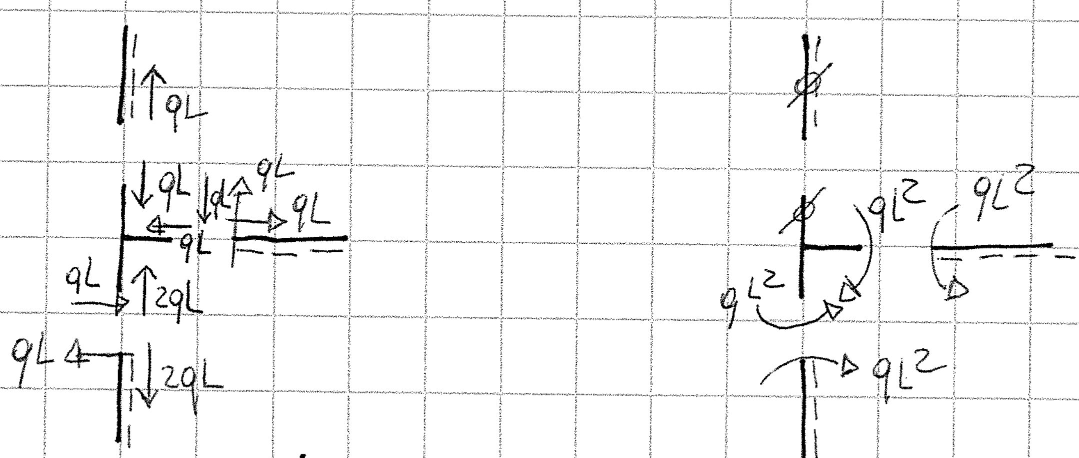
Taglio $\boxed{T}$



Momento flettente $\boxed{M}$



Verifica dell'equilibrio del nodo triplo E



$\Sigma F_x = qL - qL = 0$ OK

$\Sigma F_y = 2qL - qL - qL = 0$

$\Sigma M = qL^2 - qL^2 = 0$ OK

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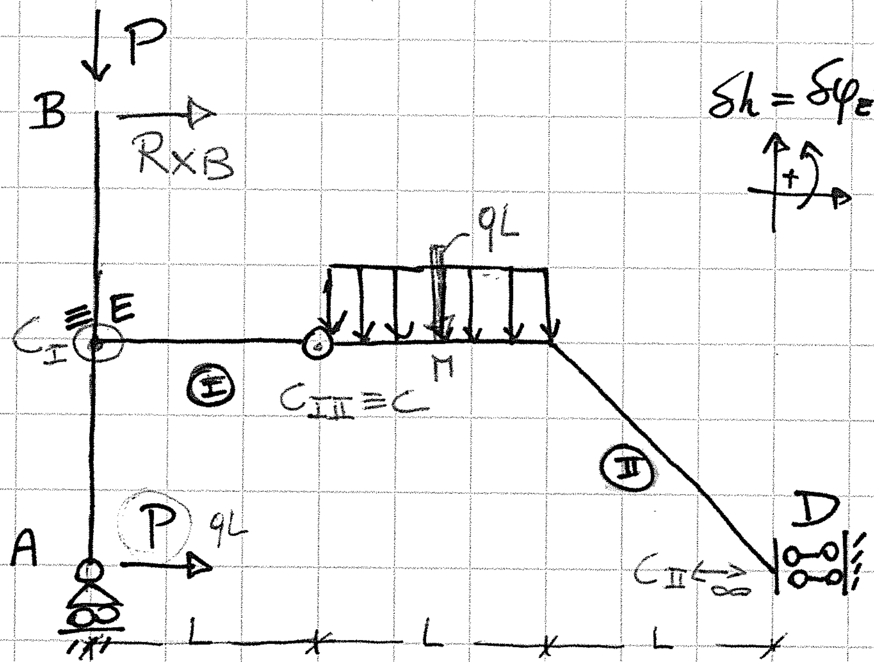
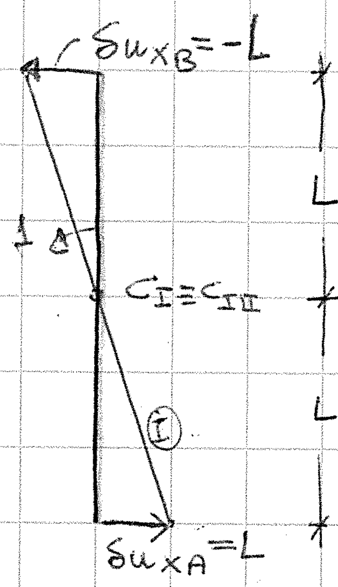
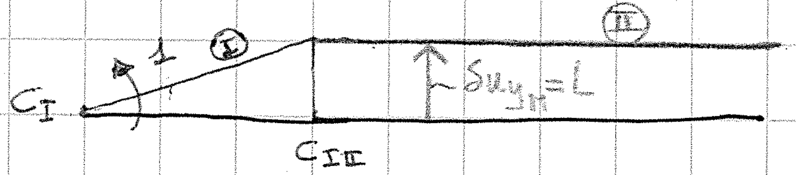
ES #2



$\delta h = \delta\varphi_E = 1$



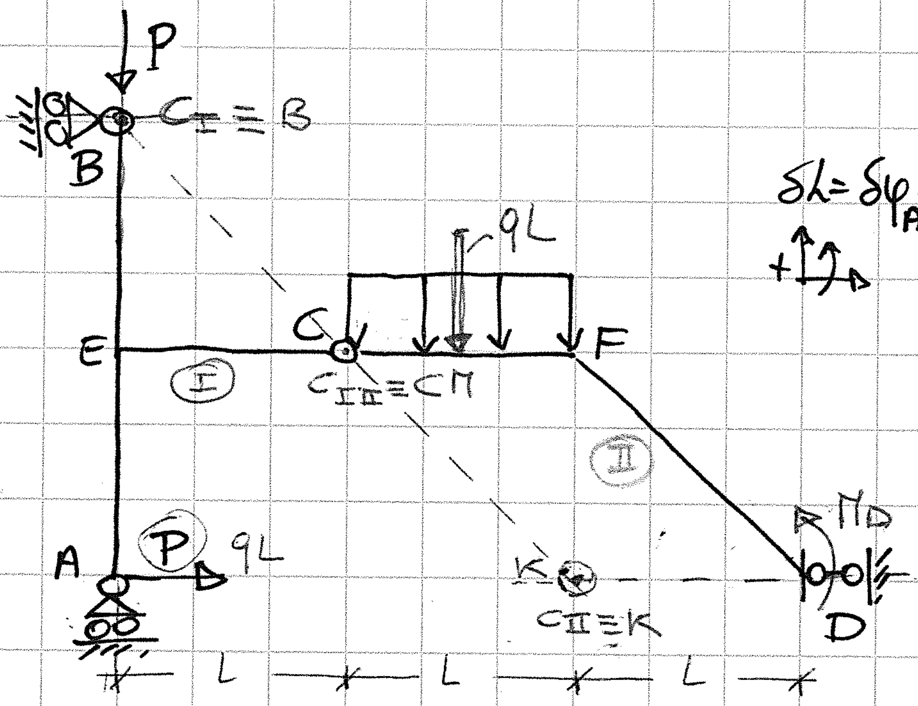


$$\underline{C_c}^T \underline{F} = \underline{0}$$

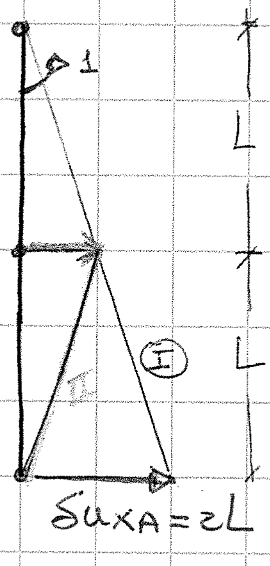
$$qL^2 - R_{xB}L - qL^2 = 0$$

$$\Downarrow$$

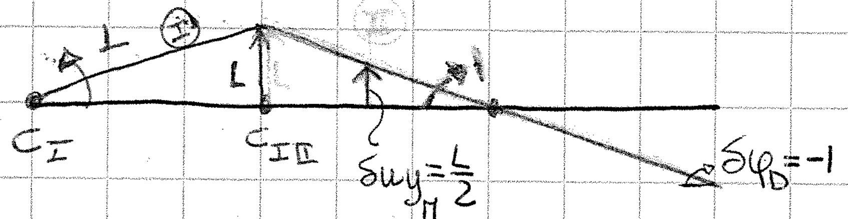
$$\boxed{R_{xB} = 0}$$



$\delta h = \delta\varphi_A = 1$



$$\underline{C_c}^T \underline{F} = \underline{0}$$



$$-q\frac{L^2}{2} + 2q\frac{L^2}{2} - M_D = 0$$

$$\Downarrow$$

$$\boxed{M_D = \frac{3}{2}qL^2}$$

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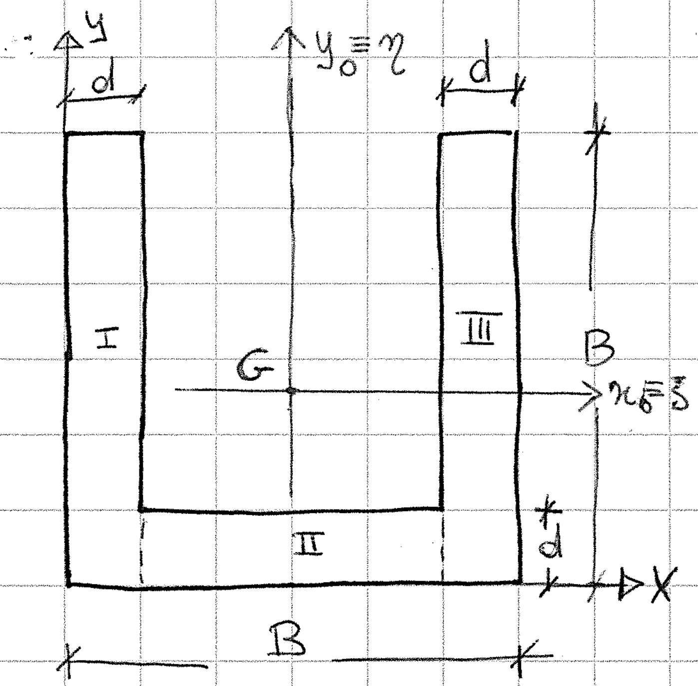
Es # 3



$B = 6\,cm$
$d = 1\,cm$

La figura presenta un asse di simmetria verticale $\Rightarrow x_G = \frac{B}{2} = 3\,cm$

$y_G = \frac{S_x}{A}$

$S_x^{I} = S_x^{III} = d \cdot B \cdot \frac{B}{2} = 18\,cm^3$; $S_x^{II} = (B-2d)\cdot d \cdot \frac{d}{2} = 2\,cm^3$

$A^{I} = A^{III} = d \cdot B = 6\,cm^2$; $A^{II} = (B-2d)\,d = 4\,cm^2$

$$y_G = \frac{2 S_x^{I} + S_x^{II}}{2A^{I} + A^{II}} = 2{,}375\ cm$$

$I_{x_0} \equiv I_\xi = 2 I_{x_0}^{I} + I_{x_0}^{2}$

$I_{x_0}^{I} = \frac{1}{12} B^3 d + A^{I}(y_G - y_G^{I})^2 = \frac{1}{12} 6^3 \cdot 1 + 6 \cdot 1 \cdot (2{,}375 - 3)^2 = 18 + 2{,}344 = 20{,}344\ cm^4$

$I_{x_0}^{II} = \frac{1}{12}(B-2d)d^3 + A^{II}(y_G - y_G^{II})^2 = \frac{1}{12} 4 \cdot 1^3 + 4 \cdot 1 (2{,}375 - 0.5)^2 =$
$= 0{,}3\bar{3} + 14{,}062 = 14{,}396\ cm^4$

$$\boxed{I_\xi = 2 \cdot 20{,}344 + 14{,}396 = 55{,}084\ cm^4}$$

$I_\eta = I_{y_0} = 2 I_{y_0}^{I} + I_{y_0}^{II}$

$I_{y_0}^{I} = \frac{1}{12} B d^3 + A^{I}(x_G - x_G^{I})^2 = \frac{1}{12} 6 \cdot 1^3 + 6 \cdot 1 (3 - 0{,}5)^2 = 0{,}5 + 37{,}5 = 38\ cm^4$

$I_{y_0}^{II} = \frac{1}{12}(B-2d)^3 d = \frac{4^3}{12} \cdot 1 = 5{,}3\bar{3}$

$$\boxed{I_\eta = 2 \cdot 38 + 5{,}33 = 81{,}3\bar{3}\ cm^4}$$